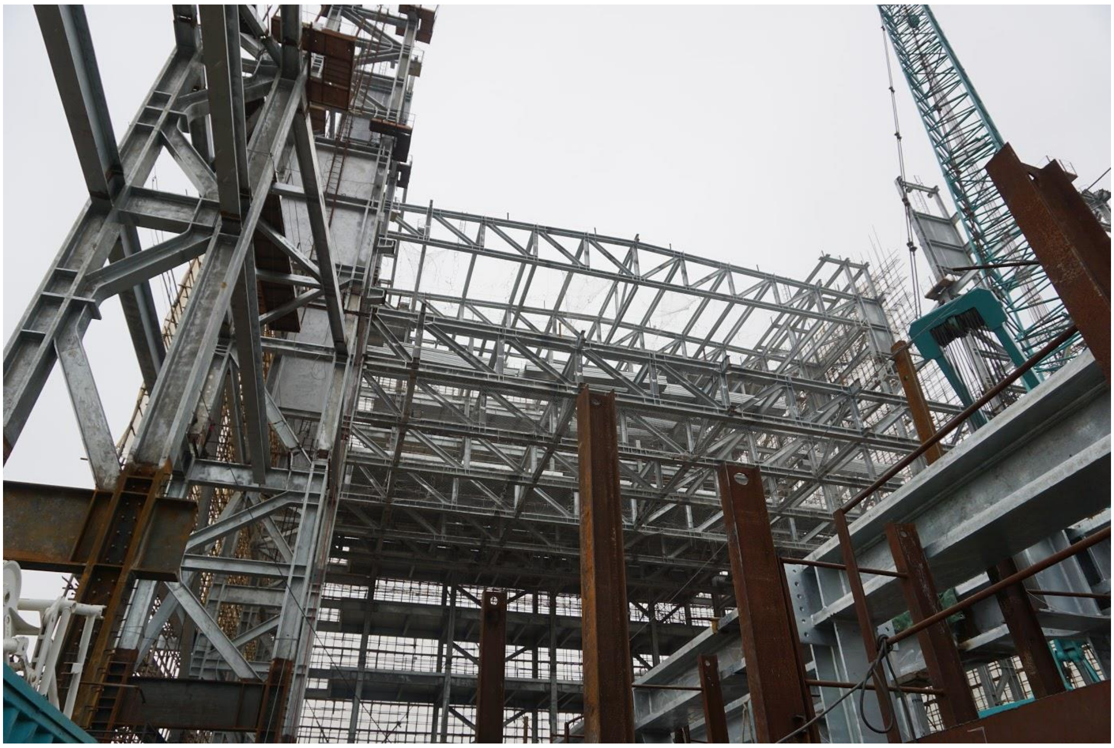

Handbook for Code of practice for structural use of steel 2011

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Preface by Chairman

The evolution of the latest Code of Practice for Structural Use of Steel 2011 follows closely with the latest design practice, the desire for more exact analytical approach and the deeper understanding of the effect on stability, and most importantly, the aim to achieve a safe and cost effective design. Since the issuance of the Code, the practicing engineers are presented with a concept on stability that revolutionizes the approach for the analysis and design of steel structures. Many look for explanatory materials, analytical and design software to help them in their course of design. Being a more sustainable material and able to meet the latest design requirements in high-rise and long span structures, structural steel is rapidly gaining popularity. The pressure is mounting for the practicing engineer in the need to design and to comply with the latest Steel Code.

This handbook is not only a supplement to the Code and the Explanatory Materials. It also provides you with the background of the code requirements, guides you to correlate the design assumptions and the actual structural behavior, and illustrates its application with step-by-step design examples including a detailed narrative design approach and application in second order analysis. This is without doubt a dreamed gift to the practicing engineers.

On behalf of the Structural Division of the Hong Kong Institution of Engineers, I express our most sincere gratitude to the Working Committee led by Ir Professor Chan Siu Lai, and everyone who has contributed in preparing this handbook. The making of this handbook has not come easy, it has taken up the Working Committee a lot of their own time. It is entirely their dedication and effort in sharing their knowledge and knowhow that make this handbook possible. I sincerely wish you can enjoy and benefit from this handbook

Ir Edward CHAN Sai Cheong
Chairman,
Structural Division
Hong Kong Institution of Engineers

April 2018

Forewords

In 2002, a contract was initiated by the Buildings Department of the Hong Kong SAR Government to draft a limit state design code for steel structures used in Hong Kong SAR region. The Hong Kong Polytechnic University and Ove Arup and Partners (Hong Kong) Limited by then jointly formed a joint venture to bid for the project which was awarded in July of the same year. The first author was involved as one of the principal consultants of this project and this book is written with an aim of assisting the users of the Code of which the official name is "Code of Practice for the Structural Use of Steel 2005/2011" published by the Buildings Department of the Hong Kong SAR Government. The code can also be downloaded at web http://www.bd.gov.hk/english/documents/index_crlist.html.

This book is written for use with the Code of Practice for the Structural Uses of Steel Hong Kong 2005 and 2011 versions (The HK Code) which are under the direction of a modern limit state design philosophy, the simulation-based design (SBD) concept which is actually embedded in the second-order direct and advanced analysis referred in many other national codes. SBD makes use of the first and second variation of the energy principle for checking of strength and stability and it encompasses various non-linear analyses but excludes the first-order linear, rigid plastic and elastic P-Δ-only second-order direct analyses. Undoubtedly, this book is not only a design text, it is also written in the hope as a guidebook on the use of second-order direct and advanced analysis to any code with provision of second-order direct analysis. To the authors' knowledge, a comprehensive design guide on the codified use of second-order direct analysis is not yet available. When using the SBD, the simple difference between various codes will be on the use of imperfection factors and notional forces or other means of disturbance. This argument is based on the fact that full second-order direct analysis in all codes are to reflect structural behaviour and SBD as a realistic simulator in second-order effect, practically fit the bill. This feature cannot be established when the codes are prescriptive and the formulae are empirical.

The authors gratefully acknowledge the supports by the Research Grant Council of the Hong Kong SAR Government for drafting of this handbook which has incorporated the valuable comments by the advisory committee of the Joint Structural Division of the Hong Kong Institution of Engineers in year 2014 to 2016 below.

Ir CHAN Chi-kong, Ir Edward CHAN Sai-cheong, Ir Kenneth CHAN, Ir Jacky CHIONG Kam–yueng, Ir Prof KUANG Jun-shang, Ir CHAN Siu-tack, Ir Paul LEE Kai-hung, Ir Paul TSANG Sau-chung, Ir Prof Paul PANG Tat-choi, Ir LAU Chi-kin, Ir Dr Eddie LAM Siu-shu, Ir Jenny Lau, Ir Paul LEE, Ir Prof Andrew LEUNG, Ir LAM King-kong, Ir Benny LAI Siu-lun, Ir Ken NG Kin-shing, Mr. Simon Pang, Ir Thomas WONG, Ir Albert LEUNG Wing-keung, Ir LEUNG Kwok-tung, Ir LUI Yuen-Tat, Ir Johnny SHING, Ir Martin TSOI Wai-tong, Ir Ben TSE Wai-keung, Ir TSE Kam-leung, Ir Dr. Simon WONG and Ir Prof. Ben YOUNG. Special thanks should also go to Dr. Z.H. Zhou for his contributions to the background research of this book.

Cover:
Steel building "Centro Polidesportivo" in Macau.
Architect : Eddie Wong & Associates.
Structural Engineer : Alpha Consulting Limited.
Designed by Second-order Direct Analysis (or simly Direct Analysis) without effective length.

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Chapter 1 Introduction to limit state design

1.1 Background

Code of Practice for the Structural Uses of Steel Hong Kong (abbreviated as HK Code in this book) was published and released by the Buildings Department in replacement of the British Standard BS5950 (1990) used in Hong Kong. This book describes the use of HK Code but the final interpretation should follow clauses in the HK Code rather than in this book.

In modern steel structural design, computer software is commonly used even though we always advocate double-checking by hand as well as analysis method using physical models to study the structural behaviour for checking, scheming and framing. In HK Code and this guidebook, the structural analysis and design software NIDA Version 9 (2015) or above is used, which fullfills the requirements included in the code. For example, the member imperfection in *Table 6.1* of HK Code or *Table 5.1* in Eurocode 3 (2005) can be input explicitly in NIDA Version 9 while such option is not available in most other software available in the market, thus greater caution should be given if such software is to be used.

1.2 Scope of this book

This book describes the design of hot-rolled steel sections and cold-formed steel hollow sections. It covers mainly building structures. Other types of structures and other common structural forms as referenced by other supporting building codes are also admitted in this guidebook to help the readers achieve a more economical and safer design.

Structural elements in a steel structure refer to members designed and constructed to assist the structure in resisting external loads. This book encompasses the design of steel structures against safety and serviceability or the ultimate and serviceability limit state design. This book is aimed for a basic guide for engineers involved in the design and it covers the modern system based design based on second-order direct nonlinear analysis as well as conventional first-order linear analysis and design using the effective length method.

A detailed coverage of all topics in steel structure design is not only impossible in the length of a single book, it also impairs its readibility. Therefore, this book provides some of the most basic information and guidance on structural design of hot-rolled steel sections, cold-formed steel hollow sections and structures. A more in-depth design of various specific and specialist structures may still require the engineers to carry out research and studies on the topic. For example, the design of specialist scaffolding system may involve the assessment and subsequent assumption of joint stiffness of sleeve between scaffolding modules which can be found in other design codes such as BS EN 12810 and relevant research papers. They are not within the scope of this book.

1.3 Aim of structural design

The aim of structural design is to produce a structure of adequate level of safety and serviceability during its design life with a satisfactorily low probability of violating the limit states. The structure should be fit for its intended usage during its design life,which is generally taken as 50 years for “permanent” structures. For temporary structures and more sensitive structures may respectively require a higher and lower probability of failure and a shorter and longer length of design lives. For example, the design life of temporary structures can be much shorter than for permanent structures because the chance of having a wind speed greater than wind over 50 year return period is smaller when a structure is only used for, say, 2 years as temporary structures. Also, the chance of having accidentally large live load is reduced.

As stated in *Clause 1.2.1* of HK Code, the explicit aims of structural design are made as follows.

a) Overall Stability against overturning, sliding or global buckling under the design loads.

b) Strength against collapse under normal loads and imposed deformations and during construction with an acceptable level of safety.

c) Integrity, ductility and robustness against abnormal loads from extreme events without suffering disproportionate collapse, in which alternative load paths may be established.

d) Fire resistance.

e) Serviceability under all normal loads and imposed deformations.

f) Durability.

g) Maintainability during its design working life.

h) Buildability.

i) Economy: The structure should fulfill the above requirements in an economical manner.

1.4 Limit state design

The limit state design (LSD) was first introduced and became widely used around early 80's and it is aimed to make sure the factored resistance greater than factored design load as,

$$\gamma R \geq \gamma_l F \tag{1.1}$$

in which γ and γ_l are respectively the resistance and load factors, R is the resistance of the structure and F is the external load.

It may be useful to make a reference to the older design philosophy. In contrast to LSD, allowable stress design code (ASD) is an old design code which controls stress only and it becomes more difficult to apply to large and slender structures where safety is not controlled solely by stress, but also by stability. As ASD applies the factor of safety to material yield stress such as multiplying the yield stress by a material factor, its control of safety in a structure failed predominantly by buckling becomes complicated and inconsistent. ASD cannot control the variation of loads in a simple manner and it is becoming less used in practice.

There are mainly two limit states, namely the ultimate and the serviceability limit states. Ultimate limit state (ULS) is arrived when a structure fails or becomes incapable of taking the loads. Serviceability limit state (SLS) is a limiting state when the structure is unfit for use by the users of the structure. For obvious economical reason, the engineer does not impose the same margin of arriving at a particular limit state and this margin or factor of safety depends on the consequence of reaching the limit state. As the consequence for ultimate limit state, which implies structural failure, the load factors as a means of controlling the safety margin are normally larger than the factors for serviceability limit state, with the exception that a smaller load factor is on the favourable side such as overturning. *Table 2.1* of HK Code reproduced below shows various limit states under these two principal categories. The use of factors of safety as load and material factors is to account for the variation in different aspects of structural deficiency such as,

Load and material properties variation
Fabrication and erection minor errors in shop and on site
Connection detailing
Design and analysis assumptions and
Rolling and fabrication tolerance

Ultimate limit states (ULS)	Serviceability limit states (SLS)
Strength (including general yielding, rupture, buckling and forming a mechanism)	Deflection
Stability against overturning and sway stability	Vibration
Fire resistance	Wind induced oscillation
Brittle fracture and fracture caused by fatigue	Durability

Table 1.1 Limit states

1.4.1 Ultimate limit state

As its name implies, ultimate limit state (ULS) refers to the ultimate strength and stability of a structure against failure and thus it adopts a larger factor of safety through the load factor in the design. Recognizing that loading and material properties are probabilistic based, the design ensures a smaller probability of violation of the limit state through the use of larger load factors. *Table 4.4* of HK Code indicates various values of partial load factors used.

1.4.2 Serviceability limit state

A structure becomes unfit for use when one or more limit state is violated. The common serviceability limit state includes the deflection and deformation, vibration, repairable damage due to fatigue and corrosion and durability not leading to immediate collapse.

1.4.2.1 Deflection limit state

Deformation is commonly considered as an intolerable serviceability limit state. It affects the cracking of finishes, makes occupants uncomfortable and it is also used as a means of preventing vibration. Normally unfactored live and wind loads are used for the calculation of deflection. Typical and suggested deflection limits are given in *Table 5.1* of HK Code. Deflection limits of tall buildings are more related to the comfort of occupants and the following section shows the acceleration limits for tall buildings.

1.4.2.2 Vibration limit state

Excessive vibration leads to human discomfort. Worst of all, resonance leads to a structural response in phase with exciting disturbance such as wind or machine vibration for which the consideration should be under the ultimate limit state. In HK Code, the table under *Section 5.3.4(b)* gives recommended limiting peak acceleration in a high-rise building. Alternatively, the present HK Code provides simple frequency check whilst the ISO (2003) gives the acceleration limit as a function of the structural natural frequency which is a more complicated means of assessing human response due to building vibration.

1.4.2.3 Human-induced vibration

Long span floors and beams may be susceptible to human induced vibration. A conservative prevention of the occurrence is to design a beam to have a natural frequency greater than 5 Hertz. The newer version of the HK Code lowers this requirement to 3 Hertz. For more detailed study of beam vibration, ISO (2003) or other guides should be referred.

1.4.2.4 Corrosion and durability

Other serviceability limit states include fatigue, corrosion and durability. Steel will rust and corrode only in the presence of oxygen and water and therefore steel burry one meter below ground normally has no problem in corrosion because of lack of oxygen. When under bad environmental condition such as chlorides near sea and sulphide in industrial area, corrosion is more serious. Careful detailing prevents corrosion in many occasions such as prevention of ponding and debris trap.

To prevent corrosion leading to early structural defects, the expected design life is estimated and coating, painting, galvanizing, cathodic protection, coverage by concrete or use of thicker section plate thickness can be considered. In general, painting and application of protection measures are best to be done in shop rather than on-site. However, this may not be possible for some applications such as protection of corrosion around region of site weld. In assessing the degree of protection, the environmental exposure condition and the ease of maintenance are required to be considered. *Table 5.2* and *Clause 5.5.1.2* of the HK Code provide basic consideration of these issues. Monitoring is sometimes important in confirming the assumption of durability in steel members.

When metal is subject to repeated load, fatigue failure may occur. Design methods for fatigue are based on the S-N curves such as the one indicated in *Figure 2.1* of the HK Code.

Failure due to low cycle repeated loads of 10 to 100 cycles happens occasionally in some structures like cranes and scaffolds. Inspection and scrapping of old structures or their components may be needed as a management process for prevention of unexpected failure.

1.4.2.5 Brittle fracture

Brittle fracture for steel may become important under the action of low temperature, applied tension, thick steel plates and sudden change in stress and resistance against brittle fracture can also be enhanced by proper detailings. The steel quality in this aspect is controlled by its sub-grade governing impact resistance under certain tempeartures. Commonly, grade J0 is used for thin plates and higher sub-grades need to be used for thicker plates. For prevention of laminar tearing, higher Z-grade steels need to be used. The HK Code and specialist literatures should be consulted for design of steel structures made of thick steels which are becoming more popular at the time of writing this handbook. The maximum thickness formulae and tables under *Clause 3.2* in the HK Code can be referenced in selecting the maximun steel thickness.

1.5 Load and resistance factors

In the limit state design, loads are commonly amplified to account for load variation and as a factor of safety. Load combination will be applied to cater for various scenarios. The followings are common combined load cases for structural design and Table 1.2 shows the load factors.

Load combination 1: Dead load, imposed load (and notional horizontal forces)
Load combination 2: Dead load and lateral load
Load combination 3: Dead load, imposed load and lateral load

Load combination (including earth, water and temperature loading where present)	Load Type						
	Dead G_k		**Imposed** Q_k		**Earth and water** S_n	**Wind** W_k	**Temperature** T_k
	Adverse	Beneficial	Adverse	Beneficial			
1. dead and imposed	1.4	1.0	1.6	0	1.4	-	1.2
2. dead and lateral	1.4	1.0	-	-	1.4	1.4	1.2
3. dead, lateral and imposed	1.2	1.0	1.2	0	1.2	1.2	1.2

Table 1.2 Load factors for different load combinations

In the Table, the adverse and beneficial effects refer to a condition where loads are exacerbating and assisting a structure against failure, such as vertical load at the centre of a building will be beneficial against overturning.

1.6 Structural integrity and robustness

A new requirement is stipulated in the new codes like the Eurocode 3 (2005) and the HK Code (2011). The implementation of the clauses here requires engineering judgment and design experience. In essence, a structure should not have progressive collapse when a single member fails. This can be done by provision of ties for general and especially edge columns. Also connections should be designed to take tensile force such that the failure of a lower column will be compensated by the column above when the connection is able to take tension. To achieve this, *Clause 2.3.4.3* of HK Code should be referred.

1.7 Progressive and disproportionate collapse

Progressive collapse refers to failure leading to a sequence of element collapse and disproportionate collapse is defined as collapse to an extent disproportionated to the cause. In general, the checking should ensure local failure will not lead to global collapse. A steel and steel-concrete composite structure or any structure should be designed to avoid this occurrence. The checking should only be conducted using the second-order direct analysis specified in *Clauses 6.8* and *6.9* of the HK Code (2011) using an authority approved software because of the important consequence of this type of failue. The load factors could be taken as those recommended in other codes below. The global collapse can be considered as failure of an area more than 15% of the floor area or 70 m^2 (whichever is less).

0.35 for dead load and 0.4 for live load with 1% of total loads as horizontal notional force. Wind load is not required to be considered.

Tying members should be able to resist 75kN or 1% of the factored vertical dead and imposed loads of the columns being tied in order to prevent the columns being separated from the building or structure.

Chapter 2 Steel as Engineering Material

2.1 Materials

What is steel? Steel is iron added with carbon with content close to 0, corresponding to very slight amount to 2%. Carbon content has a significant influence on the characteristics of the metal.

There are two major types of steel as alloy steels and non-alloy steels. Alloy steel refers to chemical elements other than carbon added to the iron in accordance with a minimum variable content for each. For example: 0.50% for silicon, 0.08% for molybdenum, 10.5% for chrome. An alloy of 17% chrome and 8% nickel is used to create stainless steel.

For iron or what we normally call low-carbon steel to-date, the carbon content is less than 0.1%. For steel this content is between 0.1% and 2% and between 2.5% and 6% for cast iron.

Material constitutes a very important component of a steel structure. The HK Code covers the control of steel material up to 460 N/mm^2, with use of higher grade steel based on a performance-based approach. Because Hong Kong is an international city, it accepts steel from various countries of greater population size and reputation in making quality steel. These countries are Australia, China, Japan, Europe including Britain and USA. Steel is commonly of type carbon or carbon-manganese steel (mild steel), high strength low alloy steel and high strength, quenched and tempered alloy. High strength, quenched and self-tempered alloy steel is not commonly used and will not be further elaborated. Shown in Figure 2.1 is the common stress vs strain curve of mild steel.

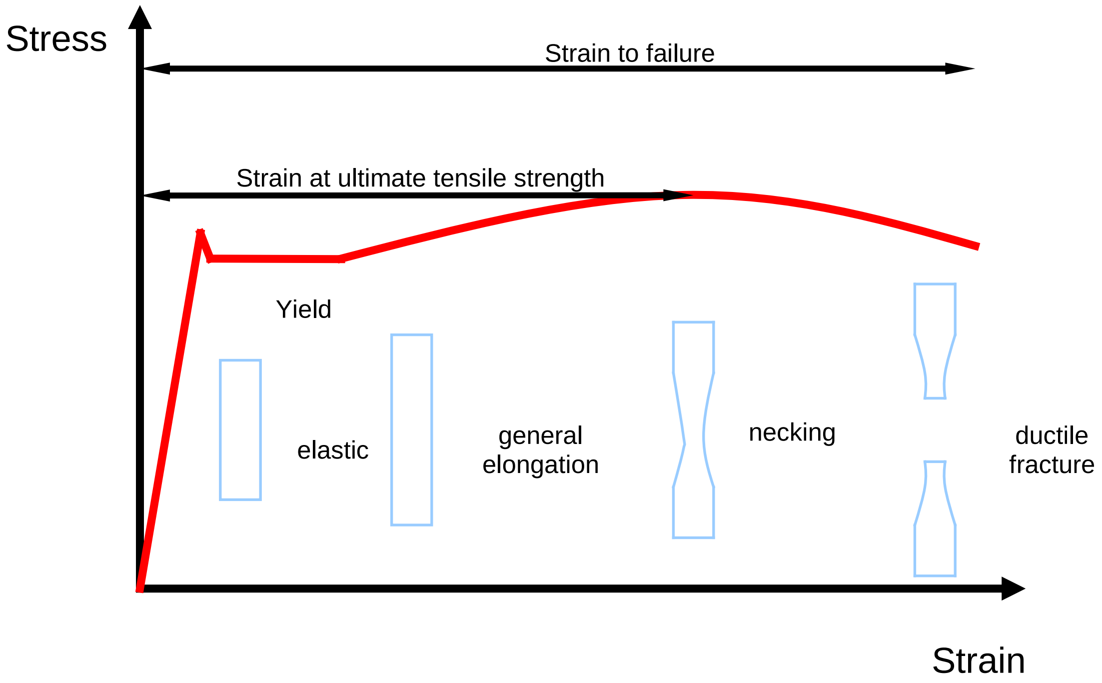


Figure 2.1 Stress vs. Strain Relationship for ductile steel

Irrespective of the grade of steel, the Young's modulus of elasticity, Poisson ration and coefficient of thermal expansion for all steel grades are the same as follows.

E	$= 205$ kN/mm^2	(Young's modulus)
ν	$= 0.30$	(Poisson's ratio)
α	$= 14 \times 10^{-6}$ /°C	(Coefficient of linear thermal expansion)

In accepting or rejecting the use of steel material, the mill certificate is referred and various contents of chemical are inspected. Many elements must be controlled below a certain percentage otherwise one or more properties in strength, weldability, durability or ductility is not warranted.

2.2 Grades of steel

In general, we have the following common grades of steel. The design strength is normally taken as the strength for the steel plate of thickness 16mm.

Low carbon or carbon-manganese steel (mild steel) like S275 of yield 275 N/mm^2
High strength low alloy steels like S355 steel of yield 355 N/mm^2
High strength, quenched and self-tempered alloy steel of yield 500 N/mm^2
High strength, quenched and tempered alloy plates of yield 690 N/mm^2
Alloy bars for tension only of yield 1000 N/mm^2
High carbon hard-drawn wire for cables of yield 1700 N/mm^2

Only the first two types of steel (i.e. Low carbon and High strength low alloy steels of yield between 275 to 355 N/mm^2) are commonly used because other steel types are brittle, contain too high the carbon content and difficult to weld. These high strength steels are more commonly used in some applications like bolts.

2.3 Designation system

In HK Code, steel grades from 5 countries are allowed to use but only the European and the Chinese grade steels are tabulated on their resistance when used in beams and columns. The commonly used grades like grade 43A and grade 50C are replaced by S275 and S355J0 steel. Below is the summary of the symbol meaning.

Taking S355J0 as an example in the new system, the symbols (S in S355J0 here) in front of the steel grade are represented by S for structural steel and E for engineering steel. The following number (355) refers to the minimum yield strength in N/mm^2 at steel plate thickness equal to 16mm. The next following letters refer to the impact value as JR, J0 and J2 are respectively the longitudinal Charpy V-notch impacts at 27 J and at 20°C, 0°C, -20°C temperature while K2 refers to impact value of 40J at -20^{0}C. For some special steels like thick steel plates under stress in transverse direction, additional property in the perpendicular direction to the surface is required and this is specified as Z grade like Z25. The following table represents some of the common conversions between the old and new system steels.

New grade	Yield (N/mm^2)	Tensile (N/mm^2)	Charpy V-notch in longitudinal direction		Old grade
			Temperature(^{0}C)	Energy (J)	
S185	185	290/510	/	/	/
S235	235	360/510	/	/	40A
S235JR			20	27	40B
S235J0			0	27	40C
S235J2			-20	27	40D
S275	275	410/560	/	/	43A
S275JR			20	27	43B
S275J0			0	27	43C
S275J2			-20	27	43D
S355	355	470/630	/	/	50A
S355JR			20	27	50B
S355J0			0	27	50C
S355J2			-20	27	50D
S355K2			-20	40	50DD
E360	360	650/830	/	/	/

Note: The strength and energy are referred to steel plate of 16mm thickness.

Table 2.1 Comparison between the new and the old grading systems for steel

2.4 Residual stress

During a rolling process at 2,300°F, the steel section is rolled to a sectional shape and during cooling, the heat dissipates but at a different rates making the section to contain a residual stress. The fibre such as those in flanges cools faster will be in compression when other parts cool slower and exert a contracting tensile force on the cooled fibre. The residual stresses in a section are in a self-equilibrium state. As the stress depends on E which is the same for all steel grades, the residual stress affects lower grade steel than high grade steel. Also, as residual stress makes the steel material to yield earlier, buckling of columns and beams is more affected by residual stress and this explains why welded columns are weaker than rolled columns which have a smaller residual stress. Generally speaking, the thicker a section, the larger its residual stress and its pattern for rolled and welded I-sections is simplified as follows.

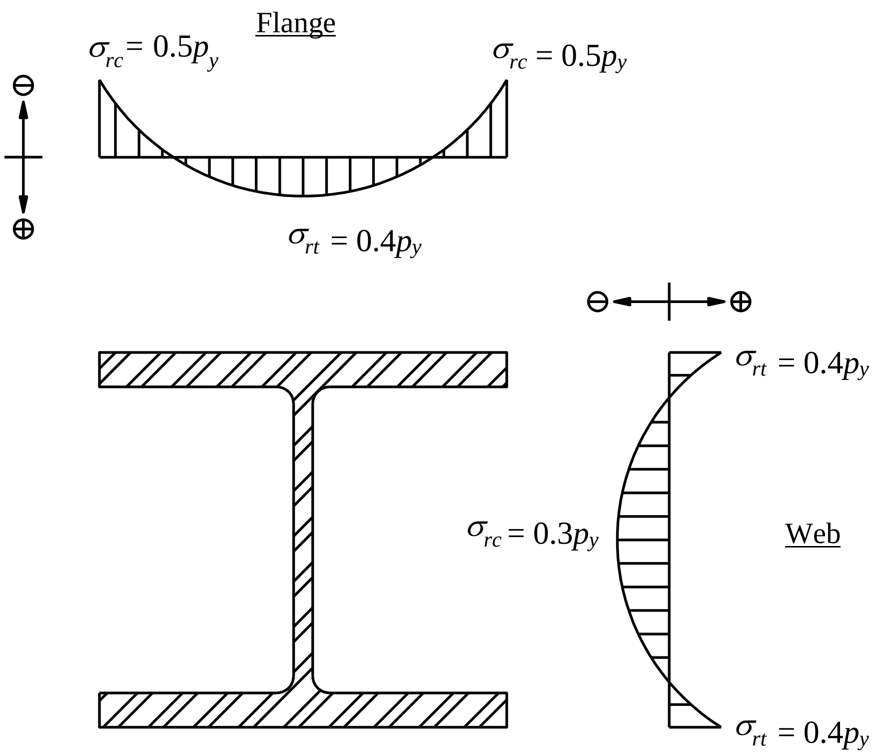


Figure 2.2 Residual stress in a rolled I-section

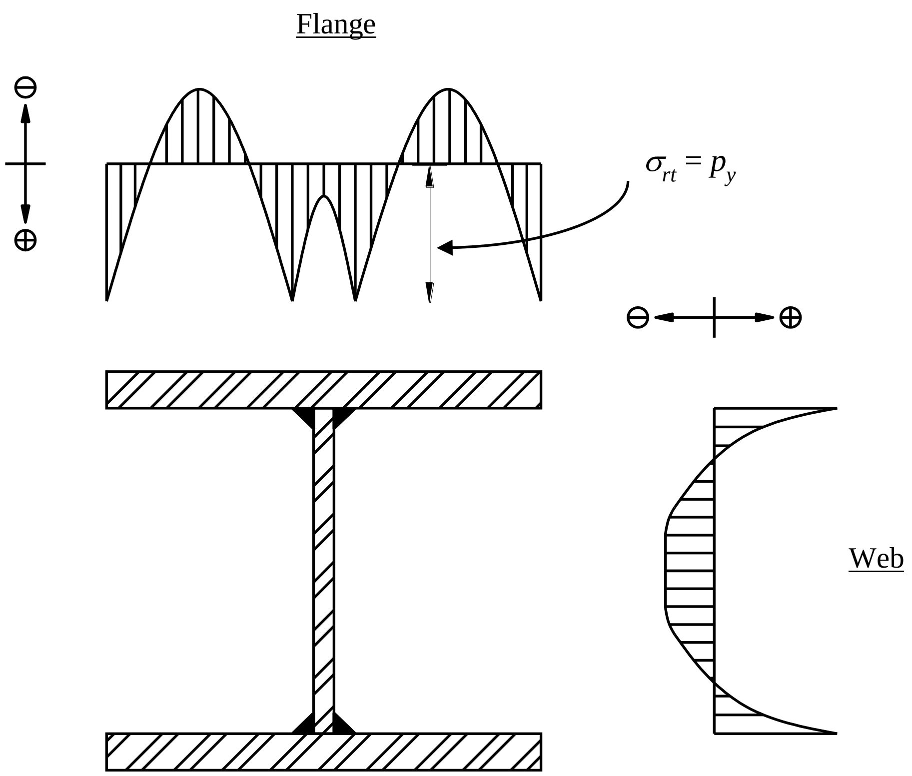


Figure 2.3 Residual stresses in a welded I-section

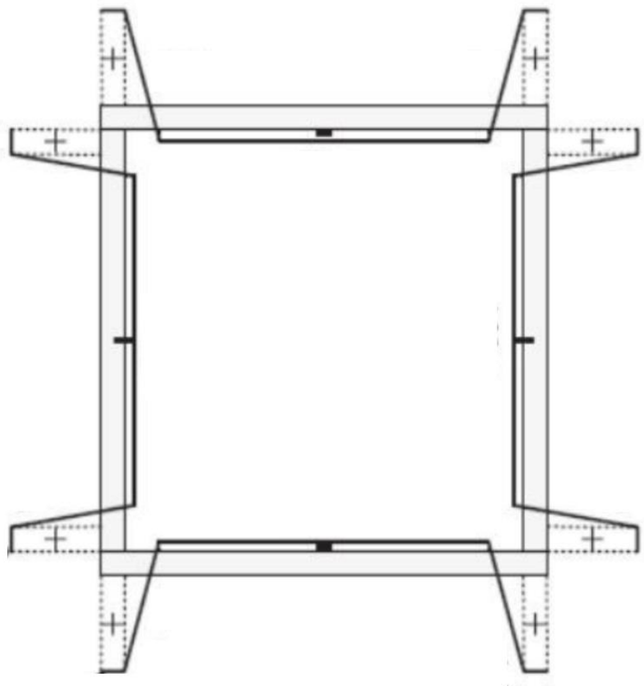


Figure 2.4 Idealised residual stress for box sections

Residual stresses for some sections have been idealized for practical design. It is now possible to make use of residual sections across a section and geometrical imperfection commonly taken as 0.1% of a member length to obtain the buckling resistance or buckling curves of any section (see Li, Liu and Chan, 2015).

Rolling creates residual stress but local welding also generates residual stress, which can be a problem in welding of thick sections or flame-cutting of a section. The pattern of residual stress in a welded section is indicated in Figure 2.3. Pre-heating or heating in the region after welding in order to allow the zone to cool more uniformly will reduce the residual stress. This process is necessary for welding of thick sections.

Cold straightening is a process of meeting the straightness requirement in codes, but it will induce a residual stress in the section and also changing the grain size of the section, making it to have a higher strength but lower ductility. This explains why corners in a hollow section normally have higher strength and lower ductility. Welding should be avoided in the area when cooling work took place.

2.5 Chemistry of steel

Carbon (or carbon-manganese) steel is normally referred to as mild steel. Its composition is iron, carbon, manganese with restricted amount on phosphorus and sulphur and their excess of which are detrimental to weldability and/or durability of steel. Increasing the content of carbon will improve the yield strength, but will decrease the weldability and ductility. S275 belongs to this category of steel.

High strength low alloy steel was developed over the past 3 decades and it is the most widely used steel grade. The strength of this steel material is increased by lowering carbon but increasing other alloys contents so that the toughness, ductility and strength can be improved. S355 steel belongs to this category of steel

High strength alloy steel quenched and tempered alloy steel is the commonly used steel with highest strength. It is commonly available in the form of plates and the high strength property is achieved by a combined lower carbon content replaced by alloys and a quenching (rapid cooling) process. The steel is of very fine grain size and very hard and therefore they are very suitable for making bolts and nuts where hardness is very important in making rigid connection at the teeth and notch of the threaded area of bolts and nuts. Tempering and re-heating improve the ductility and other performance of steel. The steel material is very good for fabrication and welding.

In control of weldability of steel in HK Code, the content of chemicals, carbon, sulphur and phosphorus are limited. Carbon equivalent value given in Equation (2.1) below should be satisfied and the carbon content should not be greater than 0.24%, the sulphur and the phosphorus content should not be greater than 0.035 individually.

Norme Standard Norm	Nuances Grades Güten	C max. % Epaisseur nominale (mm) Nominal thickness (mm) Nenndicke (mm)			Mn max. %	Si max. %	P max. %	S max. %	N max. %	Al[1] min. %	Nb max. %	V max. %	CEV[1] max. % Epaisseur nominale (mm) Nominal thickness (mm) Nenndicke (mm)			
		≤16	>16 ≤40	>40									≤16	>16 ≤40	>40 ≤63	>63 ≤150
EN 10025 : 1993[D]	S 235 JRG2	0.17	0.17	0.20	1.4	-	0.045	0.045	0.009[E]				0.35	0.35	0.38	0.38
	S 235 J0	0.17	0.17	0.17	1.4	-	0.040	0.040	0.009[E]				0.35	0.35	0.38	0.38
	S 275 JR	0.21	0.21	0.22	1.5	-	0.045	0.045	0.009[E]				0.40	0.40	0.42	0.42
	S 275 J0	0.18	0.18	0.18	1.5	-	0.040	0.040	0.009[E]							
	S 355 JR	0.24	0.24	0.24	1.6	0.55	0.045	0.045	0.009[E]				0.45	0.45	0.47	0.47
	S 355 J0	0.20	0.20[D]	0.22	1.6	0.55	0.040	0.040	0.009[E]				0.45	0.45	0.47	0.47
	S 355 J2G3/G4	0.20	0.20[D]	0.22	1.6	0.55	0.035	0.035	-				0.45	0.45	0.47	0.47
	S 355 K2G3/G4	0.20	0.20[D]	0.22	1.6	0.55	0.035	0.035	-				0.45	0.45	0.47	0.47
EN 10113-3: 1993[D]	S 355 M	0.16	0.16	0.16	1.6	0.50	0.035	0.030	0.015	0.02	0.05	0.10	0.39	0.39	0.40	0.45
	S 355 ML	0.16	0.16	0.16	1.6	0.50	0.030	0.025	0.015	0.02	0.05	0.10	0.39	0.39	0.40	0.45
	S 420 M	0.18	0.18	0.18	1.7	0.50	0.035	0.030	0.020	0.02	0.05	0.12	0.43	0.45	-	-
	S 420 ML	0.18	0.18	0.18	1.7	0.50	0.030	0.025	0.020	0.02	0.05	0.12	0.43	0.45	-	-
	S 460 M	0.18	0.18	0.18	1.7	0.60	0.035	0.030	0.025	0.02	0.05	0.12	0.45	0.46	-	-
	S 460 ML	0.18	0.18	0.18	1.7	0.60	0.030	0.025	0.025	0.02	0.05	0.12	0.45	0.46	-	-

Table 2.2 Chemical content requirements in HK Code

2.6 Strength

The design strength shall be the minimum yield strength but not greater than the ultimate tensile strength divided by 1.2. Steel grade number normally refers to the approximate or nominal design strength and the alphabet refers to the resistance against impact Charpy test. Thicker plates normally need a higher resistance against impact Charpy test.

2.7 Resistance to brittle fracture

The minimum average Charpy V-notch impact test energy at the required design temperature is specified in *Clause 3.2* of HK Code. When thick steel is used or when it is used in cold weather, the Charpy test will check whether or not the steel material will exhibit brittle fracture. For example, in *Table 3.7* of HK Steel Code, the maximum thickness is specified as the steel material which can pass a Charpy test of 27J at a specified temperature.

2.8 Ductility

The elongation on a gauge length of $5.65\sqrt{S_0}$ is not to be less than 15% where S_0 is the cross sectional area of the section. Steel of low elongation cannot be used because of lack of ductility prohibiting stress re-distribution. For example, stress around an opening has a high stress concentration that steel material needs to sufficiently ductile.

2.9 Weldability

Carbon increases the yield strength of steel, but reduces its weldability. In HK Code, the carbon equivalent value (CEV) should not be greater than 0.48% and the carbon content should not exceed 0.24%. The carbon equivalent value can be calculated as follows.

$$\mathrm{CEV} = \mathrm{C} + \frac{\mathrm{Mn}}{6} + \frac{\mathrm{Cr} + \mathrm{Mo} + \mathrm{V}}{5} + \frac{\mathrm{Ni} + \mathrm{Cu}}{15} \quad (2.1)$$

The design strength p_y of steel is not constant even for the same grade of steel. The thicker steel contains lower design strength because of residual stress which is present when the materials in different locations of a steel section cool at a different rate resulting in the building up of residual stress. For welded columns with design strength below 460N/mm^2, we need to reduce the design strength by 20N/mm^2 because of greater residual stress. This reduction should further be increased to 30N/mm^2 for higher steel grade. Web has greater design strength than flanges that testing of steel strength may be taken from flange rather than from web for more critical test. Table 2.3 below adopted from *Clause 3.1.2* of HK Code shows the design strength for steel specified in the British system.

	Thickness less than or equal to (mm)	Design strength p_y (N/mm^2)
S235	16	235
	40	225
	63	215
	80	215
	100	215
	150	205
S275	16	275
	40	265
	63	255
	80	245
	100	235
	150	225
S355	16	355
	40	345
	63	335
	80	325
	100	315
	150	295
S460	16	460
	40	440
	63	430
	80	410
	100	400

Table 2.3 Design strength p_y of steel material

Design strength of steel grades from countries of China, Japan, Australia and USA should be referred to HK Code.

2.10 Used steels

For sustainability construction, used steel is normally allowed for temporary structures such as scaffolds, hoarding and excavation and lateral support works. However, the traceability of the mill certificates and structural conditions of these used steel sections have to be strictly enforced so as to ensure structural integrity and safety. Re-used temporary structures attract a higher collapse rate than new ones and care should be taken. In Hong Kong practice, certified copies of mill certificates endorsed by the respective registered contractors shall be properly kept on site for audit checking by the engineers. The structural conditions of the used structural steel sections shall also be assessed in accordance with acceptable standard like BSEN 10034 should the original design stresses of the steel sections be adopted in design analysis.

When the above conditions are not saisified, the allowable design stress of the used steel members shall follow the un-certified steel requirement (i.e. 170 MPa) given in the Clause 3.1.4 of the HK Code. Sectional dimensions should also be measured and determined to ensure plate thicknesses and other dimensions are adequate and not reduced by corrosion.

Chapter 3 Framing and Load Path

3.1 Introduction

Structures are erected to protect and support people, equipment etc like buildings and to allow transportation like bridges. Different framing systems are derived to achieve these aims under the consideration of economy, safety, speed of construction and environment. The principle of designing a structure is to carry load from gravity or from wind or seismic motion safely to the foundations. Failures due to buckling, overturning type of instability, fracture and yielding should be avoided with additional use of load and material factors to account for unexpected event and variation in loads and material properties.

3.2 Common types of steel frames

For steel structures, engineers normally adopt the following frame systems.

Braced frames
Frames with shear or core wall
Moment frames like portal frames
Shell structures
Long span trusses systems and
Tension systems

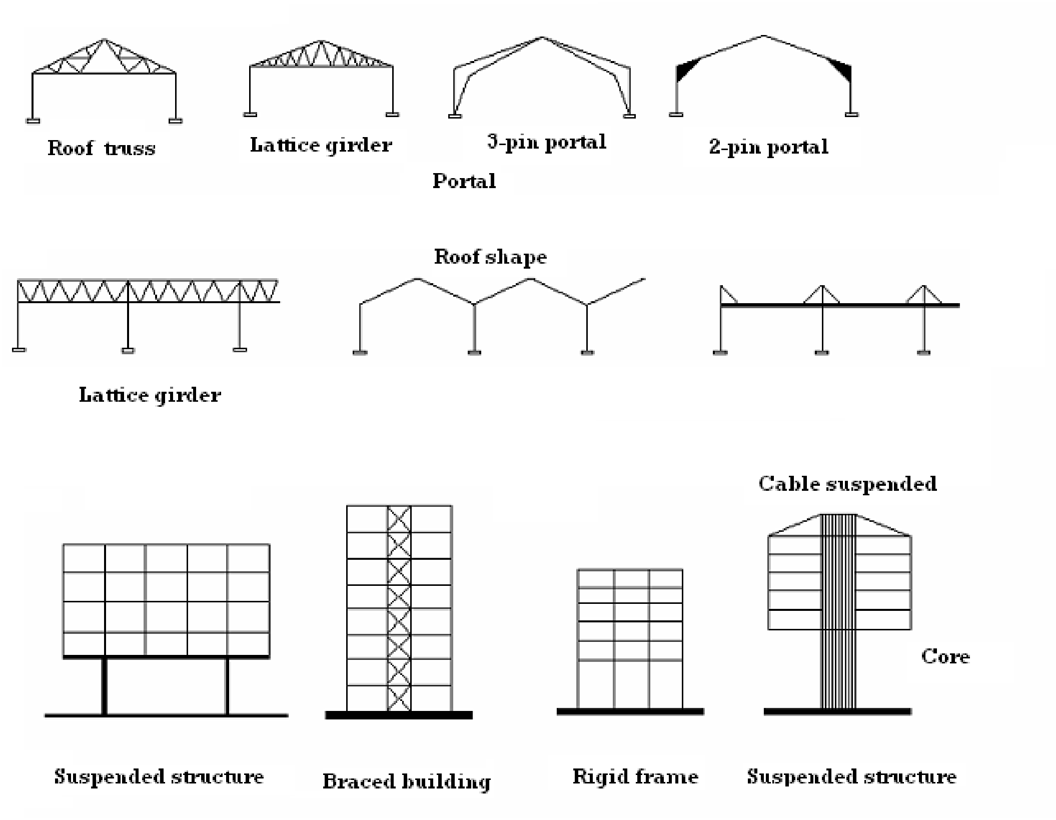


Figure 3.1 Typical structural schemes

Depending heavily on the site condition and purpose of use, these systems have their advantages and limitations. In essence, we need to have a stiffer and high strength structural system to resist large forces, such as braces and shear walls to resist wind loads and columns to resist large gravitational force from the weight of the structure. The load paths should be clearly defined so that we visualize how loads are transferred from slabs, beams to columns and foundations.

3.3 Typical lateral force resisting systems

A structure can be designed and constructed by using different lateral force resisting systems. The connection design should follow the design assumption such as one should design a connection to resist moment if a rigid moment joint assumption is made in a frame design. On the other hand, the connection should be designed to resist shear and direct force only if the connection is assumed as pinned. In this case, the connection should further be designed to allow rotation with minimum moment resistance.

3.3.1 Simple construction

The concept of "simple construction" is to design the structure to be composed of members connected by nominally pinned connections and the lateral forces are then taken by other structural systems like bracings, shear walls and core walls. The joints should be assumed not to take the moments in the design and sufficient ductility is allowed. For example, we should use angle cleat bolted or fin plate connections at webs to prevent the connection taking too much moment. The lateral force is taken by a structurally independent system such as bracing system and shear wall so that the frame is required to take vertical loads only.

3.3.2 Continuous construction

In continuous construction, the frame is to resist lateral force by moment joints. The vertical and horizontal forces and moments are transferred between members by moment connections. The disadvantages of this method include high connection cost and larger member size. Very often, the lateral drift or deflection quite easily exceeds the deflection limit or the frame is prone to sway. However, it does not need an independent lateral force resisting system and thus save space and cost of constructing these systems.

3.3.3 Braced frames

A steel frame can be stiffened laterally by addition of braces which resist the loads by an efficient axial force system. This type of frames is normally lighter than the continuous construction using the moment frames, but the frames require braces which are not welcomed by occupants. Therefore, in many commercial and domestic buildings, moment frames are preferred. For high framed structures beyond approximately 10-storey high, the use of moment frames will become too expensive with the very large member sizes and braced frames or frames with other lateral force resisting systems like simple construction with shear walls are more commonly used. The bracing can be replaced by other lateral stiffening systems like shear walls, core walls and outriggers.

Figure 3.2 A braced frame

Either rigid or pinned joints can be assumed in braced frames and this affects the moment and force distribution. For moment connections, the joints should have sufficient rotational stiffness and moment capacity to transfer bending moment. For pinned connections, the joints should be ductile and detailed to avoid taking of moment. Rotational capacity of joints becomes more important here.

3.4 Load sharing

An important process for structural analysis and design is the assumption of load sharing. Weight of human beings on a slab will be distributed to the supporting beams and then transferred to columns and finally to foundation. The planned passing of load will affect the member size, safety and finally economy of the structure and therefore a sensible assumption should be made. The ductility of material and robustness of framing system may assist to distribute load in order to prevent failure due to local over-load but engineers should also need to assess load sharing. The mechanism of transferring loads from one part of a structure to another is generally termed as load path and a good structure normally has a clear load path for load transfer.

3.4.1 Live, dead and wind loads

Loads acting on the structural members in a building should be estimated at the early stage of a structural design process. Very often, the architectural requirements, locations and functions of the buildings are considered. Load estimation is an important exercise in a design process for economy and safety.

Realistic and possible loads and load combinations should be considered in the design life of a structure. In limit state design principle, loads are normally considered as the maximum load expected to occur in the life span of a structure. In statistical terms, characteristic loads have 95% probability of not being exceeded in a building life. However, this statistical value is only an assumption or a concept since record can hardly be obtained for many buildings which are different in function than those constructed decades ago.

Structures are designed to take the loads, such as dead, live and wind loads with a certain degree of confidence. Therefore, load estimation becomes an important exercise in determining the member size or even the structural schemes. For common steel buildings, the loads are transferred from slab panels to beam members and to columns and foundations. For some special framing, the columns can be designed to be in tension to hang the loads onto trusses at higher levels.

The load associated with the self-weight of the structure and its permanent elements like concrete floor, self-weight of beam and column member, utilities and finishes, is classified as dead load. Since dead load depends on the sizes of members which is not known in advance, its magnitude is an estimation only. If a large difference exists between the estimated and computed values of dead load, the designer should revise the design again.

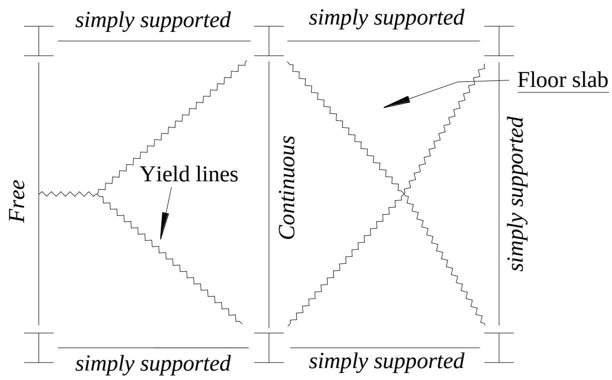
Variable loads that can be applied on or removed from a structure are termed as live loads. Live loads included the weight of occupants, furniture, machine, and other equipment. The values of live loads are specified by codes for various types of buildings and they represent a conservative estimate of the maximum load, occurred in the expected life of the structure.

Air motion or wind exerts pressure which may damage a structure. Since the speed and direction of wind are varied, the exact pressure or suction applied by winds to structures is difficult to assess accurately and they again are obtained by statistics. Furthermore, the actual effect of wind on a structure depends on the wind velocity, structure shape and surrounding configuration from ground profile and influence from adjacent structures. Thus, wind coefficients are available to determine more precisely the wind effect on structures. Values of wind coefficients for typical buildings are available in wind codes and structures with special geometry may require a wind tunnel test to determine accurately the wind coefficients. Wind tunnel test is sometimes called for assessing the wind load on a structure and on foundations.

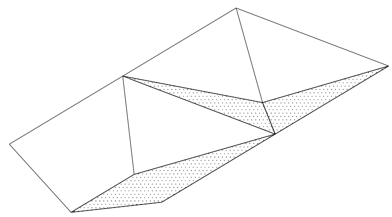
3.4.2 Load distribution

The load *w* acting on the slab is generally assumed to be uniform, even though we expect some non-uniformity of load can occur on a floor. However, for some cases, the loadings can be so concentrated that the assumption is insufficiently accurate. For instance, the weight of partition wall and machine rest only on a small area and uniform load assumption is in gross error under this condition.

For uniform load *w* on slab resting on the supporting beam members, the load distribution on beams follows the yield line pattern of the slab based on a plastic collapse mechanism. At plastic collapse of the slab, the loads within the collapsed portion of slab will be transferred to the connected beam as shown in Figure 3.3. Therefore, the pattern of yield lines is assumed to be the same as the pattern of loading shared by the connected beam members. The pattern of yield lines depends on the types of boundary conditions and geometry of floor slab as shown in Figure 3.3(a) for a general case. Also shown in Figure 3.3(b) is the deformed shape of floor constructed from the yield lines of the slab. It can be visualized that beams on the longer edges of the slab take greater loads as the same deflection at centre of the slab causes larger moment and force at supports spacing across shorter span.



a) Yield lines on floor slab for different support conditions



b) Collapse mechanism based on yield lines

Figure 3.3 Pattern of yield lines of general cases

For simplicity, the yield line is assumed to be the angle bisector at the corner of a slab, when assuming the supporting conditions of the floor slab are identical for load sharing. The effect of actual boundary conditions of floor slab is ignored. For the case of a one-way slab, the slab spans in one direction and it behaves like a beam member with larger width. This assumption is normally made when the aspect ratio of the floor is larger than two in which case the slab is narrow. Obviously, the one-way slab assumption is made when the connection details or member stiffness vary significantly, such as the stiffness of a pair of opposite beams is much greater than the other pair of beams. Apart from this simple condition, a two-way slab is also commonly assumed and designed as it is more economical and loads are shared by all four beams. The loads distributed to the supports are respectively illustrated in Figure 3.4(a) and (b) for one-way and two-way slab.

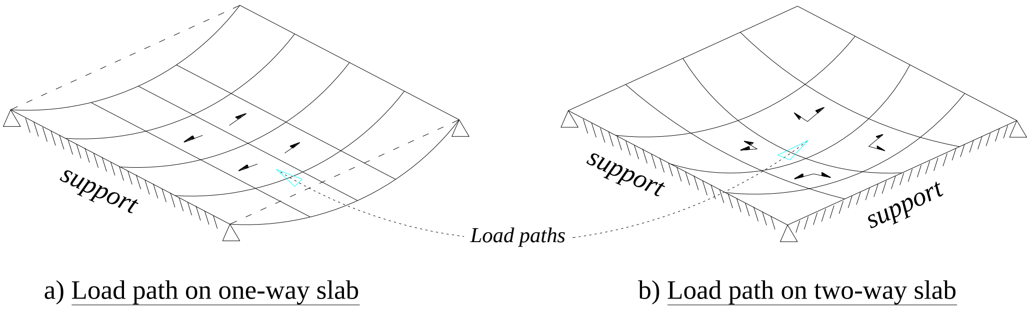


Figure 3.4 Load paths in one-way slab and two-way slab

When the slab is square and supported by four beam members as shown by solid lines in Figure 3.5, the loadings *w* (kN/m^2) on the triangular collapsed portion of slab spread to the beam members. Hence the beam is then subjected to a triangularly distributed load as shown in Figure 3.5. This slab is then a two-way slab, where the load spreads in both directions. The distribution is based on identical boundary conditions, the spreading angle at the corner is 45° as indicated by the dotted lines in Figure 3.5, which is also equivalent to the yield line pattern.

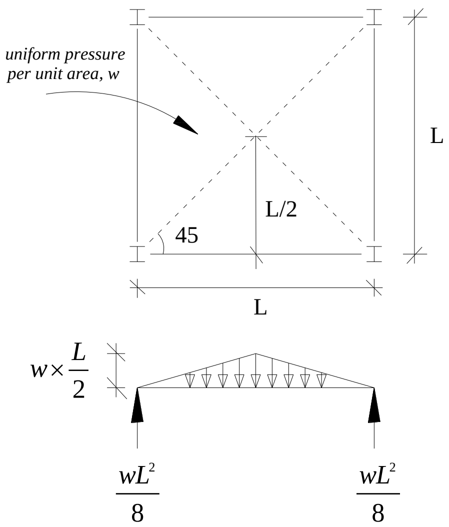


Figure 3.5 Square floor slab

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In general, the length and width of floor slab are not equal such that $L_1 \neq L_2$, and the length L_2 of floor slab is less than twice of the width L_1 as shown in Figure 3.6. The load imposed on shorter beam member should also be triangular, whereas the loading on longer beam is trapezoidal. The maximum unit distributed load on each beam should be pressure w times the distance to beam $L_1/2$, as $wL_1/2$ (kN/m) and this load sharing in a two-way slab is also considered as two way.

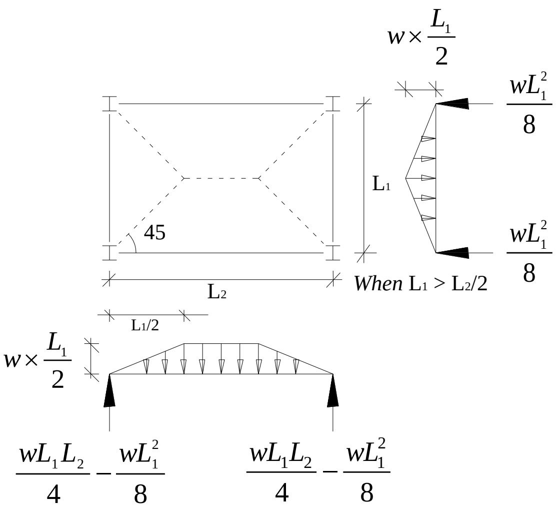


Figure 3.6 Rectangular floor slab

Consider the case of a secondary beam dividing the slab discussed above into two parts as shown in Figure 3.7, the length L_2 of each floor slab is not greater than twice the width L_1. The load spreading on main beam along transverse direction still remains trapezoidal. However, the loading distribution on the main beam in longitudinal direction will comprise of two triangular distributed loads from the slab and a point load transmitted by the secondary beam. Maximum distributed load on each beam should also be $wL_1/2$.

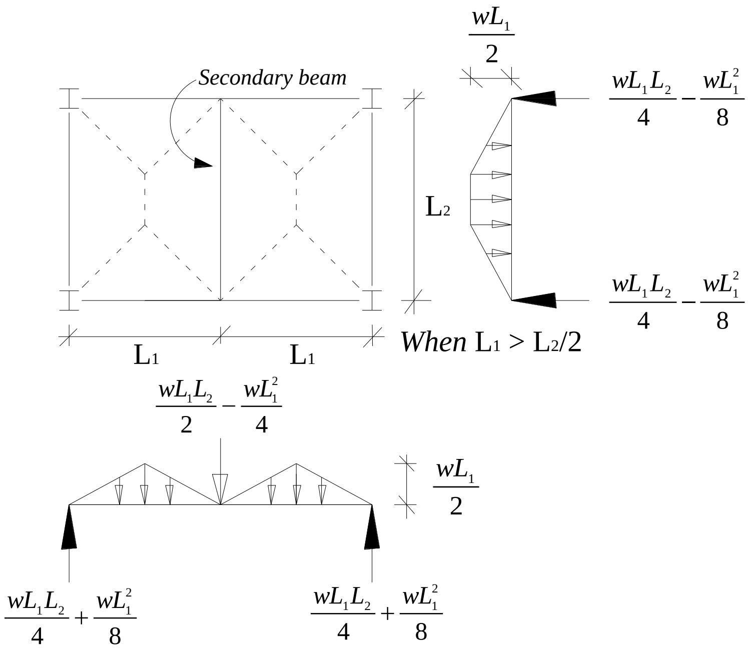


Figure 3.7 Two rectangular floor slabs

When the width L_1 of floor slab is very short, which is commonly assumed when the length L_2 is longer than twice of the width L_1 as shown in Figure 3.8, the load is assumed to spread in a shorter direction and there will be no loading distributed to shorter beam member because the triangular loads on the shorter beams are small here. The floor slab is regarded as the one-way slab, which is convenient to design. The yield line is simply a straight line dividing the floor slab into two equal parts.

Consider another case of a panel being split by two secondary beams to become three slabs as shown in Figure 3.9. The length L_2 of each floor slab is longer than twice of the width L_1. Each slab becomes a one-way slab. In this case, the beam supporting the dividing beams is considered as being loaded by point loads as shown in Figure 3.9

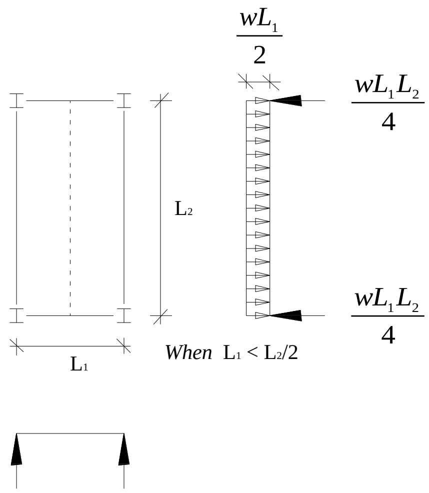


Figure 3.8 Floor slab of one-way slab

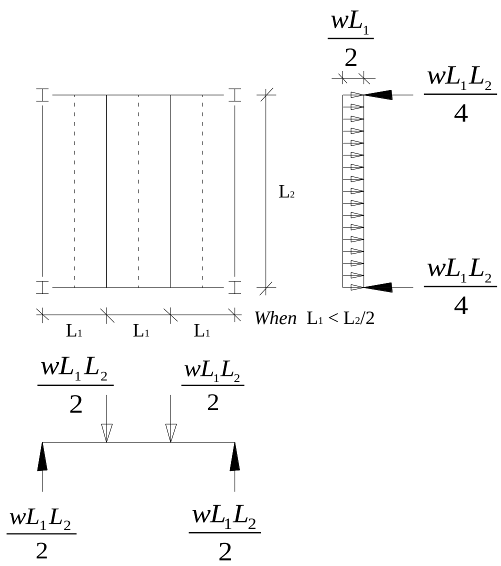


Figure 3.9 Combined one-way slab

The load on the floor is transferred to the beam member and then to column. Loading on column should be the summation of reactions of the connected beam members at each floor level. Alternatively, the axial loads on column can be simply determined from the loaded area multiplied by pressure *w* as shown in Figure 3.10 for different cases at various levels. In Figure 3.10, the loaded column is indicated by a circle and loaded area is shaded. The load area supported by a column should be obtained according to the load paths of connected beams discussed in the previous sections.

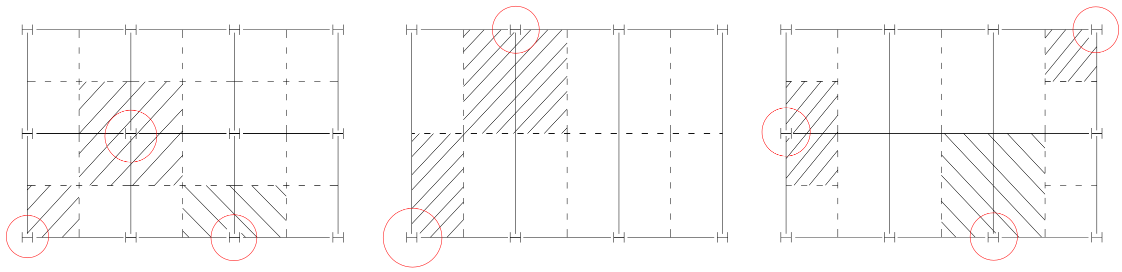


Figure 3.10 Loading taking by columns in different floor systems

When the beam-column connection is designed as moment connection, the moments from beam transmitted to the column should be considered. It is convenient to design a simple structure, which implies all beam-column connections in a structure are pinned. However, load eccentricity is required to be considered here. Alternatively, when the moment connection allowing full transfer of moment is assumed, the moment due to load eccentricity is not required to be considered.

Chapter 4 Section Classification and Local Plate Buckling

4.1 Introduction of local plate buckling

When thin plates are in compression, local plate buckling may occur. The local plate buckling resistance depends on the stress distribution along the plate, boundary condition of the plate, material design strength, presence of ribs, if any, geometry of the plate (i.e. width-to-thickness ratio) and initial imperfection in plates.

As it is uncommon to use hot rolled members with sections classified as slender, HK Code only provides effective stress method for the local plate buckling check and it refers to Chapter 11 for the effective width method. As the application of the formulae in Chapter 11 is limited to 8mm thick plate, *Clause 7.6* of HK Code further refers to other literatures for the checking by the effective width method and Eurocode 3 (2005) is considered as one of the literatures appropriate for checking of hot rolled slender sections by the effective width method for plates thicker than 8mm.

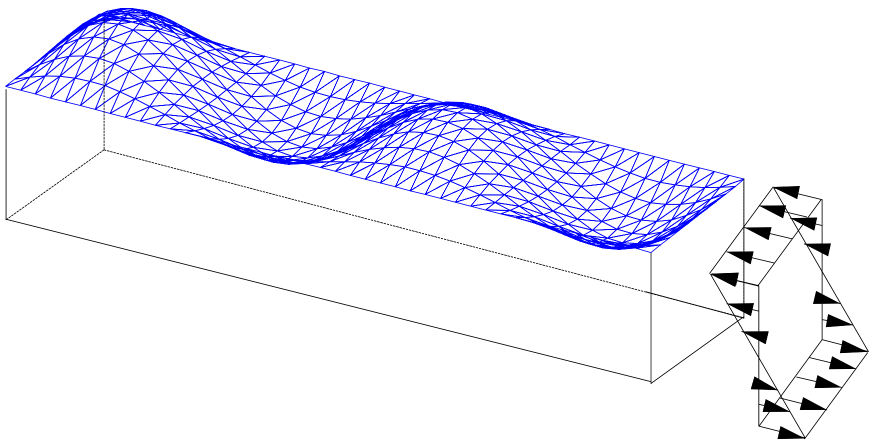


Figure 4.1 Local plate buckling simulated by the NIDA-9, non-linear frame and shell analysis and design software

In the HK Code, two types of elements are generally considered in classifying for plate boundary condition, namely the internal and outstanding elements. Internal elements refer to the plate elements or components with both longitudinal edges supported by other plate elements such as webs of box or I-sections. Outstanding elements refer to plate elements or components with only one edge supported by other plate elements such as flanges of an I-section.

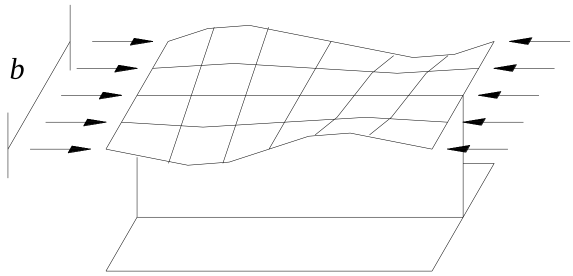


Figure 4.2 Internal and outstanding plate elements in an I-section

4.2 Cross section classifications

Plate buckling is controlled and classified by the breadth to thickness ratio (*b/t*). Thicker plates or plates with smaller breadth are less likely to buckle than the thinner plates or plates with larger breadth. Plates with stiffeners will reduce the breadth by the distance between longitudinal stiffeners and thus increase the buckling resistance. Transversely placed stiffeners are not effective in reducing the local plate buckling resistance as they are unable to stiffen the long plate elements unless they are very closely spaced.

In HK Code, the breadth is generally measured as the width of flange or webs as in *Figure 7.1* of the HK Code. There are 4 types of element class, being Class 1 for plastic sections, Class 2 for compact sections, Class 3 for semi-compact sections and Class 4 for slender sections. The graphical representation of the resistance of these 4 classes of element against member rotation is indicated in Figure 4.3.

Class 1: Plastic Cross Sections
Plastic hinge can be developed with sufficient rotation capacity.

Class 2: Compact Cross section
Full plastic moment capacity can be developed but local buckling will occur soon after the formation of plastic hinge. Thus, it is allowed to possess a plastic hinge in an elastic design but it is not allowed to do so when used in a plastic design. However, all members must be at least compact cross sections when used in a plastic design.

Class 3: Semi-compact Sections
Extreme fiber may yield but local buckling prevents it from plastic moment formation. Both Classes 3 and 4 cannot be used in plastic design.

Class 4: Slender Sections
Sections under compression or bending that do not meet the limits for Class 3 sections. The section may buckle before extreme fiber yields.

The purpose of the above classification is to calculate the load carrying capacity of the structural members, which depends on the failure mode (yielding, buckling or combined elasto-plastic buckling). For slender section in Class 4, the member sectional properties or design strength shall be reduced to account for the local buckling effect.

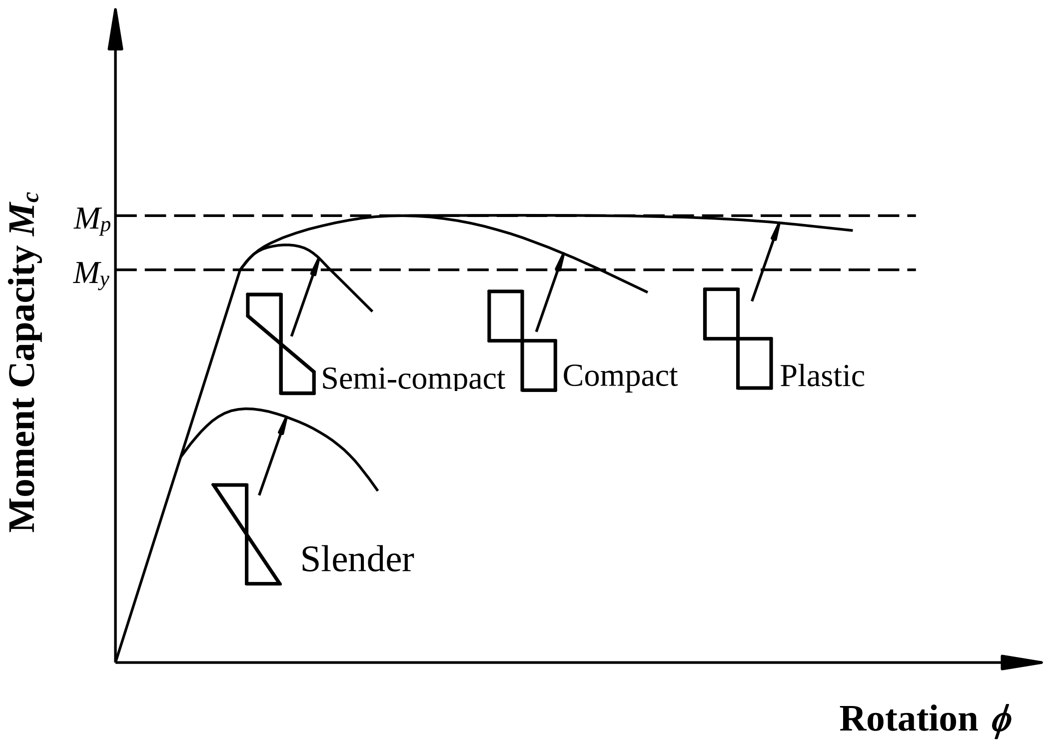


Figure 4.3 Local buckling of various classes of plate element in sections

4.3 Limiting width-to-thickness ratio

There are three main methods for the design of sections against local plate buckling, namely the effective width method, the effective stress method and the numerical finite element method. The effective width method is widely adopted in newer design codes and the width of a section is reduced to an "effective" width. As it sometimes depends on the stress and thus the load case so it is more tedious in general applications but it is considered to be more economical. The effective stress method reduces the design strength to account for local buckling and it is simpler to use. The numerical finite element method is most exact but sometimes involves analysis expert for an accurate solution.

The section classification is carried out by the limiting *b/T* ratio in *Table 7.1* for non-RHS and non-CHS sections and *Table 7.2* for RHS and CHS (RHS Rectangular hollow sections and CHS Circular hollow sections). To unifying the use of the equations to various steel grades, a parameter, $\varepsilon = \sqrt{275/p_y}$, is used to factor the limiting ratio.

In the *Tables 7.1* and *7.2*, the stress ratio r_1 and r_2 are the stress ratios given in Equations (4.1) to (4.4) as,

For typical H-sections with equal flange, r_1 and r_2 are determined as,

$$r_1 = \frac{F_c}{dtp_{yw}} \qquad \text{but} \quad -1 < r_1 \le 1 \tag{4.1}$$

$$r_2 = \frac{F_c}{A_g p_{yw}} \tag{4.2}$$

For typical RHS or welded box sections with equal flanges, r_1 and r_2 are determined as,

$$r_1 = \frac{F_c}{2dtp_{yw}} \qquad \text{but} \quad -1 < r_1 \le 1 \tag{4.3}$$

$$r_2 = \frac{F_c}{A_g p_{yw}} \tag{4.4}$$

where

A_g = gross cross-sectional area
d = web depth
F_c = axial compression (negative for tension)
p_{yw} = design strength of the web (but $p_{yw} \le p_{yf}$)
t = web thickness

For other sections such as unequal flange sections, the code should be referred and for other complex shape sections, a finite element buckling analysis NIDA-9 (2015) can be used.

4.3.1 Effective width method

4.3.1.1 Effective width of flange element under uniform compression

If the plate section of member is classified as Class 4 slender section according to the above mentioned classification method, it represents the local plate buckling may probably occur on the plate section. The effective section properties should be evaluated such that the corresponding member resistance, such as section modulus or cross-section area, can be computed accounting for the local plate buckling effect.

In the evaluation of section properties for slender section, the effective width of slender section including flange or web should be determined pursuant to *Clause 11.3* of HK Code. There are two types of section. One is section, whose thickness is between 1mm to 8mm, and the other is sheet profile, whose thickness ranges from 0.5mm to 4mm. When any thickness of the section is greater than 8mm, the effective width method for such member section should accord to other literature or Eurocode 3 (2005). For hot-rolled member sections, their plate section is most likely classified as section. In other case, the section type should be sheet profiles for floor decking, roof and wall cladding commonly.

The type of section is only used in determination of effective width of flat stiffened flange section under uniform compression. The stiffened and unstiffened elements are defined by their support conditions. The internal element, which includes internal flange or web, used in classification *Tables 7.1* and *7.2* is same as stiffened element named in *Clauses 11.3* of HK Code. On the other hand, the outstand element, which comprises outstand flange, should be equivalent to the unstiffened element.

It should be emphasized that the dimension of the plate section in *Chapter 11* of HK Code is defined by the mid-line section in the subsequent effective width method, which is disparate from the dimension of element section used in *Section 7* of HK Code according to *Tables 7.1* and *7.2*. When the slender hot-rolled section properties are calculated based on the effective width method, which is a less conservative approach for hot-rolled section, the dimension of element section should be according to *Tables 7.1* and *7.2* of HK Code.

After classification of section type, the determination of effective width should also depend on the loading cases including uniform compression case and bending case. In the following calculations of effective width of element are confined to the flange element section under uniform compression. The effective width of element under uniform compression is given as,

$$b_e = \beta b \tag{4.5}$$

and β is given by,

$$\beta = \sqrt{\frac{p_{cr}}{p_e}}\left(1.0 - 0.22\sqrt{\frac{p_{cr}}{p_e}}\right) \tag{4.6}$$

The local buckling strength p_{cr} of the element is given as Equation (4.7).

$$p_{cr} = K\frac{\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2 \quad (4.7)$$

where E is elastic modulus of element, t and b are the net thickness and the width of the element respectively and K is relevant local buckling coefficient depending on the support conditions of flange element, such as stiffened and unstiffened element and intenal and outstand element, ν is the Poisson's ratio taken as 0.3. It should be noted that the gross section, such the width b and depth d, should be defined by the mid-line dimension in *Clause 11.3.1* of HK Code.

Equation (4.7) is the local buckling strength of the element. For different element section types and support conditions, the local buckling strength p_{cr} of the element is also different relying on the different value of relevant local buckling coefficient K. The unstiffened element, which is supported at one edge, is more vulnerable to the local plate buckling by comparing with the stiffened element, which is supported by both edges, as the supporting condition can cater the additional section capacity of the section for post-buckling or load redistribution effect. Therefore, the buckling coefficient K for stiffened flange or internal element under uniform compression can be expressed as,

$$K = 5.4 - \frac{1.4h}{0.6+h} - 0.02h^3 \quad (4.8)$$

where h is equal to the ratio between depth of web d_w and width of flange b, i.e. $h = d_w/b$, d_w is the sloping distance between the intersection points of a web and the two flanges and b is the flat width of the flange. It should be pointed out that the buckling coefficient K of stiffened flange element for sheet profiles is neglected herein, because it is uncommon that the thickness of hot-rolled section is less than 4mm. Alternatively, the value of the buckling coefficient K should be conservatively taken as 4 for internal or simpl supported elements.

When the flange element is restrained at only one edge, the unstiffened flange element is prone to local plate buckling with a slight modification of coefficient in Equation 4.6 to the following.

$$\beta = \sqrt{\frac{p_{cr}}{p_e}}\left(1.0 - 0.188\sqrt{\frac{p_{cr}}{p_e}}\right) \quad (4.9)$$

The buckling coefficient K for determination of p_{cr} of unstiffened flange or outstand element should be taken as,

$$K = 1.28 - \frac{0.8h}{2+h} - 0.0025h^2 \quad (4.10)$$

Alternatively, the value of buckling coefficient K should be conservatively taken as 0.425 for unstiffened elements.

4.3.1.2 Effective width of web element under bending stress

When the element section is subjected to bending stress, the local plate buckling may not be so easy to occur than those under uniform compression. It is because compression load deteriorates the stiffness of the slender element section to cause local plate buckling. On the contrary, the tensile load in a certain extent eliminates the instability effect from compression load and therefore hinders the slender element section from plate buckling. Thus, when the web section is fully under tension, the web is considered as fully effective. Further, if the slenderness ratio of web, such as depth to thickness ratio d_w/t is smaller than or equal to 70ε, the web section is classified as fully effective.

As a result, the buckling load resistance of the element section differs under different loading conditions. When the element section is subjected to bending stress, the local plate buckling load should be determined correspondingly in the following set of formulae.

For one edge in tension as shown in Figure 4.4(a), the effective width of different portion are given as,

$$b_{e,1} = 0.76t\sqrt{\frac{E}{f_{c,1}}} \tag{4.11}$$

$$b_{e,3} = 1.5b_{e,1} \tag{4.12}$$

in which $b_{e,1}$ and $b_{e,3}$ are the portion of the effective width adjacent to the more compressed edge and tension edge respectively, $f_{c,1}$ is the larger compressive edge stress, b_t is the portion of web under tension, E is elastic modulus and t is the net thickness of the steel material. It should be remarked that if the condition of $b_{e,1} + b_{e,3} + b_t \geq d_w$ for web section attains, then the web section is fully effective against local plate buckling.

For both edges in compression as shown in Figure 4.4(b), the effective width of different portion are written as,

$$b_{e,1} = 0.76t\sqrt{\frac{E}{f_{c,1}}} \tag{4.13}$$

$$b_{e,2} = \left(1.5 - 0.5\frac{f_{c,2}}{f_{c,1}}\right)b_{e,1} \tag{4.14}$$

in which $b_{e,1}$ and $b_{e,2}$ are the portion of the effective width adjacent to the more and less compressed edge respectively, $f_{c,1}$ and $f_{c,2}$ are the larger and smaller compressive edge stress respectively. Similarly, if the condition of $b_{e,1} + b_{e,2} \geq d_w$ for web section achieves, the web is classified as fully effective.

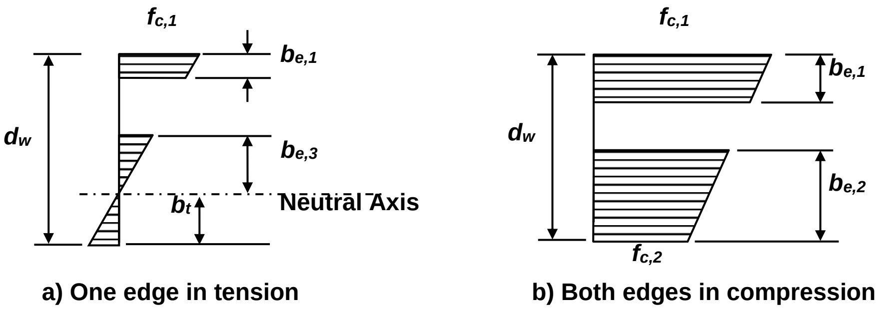


Figure 4.4 Stress distributions over effective portions of web element

4.3.2 Effective stress method

The effective width method allows for stress distribution across a section and it is more accurate in general. The effective stress method is simpler to use by reducing the design strength. In HK Code, the reduction can carried out using the following formula.

$$p_{yr} = \left(\frac{\beta_3}{\beta}\right)^2 p_y \tag{4.15}$$

in which β is the value of width-to-thickness ratio that exceeds the limiting values of β_3.

4.3.3 Finite element method

Possibly second to a direct experimental test, the finite element method is most accurate. In some cases where the geometry of a plate is irregular, or opening or ribs exists, the finite element method is the most sensible solution to the design of slender sections. Care should be taken to assume an appropriate set of initial imperfection for the plates and a nonlinear incremental-iterative analysis method is needed to trace the load vs. deflection path to locate the maximum load resistance of the plated structure. The procedure and concept for the analysis is the same as in the second-order direct analysis for frames in Chapter 10, while they use different element types as beam-column and shell elements for the structural model.

4.4 Worked examples

4.4.1 Section classification of rolled universal I-beam

Determine classification of section of beam of 254×102×22 UB in steel grade S355.

<u>Solution</u>
SECTION PROPERTIES
$D = 254.0mm$, $B = 101.6mm$, $t = 5.7mm$, $T = 6.8mm$, $d = 225.2mm$

SECTION CLASSIFICATION
Design strength, $p_y = 355N/mm^2$ for $T \le 16mm$ *(Table 3.2)*

$$\varepsilon = \sqrt{\frac{275}{355}} = 0.88$$ *(Table 7.1 Note b)*

Plastic limiting value of b/T for outstand flange of an I-section is 9ε

$$\frac{b}{T} = \frac{101.6}{2\times 6.8} = 7.47 \le 9\times 0.88 = 7.92$$ *(Table 7.1)*

$\therefore$ flange is plastic

Plastic limiting value of d/t for web of an I-section with neutral axis at mid-depth is 80ε

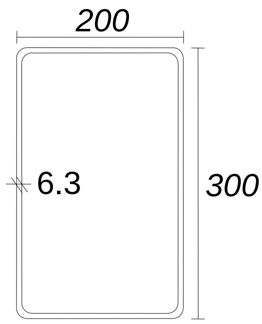
$$\frac{d}{t} = \frac{225.2}{5.7} = 39.5 \le 80\times 0.88 = 70.4$$ *(Table 7.1)*

$\therefore$ web is plastic

$\therefore$ the section is Class 1 plastic

4.4.2 Effective width method for hot-rolled RHS under uniform compression

A stocky column of 300×200×6.3 hot-rolled RHS section in steel grade S355 is under a factored compression force of 1600kN and under a small moment causing negligible stress gradient. Determine the section properties for compression capacity of the section.



<u>Solution</u>

SECTION PROPERTIES

$D = 300mm$, $B = 200mm$, $t = 6.3mm$, $A = 61.0cm^2$

SECTION CLASSIFICATION

Design strength, $p_y = 355N/mm^2$ for $t \le 16mm$ *(Table 3.2)*

$\varepsilon = \sqrt{\dfrac{275}{355}} = 0.88$ *(Table 7.2 Note b)*

Width of RHS, $b = B - 3t = 200 - 3 \times 6.3 = 181.1mm$ *(Table 7.2 Note a)*

Depth of RHS, $d = D - 3t = 300 - 3 \times 6.3 = 281.1mm$ *(Table 7.2 Note a)*

Limiting value of b/t for flange of a hot-rolled RHS is 40ε

$\dfrac{b}{t} = \dfrac{181.1}{6.3} = 28.7 \le 40 \times 0.88 = 35.2$ *(Table 7.2)*

$\therefore$ flange is non-slender

Limiting value of d/t for web of a hot-rolled RHS under axial compression is $120\varepsilon/(1 + 2r_2)$

Stress ratio, $r_2 = \dfrac{F_c}{A_g p_{yw}} = \dfrac{1600 \times 10^3}{6100 \times 355} = 0.739$ *(7.6)*

$\dfrac{d}{t} = \dfrac{281.1}{6.3} = 44.6 > \dfrac{120 \times 0.88}{1 + 2 \times 0.739} = 42.6$ *(Table 7.2)*

$\therefore$ web is slender

$\therefore$ the section is Class 4 slender

COMPRESSION RESISTANCE

By effective width method,

$K = 4$ for conservative approach *(Clause 11.3.4.4.3)*

$$f_c = \frac{1600\times10^3}{6100} = 262.3\,N/mm^2$$ *(Clause 11.3.4.4.1)*

For flange,

$$p_{cr} = 0.904EK\left(\frac{t}{b}\right)^2 = 0.904\times205000\times4\times\left(\frac{6.3}{181.1}\right)^2 = 897.1\,N/mm^2 \quad (11.11)$$

$$\rho = \frac{f_c}{p_{cr}} = \frac{262.3}{897.1} = 0.292 > 0.123 \quad (11.10)$$

$$\beta = \left\{1+14\left(\sqrt{\rho}-0.35\right)^4\right\}^{-0.2} = \left\{1+14\left(\sqrt{0.292}-0.35\right)^4\right\}^{-0.2} = 0.996 \quad (11.9b)$$

$$\therefore b_e = \beta b = 0.996\times181.1 = 180.4mm \quad (11.8)$$

For web,

$$p_{cr} = 0.904\times205000\times4\times\left(\frac{6.3}{281.1}\right)^2 = 372.3\,N/mm^2 \quad (11.11)$$

$$\rho = \frac{262.3}{372.3} = 0.705 > 0.123 \quad (11.10)$$

$$\beta = \left\{1+14\left(\sqrt{0.705}-0.35\right)^4\right\}^{-0.2} = 0.889 \quad (11.9b)$$

$$\therefore b_e = 0.889\times281.1 = 249.9mm \quad (11.8)$$

Effective Area, $A_{eff} = (180.4+249.9)\times6.3\times2 = 5422mm^2$

Compression Resistance, $P_c = A_{eff}\,p_y = 5422\times355 = 1924.8kN > F_c$ *(OK)*

4.4.3 Effective stress method of slender section

Determine the reduced design strength of a 305×457×127 Tee in S275 steel.

<u>Solution</u>

SECTION PROPERTIES

$D = 459.1mm$, $B = 305.5mm$, $t = 17.3mm$, $T = 27.9mm$, $A = 161cm^2$

SECTION CLASSIFICATION

Design strength, $p_y = 265N/mm^2$ for $16mm < T \le 40mm$ *(Table 3.2)*

$$\varepsilon = \sqrt{\frac{275}{265}} = 1.02$$ *(Table 7.1 Note b)*

Plastic limiting value of b/T for outstand flange of a T-section is 9ε

$$\frac{b}{T} = \frac{305.5}{2 \times 27.9} = 5.47 \le 9 \times 1.02 = 9.18$$ *(Table 7.1)*

$\therefore$ flange is plastic

Semi-compact limiting value of D/t for stem of a T-section is 18ε

$$\frac{D}{t} = \frac{459.1}{17.3} = 26.5 > 18 \times 1.02 = 18.4$$ *(Table 7.1)*

$\therefore$ stem is slender

$\therefore$ the section is Class 4 slender

By effective stress method,

Reduced design strength, $p_{yr} = \left(\frac{\beta_3}{\beta}\right)^2 p_y = \left(\frac{18.4}{26.5}\right)^2 \times 265 = 127.8N/mm^2$ *(7.13)*

4.4.4 Effective section modulus of rolled H-section

Classify beam section of 152×152×23 UC in Grade S275 steel, which is subjected to pure bending. And calculate the effective section moduli.

<u>Solution</u>

SECTION PROPERTIES

$D = 152.4mm$, $B = 152.2mm$, $t = 5.8mm$, $T = 6.8mm$, $d = 123.6mm$, $Z_x = 164cm^3$, $Z_y = 52.4cm^3$, $S_x = 182cm^3$, $S_y = 80.1cm^3$

SECTION CLASSIFICATION

Design strength, $p_y = 275N/mm^2$ for $T \le 16mm$ *(Table 3.2)*

$$\varepsilon = \sqrt{\frac{275}{275}} = 1 \qquad \textit{(Table 7.1 Note b)}$$

Semi-compact limiting value of b/T for outstand flange of an H-section is 15ε

$$\frac{b}{T} = \frac{152.2}{2 \times 6.8} = 11.2 \le 15 \times 1 = 15 \qquad \textit{(Table 7.1)}$$

$\therefore$ flange is semi-compact

Plastic limiting value of d/t for web of an H-section with neutral axis at mid-depth is 80ε

$$\frac{d}{t} = \frac{123.6}{5.8} = 21.3 \le 80 \times 1 = 80 \qquad \textit{(Table 7.1)}$$

$\therefore$ web is plastic

$\therefore$ the section is Class 3 semi-compact

EFFECTIVE PLASTIC MODULUS

$$S_{x,eff} = Z_x + (S_x - Z_x)\left[\frac{\left(\frac{\beta_{3w}}{d/t}\right)^2 - 1}{\left(\frac{\beta_{3w}}{\beta_{2w}}\right)^2 - 1}\right] = 164 + (182 - 164) \times \left[\frac{\left(\frac{120}{21.3}\right)^2 - 1}{\left(\frac{120}{100}\right)^2 - 1}\right] = 1421.5cm^3 \qquad (7.7)$$

But

$$S_{x,eff} \le Z_x + (S_x - Z_x)\left[\frac{\left(\frac{\beta_{3f}}{b/T}\right) - 1}{\left(\frac{\beta_{3f}}{\beta_{2f}}\right) - 1}\right] \le 164 + (182 - 164) \times \left[\frac{\left(\frac{15}{11.2}\right) - 1}{\left(\frac{15}{10}\right) - 1}\right] = 176.2cm^3 \qquad (7.9)$$

$\therefore$ $S_{x,eff} = 176.2cm^3$

$$S_{y,eff} = Z_y + (S_y - Z_y)\left[\frac{\left(\frac{\beta_{3f}}{b/T}\right) - 1}{\left(\frac{\beta_{3f}}{\beta_{2f}}\right) - 1}\right] = 52.4 + (80.1 - 52.4) \times \left[\frac{\left(\frac{15}{11.2}\right) - 1}{\left(\frac{15}{10}\right) - 1}\right] = 71.2cm^3 \qquad (7.8)$$

Chapter 5 Tension Members

5.1 Introduction

Tension members are very effective in resisting as they have no buckling problems. Also, their design is relatively simple. The capacity of a tension member is limited by the following conditions.

1. Eccentric connection e.g. angles connected in one leg only.
2. Reversed loads making the tension members under compression and then buckle.
3. Moment due to eccentric loads etc.

5.2 Tension capacity

The tension capacity of a member, P_t, is given by

$$P_t = p_y A_e \tag{5.1}$$

in which A_e is the effective area of all elements in a cross section. The effective area of each element, a_e, is given by,

$$a_e = K_e a_n \quad \text{but} \ \le a_g \tag{5.2}$$

in which K_e is the effective net area coefficient given by,

K_e = 1.2 for grade S275 steel
= 1.1 for grade S355 steel
= 1.0 for grade S460 steel

$$= \frac{U_s}{1.2 p_y} \le 1.2 \ \text{for other steel grades} \tag{5.3}$$

a_n = the net cross sectional area of leg deducted for openings
a_g = gross sectional area without deduction for openings
U_s = ultimate strength
p_y = design strength

5.3 Eccentric connections

Many sections are commonly or unavoidably connected by eccentric connections as shown in Figure 5.1 for a truss. When the axial tension is applied not through the centroid of a section, a moment exists and it can be accounted for explicitly by an application of a moment equal to the product of axial force and eccentricity and the member should be designed as a beam-column under axial force and moment. This is a conservative approach for slender members as they will deform and reduce the amount of moment. An alternative approach in clause of HK Code available for design of single and double angles is to use the effective area (A_e) for the area in a cross section to account for eccentric load.

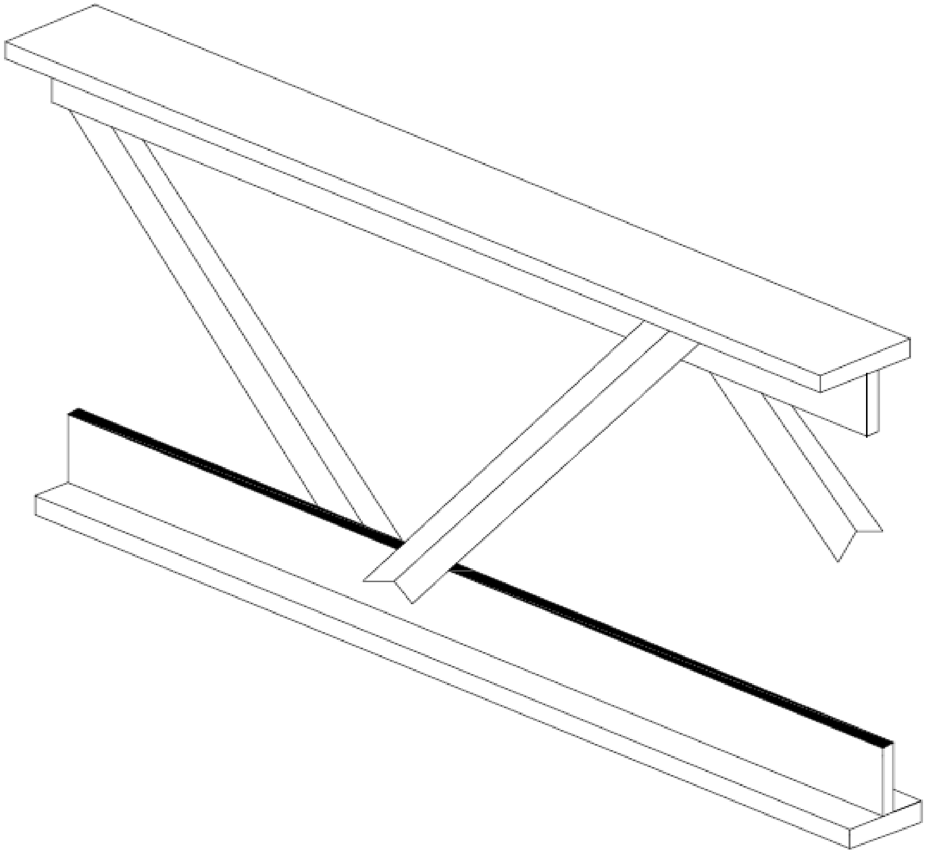


Figure 5.1 Truss with T-sections as chords and angles as webs

5.3.1 Single and double angle, channel and T-sections

In bolted connections, $P_t = p_y\left(A_e - 0.5a_2\right)$ (5.4)

In welded connections, $P_t = p_y\left(A_e - 0.3a_2\right)$ (5.5)

in which

a_2 = gross sectional area of the unconnected element in the section

= $A_g - a_1$ (5.6)

a_1 = gross sectional area of the connected element in the section

A_g = gross area of the section

A_e = effective area of the section defined in Equation 5.1

5.3.2 Double angle, channel and T-sections with intermediate connections

The effect here is less severe than the above cases.

In bolted connections, $P_t = p_y\left(A_e - 0.25a_2\right)$ (5.7)

In welded connections, $P_t = p_y\left(A_e - 0.15a_2\right)$ (5.8)

To qualify for design as double sections here, the sections must be separated by at least 2 number of solid packing pieces or battens along the combined member otherwise the combined section is required to be designed as a single section.

5.4 Non-linear analysis for asymmetric sections

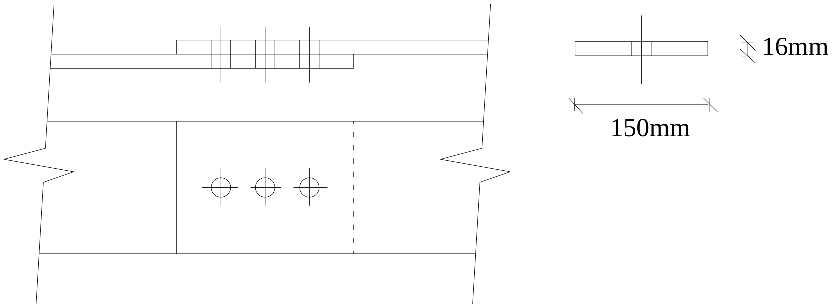
The general second-order direct analysis method can be applied to the design of angle and asymmetric sections, provided that the effects of member imperfections as well as additional effects due to eccentric connection and sectional asymmetry are accounted for. Chan and Cho (2005) tested a series of angle trusses and the test results were compared with the first-order linear, second-order direct elastic and advanced analysis, indicating a conservative design can be obtained

The eccentric connection effect can be modeled in a robust nonlinear frame analysis program by rigid links connecting the connecting and off-set nodes. The section capacity check is carried out until the section yields, no matter whether the member is in tension or in compression as the presence or absence of P-delta effects have been considered in computation of moment. Thus, the sectional check as follows can be carried out as in Equation (10.24). It will be necessary to use a reduced section capacity in the section capacity check by using the smaller sectional strength in Equation (5.1), with automatic consideration in computer program of checking the sectional tensile strength according to the connection type. A detailed discussion on the rationale is discussed in Chapter 10.

5.5 Worked Examples

5.5.1 Tension capacity of plate

Determine the design load capacity of a tension member consisted of two plates of 150×16 cross section and grade S275 steel connected by a single line of 20 mm bolts.



Solution

TENSION CAPACITY

Design strength, $p_y = 275N/mm^2$ for $T \leq 16mm$ *(Table 3.2)*

Gross area of the plate, $A_g = 150\times 16 = 2400mm^2$

Net area of the plate, $A_n = 2400 - 22\times 16 = 2048mm^2$

Effective net area coefficient, $K_e = 1.2$ for S275 *(Clause 9.3.4.4)*

Effective area of the plate, $A_e = K_e A_n = 1.2\times 2048 = 2458mm^2 > A_g$ *(9.10)*

$\therefore A_e = A_g = 2400mm^2$

If the load eccentricity is ignored, the tension capacity P_t is given by

$P_t = p_y A_e = 275\times 2400 = 660kN$ *(8.66)*

If the load eccentricity is considered, the tension capacity P_t is given by

$$p_y = \frac{P_t}{bd} + \frac{6P_t e}{bd^2}$$

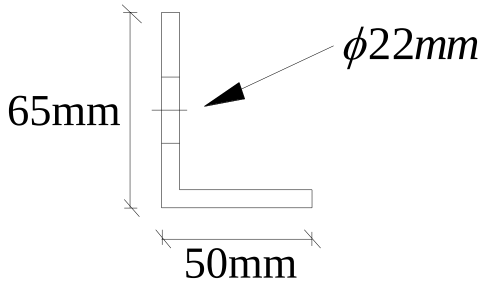
$$275 = \frac{P_t}{150\times 16} + \frac{6P_t \times 16}{150\times 16^2}$$

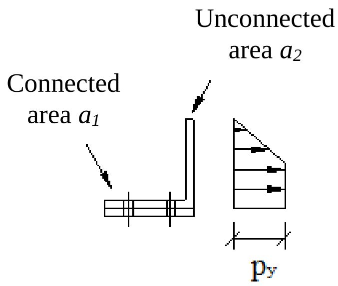
$$\therefore P_t = 94.3kN$$

This is why use of double plates to reduce moment is better.

5.5.2 Tension capacity of unequal angle

Determine the design load capacity of an angle 65×50×6 of Grade S275 Steel connected through the long leg by a single line of 20 mm bolts.





<u>Solution</u>

TENSION CAPACITY

Design strength, $p_y = 275 N/mm^2$ for $t \le 16mm$ *(Table 3.2)*

Gross area of the angle, $A_g = (65 + 50 - 6) \times 6 = 654 mm^2$

Gross area of the connected leg, $a_1 = \left(65 - \frac{6}{2}\right) \times 6 = 372 mm^2$

Gross area of the unconnected leg, $a_2 = A_g - a_1 = 654 - 372 = 282 mm^2$ *(8.69)*

Net area of the connected leg, $a_n = 372 - 22 \times 6 = 240 mm^2$

Effective net area coefficient, $K_e = 1.2$ for S275 *(Clause 9.3.4.4)*

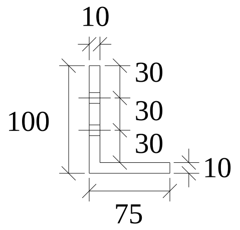
Effective area of the connected leg, $a_e = K_e a_n = 1.2 \times 240 = 288 mm^2 \le a_g = a_1$ *(9.10)*

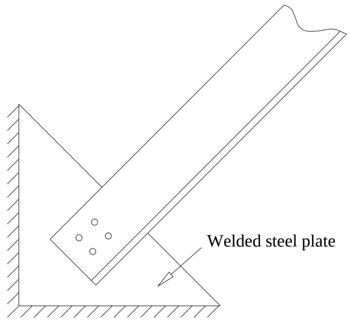
Effective area of the angle, $A_e = a_e + a_2 = 288 + 282 = 570 mm^2$

Tension capacity, $P_t = p_y (A_e - 0.5 a_2) = 275 \times (570 - 0.5 \times 282) = 118.0 kN$ *(8.67)*

5.5.3 Tension capacity of angle bracings

A single angle is used as a tension bracing as shown, which is in S275 steel material and section is unequal angle 100×75×10. The section of single angle is shown. The factored tension force supported by the tension bracing is 200kN. At the end connection, M18 bolts are used and diameter of bolt holes are then 20mm. Check the tension capacity of the tension bracing with the single angle section.





<u>Solution</u>

SECTION PROPERTIES

$D = 100mm$, $B = 75mm$, $t = 10mm$, $A_g = 16.6cm^2$

TENSION CAPACITY

Design strength, $p_y = 275N/mm^2$ for $t \leq 16mm$ *(Table 3.2)*

Gross area of the connected leg, $a_1 = \left(100 - \frac{10}{2}\right) \times 10 = 950mm^2$

Gross area of the unconnected leg, $a_2 = A_g - a_1 = 1660 - 950 = 710mm^2$

Net area of the connected leg, $a_n = 950 - 20 \times 10 \times 2 = 550mm^2$

Effective net area coefficient, $K_e = 1.2$ for S275 *(Clause 9.3.4.4)*

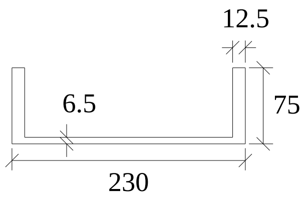
Effective area of the connected leg, $a_e = K_e a_n = 1.2 \times 550 = 660mm^2 \leq a_g = a_1$ *(9.10)*

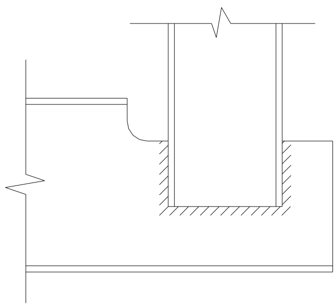
Effective area of the angle, $A_e = a_e + a_2 = 660 + 710 = 1370mm^2$

Tension capacity, $P_t = p_y(A_e - 0.5a_2) = 275 \times (1370 - 0.5 \times 710) = 279.1kN > 200kN$ *(OK)* *(8.67)*

5.5.4 Tension capacity of channel connected by welding

The channel section acts as a hanger whose section is CH230×75×26 in Grade S460 to take 150kN dead load and 500kN imposed load. The cross section of channel section is shown in following diagram. The hanger is welded to the web of the beam member, which support the floor slab as displayed. Design the tensile load carrying capacity of the hanger.





Solution

DESIGN LOAD

Factored tension force, $F_t = 1.4 \times 150 + 1.6 \times 500 = 1010kN$ *(Table 4.2)*

SECTION PROPERTIES

$D = 230mm$, $B = 75mm$, $T = 12.5mm$, $t = 6.5mm$, $A_g = 32.7cm^2$

TENSION CAPACITY

Design strength, $p_y = 460N/mm^2$ for $T \le 16mm$ *(Table 3.2)*

Gross area of the connected leg, $a_1 = 230 \times 6.5 = 1495mm^2$

Gross area of the unconnected legs, $a_2 = A_g - a_1 = 3270 - 1495 = 1775mm^2$ *(8.69)*

Tension capacity, $P_t = p_y(A_e - 0.3a_2) = 460 \times (3270 - 0.3 \times 1775) = 1259.3kN > F_t$ *(8.68)*

(OK)

Chapter 6 Restrained and Unrestrained beams

6.1 Introduction and uses of beam member

Beam member refers to structural element with loads along its length or under an action of transverse loads, making the member to be loaded principally by bending moment. Beam can be defined as a structural member to resist the transverse or lateral loads. A non-uniform moment will further create a shear along the beam which is then required to be checked for shear capacity.

In the most steel buildings, beams are used to support floors and they are then supported by columns. For typical applications, standard hot-rolled sections are used. Beams can be hot-rolled, cold-formed or fabricated from steel plates which is generally called plate girders. Hot rolled sections eliminate the need for welding steel plates but they are less flexible in adapting to various environments. Section classification system is further employed to make sure local plate buckling does not occur before the assumed moment capacity of a beam is reached or the local buckling effect is considered in the design. In some cases, stiffeners are used to reduce the breadth to thickness ratio for increasing the buckling strength of a plate section. A common practice is to fabricate stiffeners to locations under concentrated loads and supports to stiffen and strengthen locally the beam. The first part of this chapter discusses the design and in-plane behaviour of fully restrained beams with full lateral restraint and the second part of this chapter discusses the design of unrestrained beams where lateral-torsional buckling effect is required to be considered.

For beams supporting floor, beams with sufficiently large torsional stiffness such as beams of hollow sections or they are bent about the minor axis, the lateral movements are prevented such that lateral-torsional buckling is not a concern. Under this behavioral assumption, the beam can be designed as a restrained beam.

The common steel sections used in a beam member are shown in Figure 6.1 which include universal beam, compound, channel, tee, hollow, angle and other sections.

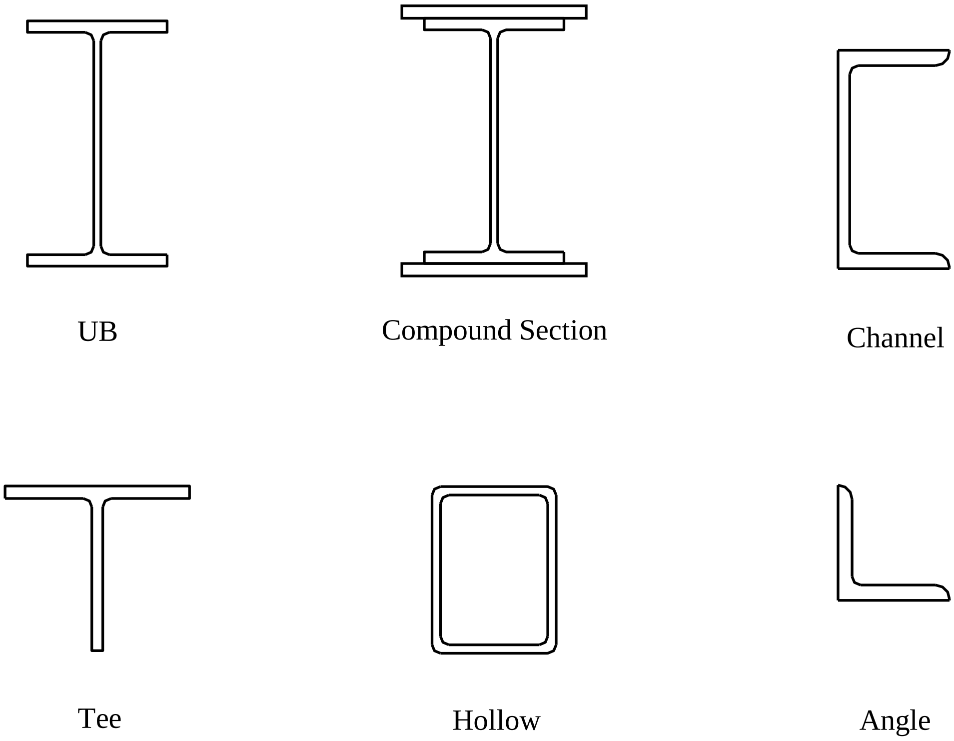


Figure 6.1 Typical sections used as beam members

Composing plate elements in a beam section are under tension, compression and combined tension and compression which respectively occur in tension flange, compression flange and web. The section classification system has different formulae to cater for the effect of boundary conditions on local plate buckling.

6.2 In-plane bending of beams

In-plane bending of beams refers to the condition of displacement of beam restrained in the loading plane. Thus, restrained beams are not affected by lateral-torsional buckling in the complete loading stage until failure. This condition can be achieved by the provision of lateral restraints along the beam that it is not allowed to deflect significantly out of the loading plane, or the length of beam is sufficiently short to warrant no out-of-plane displacement. Retrained beams are commonly used in practice for support of floors and panels which restrain the beams against its out-of-plane displacement.

6.2.1 In-plane bending of laterally restrained beams

In design of restrained beams the effect of torsional and lateral-torsional buckling is ignored and assumed to be restrained by lateral bracing or floors on top of the beams. The structural adequacy of a restrained beam is checked for section classification, adequacy of lateral restraints, shear and bending capacities, web bearing and buckling, deflection and vibration.

The in-plane bending behaviour of a beam is shown in Figure 6.2. The moment-curvature relationship of the beam is constrained in the loading plane and contained in the principal axis. Thus, the strength of a steel beam under in-plane bending is affected by material yielding only, which depends on the section properties and its yield stress p_y. The moment capacity of beam can be based on the use of the plastic or elastic section modulus for classes 1 to 4 sections of the beams. In the HK Code, the plastic section modulus is allowed for finding the moment capacity of a plastic or a compact section and the effective plastic section modulus or the elastic section modulus is used for a semi-compact section. For slender sections, the effective elastic section modulus shall be used for prevention of local buckling before attaining the design moment in a section.

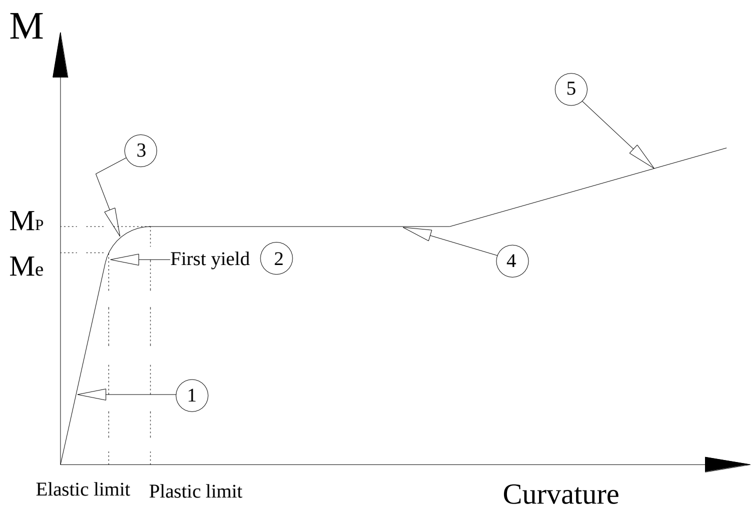


Figure 6.2 In-plane moment-curvature relationships of a beam

The stress distribution across a section in a beam under an increasing moment from an elastic stage to a plastic stage is shown in Figure 6.3. The behaviour is a conceptual and an idealized case assumed for design and the effect of residual stress is not considered here. The first yield moment M_y and plastic moment M_p in the beam section can be determined respectively by the product of elastic and plastic section modulus and the design strength.

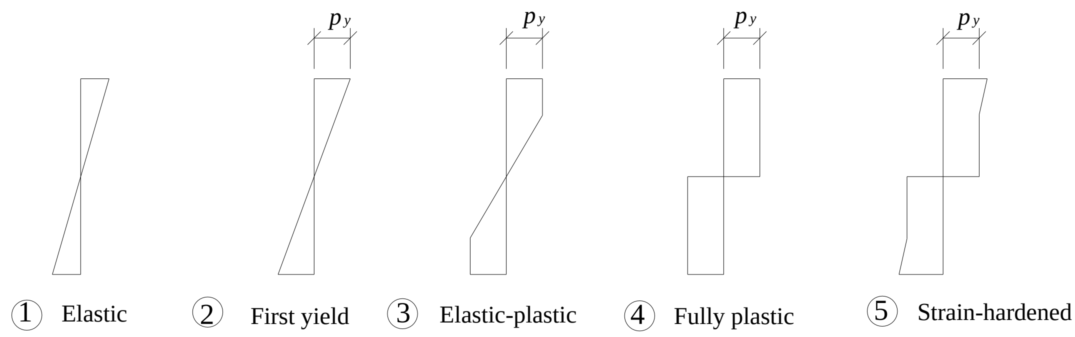


Figure 6.3 Stress distributions for the moment-curvature relationships

When a beam is loaded under an increasing bending moment in the elastic range, the strain and stress are linear before reaching the elastic limit. The elastic moment M_y refers to the moment where the extreme fiber of the beam section reaches its yield stress σ_y or p_y in general and the moment is then equal to $M_y = p_y Z$ in which Z is the elastic section modulus of the beam section. It is indicated by stages ①and ② in Figure 6.3. If moment is further increased after yielding at the extreme fiber and no local buckling occurs on the plate section of beam, the plastic moment of the beam, M_p, will be reached. The value of plastic moment is then equal to the product of design strength and plastic modulus as $M_p = p_y S$ in which S is the plastic modulus. This is referred as stage ④in Figure 6.3.

The transition from elastic to plastic moment is indicated in curve ③in Figure 6.2. The rotational capacity of the section is not a concern in elastic design, but it may be required for checking in plastic design in which structural resistance can be beyond the first plastic range.

6.2.2 In-plane elastic analysis of beams

When a beam is under uniform moment, the strain at a fiber with distance y from the centroid is equal to $\varepsilon = -\kappa y$ where $\kappa = \dfrac{d\theta}{dz}$.

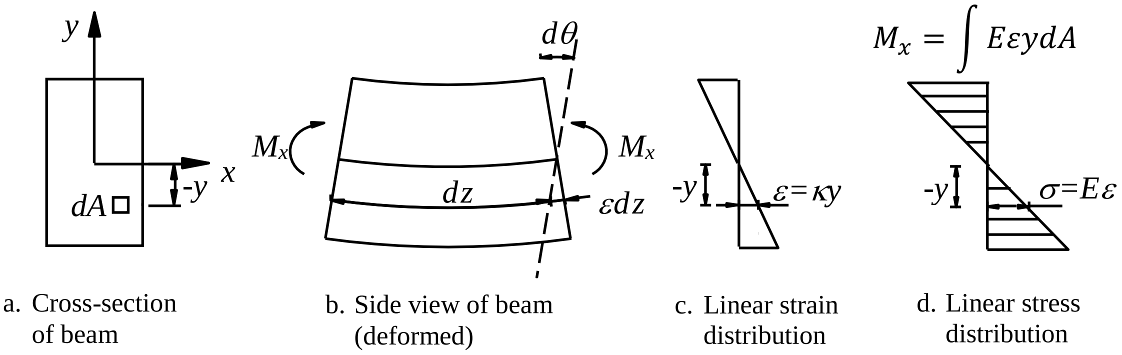


Figure 6.4 Stress distribution across a section by the elastic beam theory

The elastic stress can be obtained from the bending moment and the elastic modulus as,

$$\sigma = \frac{M_x y_{\max}}{I_x} = \frac{M_x}{Z_x} \tag{6.1}$$

where Z_x is the elastic section modulus and M_x is the bending moment about the major principal axis.

The moment capacity, M_{cx}, for a section is given by,

$$M_{cx} = p_y Z_x \tag{6.2}$$

Similarly, for bending about minor axis,

$$M_{cy} = p_y Z_y \tag{6.3}$$

in which M_{cx} and M_{cy} are respectively the moment capacity about the major x- and y-axes, p_y is the design strength and Z is the elastic section modulus.

6.2.3 In-plane plastic moment capacity of beams

When applied moment is increased further, the maximum moment capacity will be obtained from the completely yielded condition for the beam section, with half area yielded in tension and half in compression. Once the longitudinal strains of the fiber across a steel beam section exceeds the yield strain ε_y equal to p_y/E at elastic limit, the stress distribution is no longer linear under further loading but varies across the beam section as indicated in stage ③ in Figure 6.3. The section becomes elasto-plastic when resultant moment M exceeds the first yield moment as $M_y = p_y Z$ and inelastic bending stress distribution will occur. The section becomes fully plastic when the moment resultant M is equal to the full plastic moment. As no local buckling has been assumed when reaching the plastic moment capacity and therefore only plastic and compact sections are allowed to adopt the plastic modulus in calculation for the moment capacity. The plastic moment at fully yielded section can be written as,

$$M_p = p_y S \tag{6.4}$$

in which S is the plastic section modulus about the centroid axis shown in Figure 6.5, which divides the cross-section into two equal areas. This plastic neutral axis can be taken as the centroidal axis of cross-section in the absence of axial force. Equation (6.4) is derived from the force equilibrium of the fully plastic stress distribution over the section as shown in Equation (6.5). Force equilibrium is achieved so that the compression and tension on the beam section are same, which leads to the moment resultant M_p of the fully plastic stress distribution.

$$M_p = \left(p_y \sum_i A_i y_i \right)_{tension} + \left(p_y \sum_i A_i y_i \right)_{compression} = p_y S \tag{6.5}$$

in which A_i and y_i are the area of cross-section and its corresponding distance from centroid to plastic neutral axis respectively. $\sum A_i y_i$ is the first moment of area calculated using the centroidal axis of equal area of the section, which is the same as the plastic section modulus S.

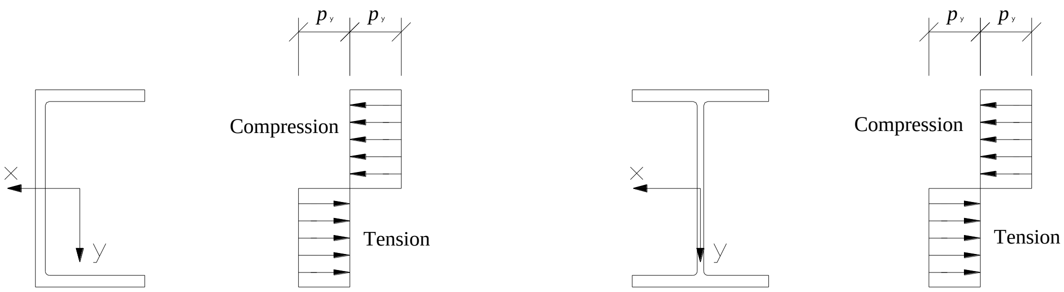


Figure 6.5 Stress distribution on symmetric beam section

Summarizing the moment capacities for sections affected and unaffected by local buckling and under low shear load condition that shear does not have interaction with the moment capacity of beams, we have the following expressions of moment capacities.

$$\begin{aligned} M_{cx} &= p_y S_x \le 1.2 p_y Z_x \text{ for Class 1 plastic and Class 2 compact sections} \\ &= p_y Z_x \text{ or } p_y S_{x,eff} \text{ for Class 3 semi-compact sections} \\ &= p_y Z_{x,eff} \text{ or } p_{yr} Z_x \text{ for Class 4 slender sections} \end{aligned} \tag{6.6}$$

Similarly, for bending about minor axis,

$$
\begin{aligned}
M_{cy} &= p_y S_y \le 1.2 p_y Z_y \text{ for Class 1 plastic and Class 2 compact sections} \\
&= p_y Z_y \text{ or } p_y S_{y,eff} \text{ for Class 3 semi-compact sections} \\
&= p_y Z_{y,eff} \text{ or } p_{yr} Z_y \text{ for Class 4 slender sections}
\end{aligned}
\tag{6.7}
$$

in which M_{cx} and M_{cy} are respectively the moment capacity about the x- and y-axes. In the effective stress method, p_y should be reduced to p_{yr} to account for the local plate buckling effect.

6.2.4 Shear capacity of beams

Shear force F_v across a section transverse to the longitudinal x-axis of a beam creates shear stresses. The distribution of the vertical shear stresses can be determined by considering the horizontal force equilibrium of an infinitesimal element of the beam in longitudinal direction shown in Figure 6.6. The horizontal shear stress τ_{zy} is in equilibrium with the horizontal change in bending stress $\delta\sigma$. Assuming an infinitesimal length for the section with δx being small, the force equilibrium is obtained by equating the horizontal force to zero.

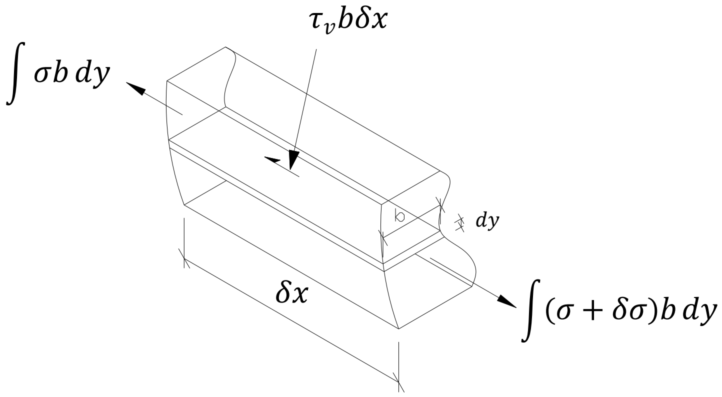


Figure 6.6 Horizontal force equilibrium of an element

$$\tau_v b\delta x = \int \delta\sigma\, b\, dy \tag{6.8}$$

$$\tau_v = \frac{\delta M}{\delta x}\int \frac{bydy}{bI} = \frac{VA\bar{y}}{bI} \tag{6.9}$$

in which $A\bar{y}$ is first moment of area for the area above the considered sectional cut, b is the width of the considered section cut and I is the second moment of area of the complete cross section.

Steel structural members are mostly made of thin-walled sections with thickness t much smaller than other dimensions like width B and depth D. As can be seen in

Equation (6.9), shear stress increases significantly at the web zone, because of the smaller thickness of web when compared with the width of flanges in a typical I-section. Also, the first moment of area above the section is larger when the section cut for shear is close to the centroid. It is therefore a good approximation to assume the web to take shear with the flanges to take moment. This assumption is sufficiently accurate with great simplification in computation. With an additional simplification made for the uniformity of shear stress across the web that the design shear stress is simply taken as the shear force divided by the web area ($d \times t$ or $D \times t$, depending on whether the section is welded or rolled section) as,

$$\tau_v = \frac{F_v}{Dt} \tag{6.10}$$

where t is the thickness of web and D is the overall depth of the I-section beam. This expression is a basis of the design formula of shear capacity of the web element.

For simplicity in design, flanges are assumed to take moment and web resists shear.

For shear areas of other sections, the following expressions from *Clause 8.2.1* of HK Code can be used.

Rolled I, H and channels sections	tD
Welded I-sections	td
Rolled and welded rectangular hollow section	$2td$
Rolled and welded T-sections	$t(D-T)$
Circular hollow sections	$0.6A$
Solid rectangular sections	$0.9A$
Others	$0.9A_O$

in which A is the cross-sectional area, A_O is the area of the rectilinear element in the cross-section with largest dimension parallel to the design shear force direction, B is the overall breadth, D is the overall depth, d is the depth of the web, T is the flange thickness, t is the web thickness.

The web in the thin-walled section behaves elastically in shear until first yielding occurs at $\tau_v = \frac{p_y}{\sqrt{3}}$. The factor $\frac{1}{\sqrt{3}}$ here is derived from the von Mises yield criterion for metal as $\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau^2 = \sigma^2$, in which σ_x, σ_y and τ_v are respectively the normal stress in x- and y-axes and shear stress. Therefore, the yield stress in shear τ can be determined from the yield stress in tension, p_y for design approach. Thus, using the design yield strength and shear area, we have the shear capacity V_c of a section as,

$$V_c = \frac{p_y A_v}{\sqrt{3}} \tag{6.11}$$

6.2.5 Interaction between shear and bending

Interaction between shear can be ignored when the external shear force is not greater than 0.6 of the shear capacity (i.e. $V \leq 0.6V_c$) and Equations (6.6) and (6.7) can be used. When the condition of low shear load is violated, the bending capacity of the beam shall be reduced using the following equations.

For Class 1 plastic and Class 2 compact sections:

$$M_c = p_y(S - \rho S_v) \leq 1.2 p_y(Z - \rho S_v / 1.5) \tag{6.12}$$

For Class 3 semi-compact sections:

$$M_c = p_y(Z - \rho S_v / 1.5) \tag{6.13}$$

or $$M_c = p_y(S_{eff} - \rho S_v / 1.5) \tag{6.14}$$

For Class 4 slender sections:

$$M_c = p_y(Z_{eff} - \rho S_v / 1.5) \tag{6.15}$$

in which

S_V is the plastic modulus of shear area A_V

ρ is given by $\left(\dfrac{2F_v}{V_c} - 1\right)^2$

V_c is the shear capacity;

F_v is the design shear.

When the web slenderness d/t is larger than 70ε for hot-rolled sections, or 62ε for welded sections, the web section should be checked for web shear buckling.

6.2.6 Web bearing, buckling and shear buckling

For typical beams under dominant bending, thickness of flanges is normally much greater than webs in order to economically put material away from the centre of the section to increase the second moment of area under the same weight. However, capacity of webs normally of smaller thickness should be ensured to avoid failure in taking shear loads. Webs shall be designed against four major modes of failure as shear buckling, web buckling under concentrated compressive loads or at supports, web bearing and crushing. A thorough discussion of web design should be referred to Chapter 9.

6.2.7 Serviceability limit state considerations

In addition to checking to the ultimate limit state for safety, a beam shall also be checked to satisfy the serviceability limit state to avoid unsatisfactory functional use. Deflection and vibration serviceability limit states are two common criteria for ensuring the stiffness of the beam will not hinder its serviceable use.

Excessive deflection leads to human discomfort and cracking of finishes. A typical value for control under normal use is span/360 and *Table 5.1* in the HK Code should be referred for other cases. In general, only unfactored live load is required to consider in calculation of deflection. Pre-cambering during fabrication of a beam to introduce an initial upward deflection can be exercised to reduce deflection from dead load to avoid ponding. Table 6.1 below shows typical maximum deflection of a beam under different load cases and conditions.

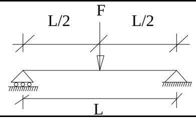
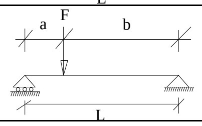
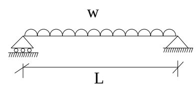
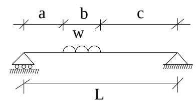
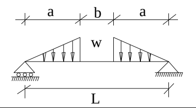
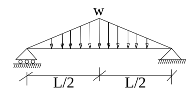
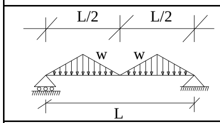
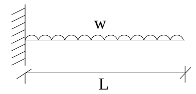
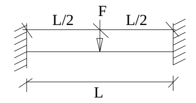
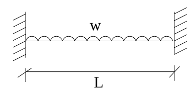
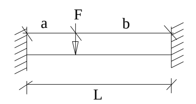
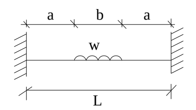
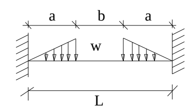
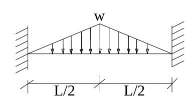
	Deflection	M_{sag}	M_{hog}
	$\frac{FL^3}{48EI}$	$\frac{FL}{4}$	0
	$\frac{FL^3}{48EI}\left[\frac{3a}{L}-4\left(\frac{a}{L}\right)^3\right]$	$\frac{Fab}{L}$	0
	$\frac{5\omega L^4}{384EI}$	$\frac{\omega L^2}{8}$	0
	$\frac{\omega b}{384EI}\left[8L^3-4b^2L+b^3\right]$ if $a=c$	$\frac{\omega}{2}\left[\left(a+\frac{b}{L}\left(\frac{b}{2}+c\right)\right)^2-a^2\right]$	0
	$\frac{\omega a^2}{120EI}\left[16a^2-20ab+5b^2\right]$	$\frac{\omega a^2}{3}$	0
	$\frac{\omega L^4}{120EI}$	$\frac{\omega L^2}{12}$	0
	$\frac{\omega L^4}{146.28EI}$	$\frac{\omega L^2}{16}$	0
	$\frac{\omega L^4}{8EI}$	0	$-\frac{\omega L^2}{2}$
	$\frac{FL^3}{192EI}$	$\frac{FL}{8}$	$-\frac{FL}{8}$
	$\frac{\omega L^4}{384EI}$	$\frac{\omega L^2}{24}$	$-\frac{\omega L^2}{12}$
	$\frac{2Fa^2b^3}{3EI(3L-2a)^2}$	$\frac{2Fa^2b^2}{L^3}$	$-\frac{Fab^2}{L^2}$
	$\frac{\omega b}{384EI}\left(L^3+2L^2a+4La^2-8a^3\right)$	$\frac{\omega b}{24L}\left(3L^2-3bL+b^2\right)$	$-\frac{\omega b}{24L}\left(3L^2-b^2\right)$
	$\frac{\omega a^3}{480EI}(15L-16a)$	$\frac{\omega a^3}{4L}$	$-\frac{\omega a^2}{12L}(4L-3a)$
	$\frac{0.7\omega L^4}{384EI}$	$\frac{\omega L^2}{32}$	$-\frac{5\omega L^2}{96}$

Table 6.1 Deflections and moments under different loading cases

Apart from serviceability check against deflection, vibration and natural frequency shall be checked and determined. A nominal minimum vibration frequency of 5 hertz is required to avoid excessive human induced vibration but other values of vibration frequency can be used with justifications. For structures under wind, a minimum 1 hertz is normally taken to ensure the structural vibration frequency is not sensitive to wind excitation.

6.3 Design procedure for in-plane bending of beams

The following steps I to VII is a typical procedure for design of beams with full lateral restraints. The steps IVa and IVb refer to two conditions of low and high shear loads.

I STEEL GRADE AND SECTION CLASSIFICATION

A beam section is classified and a reduced section property in the effective width method and the reduced stress in the effective stress method may be adopted to limit the sectional strength due to local plate buckling before reaching the designed capacity of the beam.

II LATERAL RESTRAINT

To prevent torsional and lateral movement of a beam, the compression flange of the beam should be restrained laterally while the vertical movement of the beam is permitted. The adequacy of a restraining member required in the HK Code is that the restraining member is capable of taking 2.5% of the force in the compression flange of the beam. The compression force in flange can be determined simply by dividing the maximum moment of the beam by the distance between the top and bottom flanges as,

$$F_{res} = \frac{M_{\max}}{D - T} \tag{6.16}$$

in which F_{res} is the force in flange used for designing the restraining members or ties, $M_{\max}$ is the maximum bending moment in the beam under the factored load, D and T are the depth and flange thickness of the beam.

III SHEAR CAPACITY

The shear capacity of the section shall be checked to be larger than or equal to the external shear as,

$$V_c = \frac{p_y A_v}{\sqrt{3}} \geq F_v \tag{6.17}$$

in which p_y is design strength in shear of the steel material which is equal to $\frac{1}{\sqrt{3}}$ of the design strength in tension p_y and A_v is shear area normally taken as the web area.

IVa MOMENT CAPACITY UNDER LOW SHEAR LOAD

When the applied shear is equal to or less than 60% of the permissible shear in a beam, i.e.

$$F_v \leq 0.6V_c \tag{6.18}$$

the low shear condition applies and the interaction between shear and moment can be ignored.

For plastic or compact section of beam, the plastic analysis, the moment capacity M_c for plastic and compact section are given as,

$$M_c = p_y S \leq 1.2 p_y Z \tag{6.19}$$

For semi-compact section, the full plastic moment capacity cannot be developed on beam. The moment capacity M_c should be based on the elastic modulus or effective plastic modulus and expressed as,

$$M_c = p_y Z \text{ or } M_c = p_y S_{eff} \tag{6.20}$$

For slender section, two approaches are also allowed to deal with this effect. One is the effective stress and the other is the effective section approach. Thus the moment capacity M_c is written as,

$$M_c = p_y Z_{eff} \tag{6.21}$$
$$M_c = p_{yr} Z \tag{6.22}$$

in which Z_{eff} and p_{yr} are effective elastic section modulus and reduced yield stress respectively. The effective section approach is more economical but involves more complex calculation for different load cases.

IVb MOMENT CAPACITY UNDER HIGH SHEAR LOAD

When the shear force is higher than $0.6V_c$ and not satisfying Equation (6.18) at a section, the moment capacity is lower due to interaction of shear and moment. The reduced moment capacity can be obtained from a reduction of plastic section modulus from S to $(S - \rho S_v)$ for plastic section as,

$$M_c = p_y (S - \rho S_v) \leq 1.2 p_y (Z - \rho S_v / 1.5) \tag{6.23}$$

For semi-compact section, the reduced moment capacities are determined as,

$$M_c = p_y (Z - \rho S_v / 1.5) \tag{6.24}$$
$$M_c = p_y (S_{eff} - \rho S_v / 1.5) \tag{6.25}$$

For slender section, the reduction moment capacity is given by,

$$M_c = p_y (Z_{eff} - \rho S_v / 1.5) \tag{6.26}$$

V WEB BEARING AND BUCKLING

At support and at location of a concentrated load, the web bearing capacity and buckling resistance should be checked. If no stiffeners are added, slenderness of webs should not be greater than 62ε and 70ε for welded and rolled sections respectively.

Webs shall also be checked against bearing and buckling due to concentrated loads to Chapter 9.

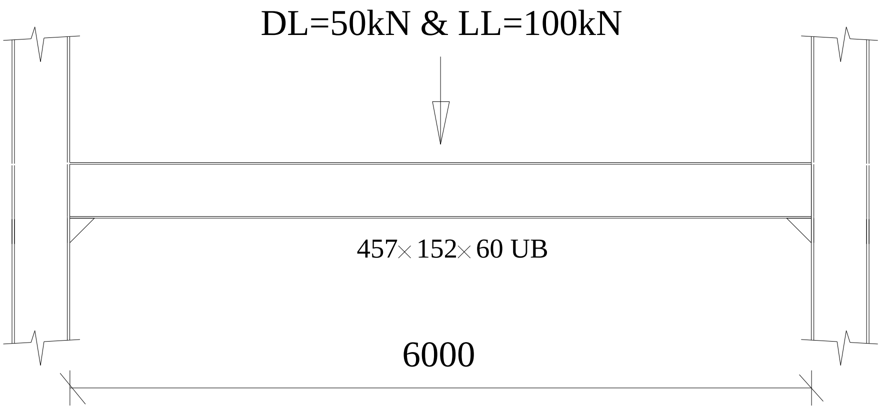
VI ADDITIONAL CHECKS TO OTHER LIMIT STATES

The serviceability of the beam is checked against relevant serviceability limit states to avoid non-functional use.

6.4 Worked examples

6.4.1 Simply supported beam under mid-span point load

Check the adequacy of a beam under an unfactored imposed load of 100kN and an unfactored dead load of 50kN at mid-span. The beam is simply supported of span 6m. The beam is a 457×152×60 UB of Grade S275 steel.



<u>Solution</u>

DESIGN LOAD

Factored point load, $P = 1.4 \times 50 + 1.6 \times 100 = 230kN$ *(Table 4.2)*

Maximum shear, $V = \dfrac{230}{2} = 115kN$

Maximum moment, $M_x = \dfrac{230 \times 6}{4} = 345kNm$

SECTION PROPERTIES

$D = 454.6mm$, $B = 152.9mm$, $t = 8.1mm$, $T = 13.3mm$, $d = 407.6mm$, $I_x = 25500cm^4$,

$Z_x = 1120cm^3$, $S_x = 1290cm^3$

SECTION CLASSIFICATION

Design strength, $p_y = 275N/mm^2$ for $T \le 16mm$ *(Table 3.2)*

$\varepsilon = \sqrt{\dfrac{275}{275}} = 1$ *(Table 7.1 Note b)*

Plastic limiting value of b/T for outstand flange of an I-section is 9ε

$\dfrac{b}{T} = \dfrac{152.9}{2 \times 13.3} = 5.75 \le 9 \times 1 = 9$ *(Table 7.1)*

$\therefore$ flange is plastic

Plastic limiting value of d/t for web of an I-section with neutral axis with mid-depth is 80ε

$\dfrac{d}{t} = \dfrac{407.6}{8.1} = 50.3 \le 80 \times 1 = 80$ *(Table 7.1)*

$\therefore$ web is plastic

$\therefore$ the section is Class 1 plastic

SHEAR CAPACITY

Shear area, $A_v = tD = 8.1 \times 454.6 = 3682mm^2$ *(Clause 8.2.1)*

Shear capacity, $V_c = \dfrac{p_y A_v}{\sqrt{3}} = \dfrac{275 \times 3682}{\sqrt{3}} = 584.6kN > V$ *(OK)* *(8.1)*

MOMENT CAPACITY

$V = 115kN \leq 0.6V_c = 350.8kN$

$\therefore$ it is low shear condition *(Clause 8.2.2.1)*

Moment capacity, $M_{cx} = p_y S_x \leq 1.2 p_y Z_x$ *(8.2)*

$$= 275 \times 1290 \times 10^3 \leq 1.2 \times 275 \times 1120 \times 10^3$$

$$= 354.8kNm \leq 369.6kNm$$

$$> M_x \quad (OK)$$

DEFLECTION

Unfactored imposed load, $P = 100kN$

Maximum deflection due to imposed load,

$$\delta = \frac{PL^3}{48EI_x} \leq \frac{L}{360}$$ *(Table 5.1)*

$$= \frac{100 \times 10^3 \times 6000^3}{48 \times 205000 \times 25500 \times 10^4} \leq \frac{6000}{360}$$

$$= 8.6mm \leq 16.7mm \quad (OK)$$

6.4.2 Design of a cantilever

Check the adequacy of a cantilever of 2.5m under an unfactored dead load of 5kN/m and an unfactored imposed load of 10kN/m along the member. The cantilever is a 254×102×25 UB of Grade S275 steel.

<u>Solution</u>

DESIGN LOAD

Factored point load, $\omega = 1.4 \times 5 + 1.6 \times 10 = 23kN/m$ *(Table 4.2)*

Maximum shear, $V = 23 \times 2.5 = 57.5kN$

Maximum moment, $M_x = \frac{1}{2} \times 23 \times 2.5^2 = 71.9kNm$

SECTION PROPERTIES

$D = 257.2mm$, $B = 101.9mm$, $t = 6.0mm$, $T = 8.4mm$, $d = 225.2mm$, $I_x = 3410cm^4$, $Z_x = 266cm^3$, $S_x = 306cm^3$

SECTION CLASSIFICATION

Design strength, $p_y = 275N/mm^2$ for $T \le 16mm$ *(Table 3.2)*

$\varepsilon = \sqrt{\frac{275}{275}} = 1$ *(Table 7.1 Note b)*

Plastic limiting value of b/T for outstand flange of an I-section is 9ε

$\frac{b}{T} = \frac{101.9}{2 \times 8.4} = 6.07 \le 9 \times 1 = 9$ *(Table 7.1)*

$\therefore$ flange is plastic

Plastic limiting value of d/t for web of an I-section with neutral axis with mid-depth is 80ε

$\frac{d}{t} = \frac{225.2}{6} = 37.5 \le 80 \times 1 = 80$ *(Table 7.1)*

$\therefore$ web is plastic

$\therefore$ the section is Class 1 plastic

SHEAR CAPACITY

Shear area, $A_v = tD = 6 \times 257.2 = 1543mm^2$ *(Clause 8.2.1)*

Shear capacity, $V_c = \frac{p_y A_v}{\sqrt{3}} = \frac{275 \times 1543}{\sqrt{3}} = 245.0kN > V$ *(OK)* *(8.1)*

MOMENT CAPACITY

$V = 57.5kN \le 0.6V_c = 147.0kN$ *(Clause 8.2.2.1)*

$\therefore$ it is low shear condition

Moment capacity, $M_{cx} = p_y S_x \le 1.2 p_y Z_x$ *(8.2)*

$$= 275 \times 306 \times 10^3 \le 1.2 \times 275 \times 266 \times 10^3$$
$$= 84.2\,kNm \le 87.8kNm$$
$$> M_x \text{ (OK)}$$

DEFLECTION

Unfactored imposed load, $\omega = 10 kN/m$

Maximum deflection due to imposed load,

$$\delta = \frac{\omega L^4}{8EI_x} \le \frac{L}{180} \quad \text{(Table 5.1)}$$

$$= \frac{10 \times 2500^4}{8 \times 205000 \times 3410 \times 10^4} \le \frac{2500}{180}$$

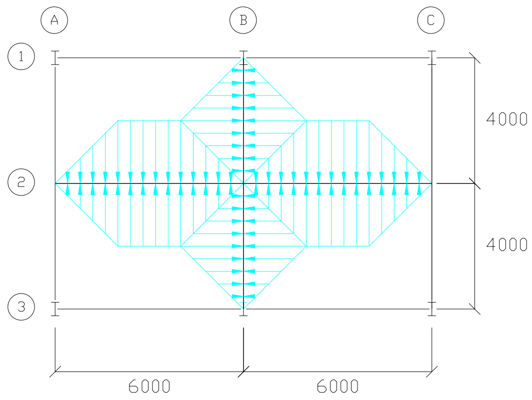
$$= 7.0mm \le 13.9mm \text{ (OK)}$$

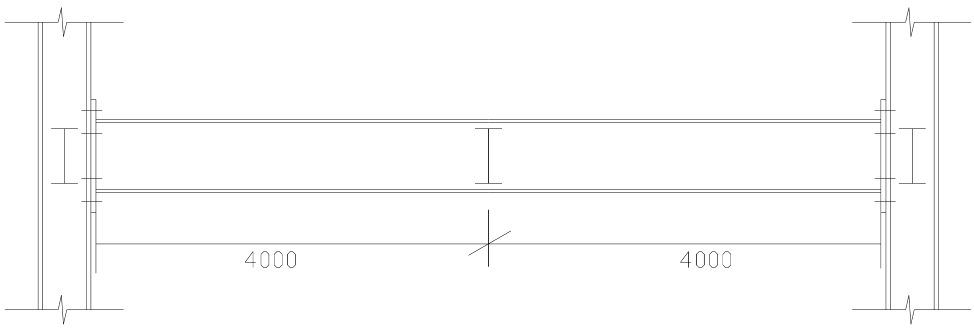
6.4.3 Design of beam in two way floor

The concrete floor system is supported by the primary and secondary steel beams as shown below. The primary beam at gridline Ⓑ is under consideration. The spread of load to the designed beam member is assumed two-way as shown. Simple connections are used to allow sufficient rotations and ductility. The design loads are given below. Design the primary beam in gridline Ⓑ using the section 686×254×140 UB of Grade S355 steel to the ultimate and serviceability limit states.

Dead load, $\omega_Q = 6kPa$

Imposed load, $\omega_G = 7.5kPa$





Solution

DESIGN LOAD

Factored distributed load on floor, $p = 1.4\times 6 + 1.6\times 7.5 = 20.4kPa$ *(Table 4.2)*

Maximum factored distributed load on beam, $w = 20.4\times 4 = 81.6\,kN/m$

Factored point load from secondary beam, $P = 20.4\times\frac{(3+1)\times 2}{2}\times 4 = 326.4kN$

Maximum shear, $V = \frac{1}{2}\times 81.6\times 4 + \frac{1}{2}\times 326.4 = 326.4kN$

Maximum moment, $M_x = \frac{1}{16}\times 81.6\times 8^2 + \frac{1}{4}\times 326.4\times 8 = 979.2kNm$

SECTION PROPERTIES

$D = 683.5mm$, $B = 253.7mm$, $t = 12.4mm$, $T = 19.0mm$, $d = 615.1mm$, $I_x = 136300cm^4$,

$Z_x = 3990cm^3$, $S_x = 4560cm^3$

SECTION CLASSIFICATION

Design strength, $p_y = 345N/mm^2$ for $16mm < T \le 40mm$ *(Table 3.2)*

$\varepsilon = \sqrt{\frac{275}{345}} = 0.89$ *(Table 7.1 Note b)*

Plastic limiting value of b/T for outstand flange of an I-section is 9ε

$\frac{b}{T} = \frac{253.7}{2\times19} = 6.68 \le 9\times0.89 = 8.01$ *(Table 7.1)*

$\therefore$ flange is plastic

Plastic limiting value of d/t for web of an I-section with neutral axis at mid-depth is 80ε

$\frac{d}{t} = \frac{615.1}{12.4} = 49.6 \le 80\times0.89 = 71.2$ *(Table 7.1)*

$\therefore$ web is plastic

$\therefore$ the section is Class 1 plastic

SHEAR CAPACITY

Shear area, $A_v = tD = 12.4\times683.5 = 8475mm^2$ *(Clause 8.2.1)*

Shear capacity, $V_c = \frac{p_y A_v}{\sqrt{3}} = \frac{345\times8475}{\sqrt{3}} = 1688.1kN > V$ *(OK)* *(8.1)*

MOMENT CAPACITY

$V = 326.4kN \le 0.6V_c = 1012.9kN$ *(Clause 8.2.2.1)*

$\therefore$ it is low shear condition

Moment capacity, $M_{cx} = p_y S_x \le 1.2 p_y Z_x$ *(8.2)*

$$= 345\times4560\times10^3 \le 1.2\times345\times3990\times10^3$$
$$= 1573.2kNm \le 1651.9kNm$$
$$> M_x \ (OK)$$

The beam supports the floor slab, which provides a full lateral restraint to the beam. Thus, the beam is not required to be checked for lateral-torsional buckling.

DEFLECTION

Maximum unfactored imposed load, $\omega = 7.5\times4 = 30kN/m$

Unfactored imposed point load from secondary beam, $P = 7.5\times\frac{(3+1)\times2}{2}\times4 = 120kN$

Maximum deflection due to imposed load,

$$\delta = \frac{PL^3}{48EI_x} + \frac{\omega L^4}{146.28EI_x} \le \frac{L}{360}$$ *(Table 5.1)*

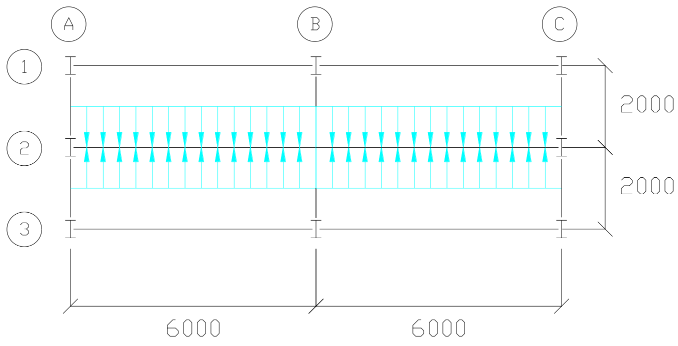
$$= \frac{120\times10^3\times8000^3}{48\times205000\times136000\times10^4} + \frac{30\times8000^4}{146.28\times205000\times136000\times10^4} \le \frac{8000}{360}$$

$$= 7.6mm \le 22.2mm \ (OK)$$

6.4.4 Design of beam at the one way typical floor system

An one-way floor system is illustrated in the figure below. A 12m long primary beam of section 457×152×52 UB of Grade S275 is simply supported in gridline ②. Check the structural adequacy of the primary beam. When the aspect ratio of concrete slab is more than 3, the loading from slab spreads to the beam member can be considered in one direction only. The loading applied including self weight of beam member on the floor system are tabulated as follows:

Services load	$0.6kPa$
Imposed load	$3.0kPa$
Finish load	$0.6kPa$
Dead load	$0.5kPa$
Wind load	$1.2kPa$



Solution

DESIGN LOAD

Load combination 1,

$$\omega = 1.4\omega_Q + 1.6\omega_G = [1.4\times(0.5+0.6+0.6)+1.6\times3]\times2 = 14.36\,kN/m$$ *(Table 4.2)*

Load combination 2,

$$\omega = 1.4\omega_Q + 1.4\omega_L = [1.4\times(0.5+0.6+0.6)+1.4\times1.2]\times2 = 8.12\,kN/m$$ *(Table 4.2)*

Load combination 3,

$$\omega = 1.2\omega_Q + 1.2\omega_G + 1.2\omega_W = 1.2\times(0.5+0.6+0.6+3+1.2)\times2 = 14.16\,kN/m$$ *(Table 4.2)*

$\therefore$ load combination 1 is critical

Maximum shear force, $V = \frac{1}{2}\times14.36\times12 = 86.2kN$

Maximum bending moment, $M_x = \frac{1}{8}\times14.36\times12^2 = 258.5kNm$

SECTION PROPERTIES

$D = 449.8mm$, $B = 152.4mm$, $t = 7.6mm$, $T = 10.9mm$, $d = 407.6mm$, $I_x = 21400cm^4$,

$Z_x = 950cm^3$, $S_x = 1100cm^3$

SECTION CLASSIFICATION

Design strength, $p_y = 275N/mm^2$ for $T \le 16mm$ *(Table 3.2)*

$$\varepsilon = \sqrt{\frac{275}{275}} = 1$$ *(Table 7.1 Note b)*

Plastic limiting value of b/T for outstand flange of an I-section is 9ε

$$\frac{b}{T} = \frac{152.4}{2\times10.9} = 7.99 < 9\times1 = 9$$ *(Table 7.1)*

$\therefore$ flange is plastic

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Plastic limiting value of d/t for web of an I-section with neutral axis at mid-depth is 80ε

$$\frac{d}{t} = \frac{407.6}{7.6} = 53.6 \leq 80 \times 1 = 80 \qquad \textit{(Table 7.1)}$$

$\therefore$ web is plastic

$\therefore$ the section is Class 1 plastic

SHEAR CAPACITY

Shear area, $A_v = tD = 7.6 \times 449.8 = 3418 mm^2$ *(Clause 8.2.1)*

$$\text{Shear capacity, } V_c = \frac{p_y A_v}{\sqrt{3}} = \frac{275 \times 3418}{\sqrt{3}} = 542.7kN > V \ \textit{(OK)} \qquad \textit{(8.1)}$$

MOMENT CAPACITY

$V = 86.2kN \leq 0.6V_c = 325.6kN$ *(Clause 8.2.2.1)*

$\therefore$ it is low shear condition

$$\begin{aligned} \text{Moment capacity, } M_{cx} &= p_y S_x \leq 1.2 p_y Z_x \qquad \textit{(8.2)} \\ &= 275 \times 1100 \times 10^3 \leq 1.2 \times 275 \times 950 \times 10^3 \\ &= 302.5kNm \leq 313.5kNm \\ &> M_x \ \textit{(OK)} \end{aligned}$$

As the floor slab provides a full lateral restraint to the beam, thus there is no lateral-torsional buckling.

DEFLECTION

Unfactored uniform imposed load, $\omega = 3 \times 2 = 6kN/m$

Maximum deflection due to imposed load,

$$\delta = \frac{5}{384}\frac{\omega L^4}{EI_x} \leq \frac{L}{360} \qquad \textit{(Table 5.1)}$$

$$= \frac{5}{384} \times \frac{6.0 \times 12000^4}{205000 \times 21400 \times 10^4}$$

$$= 36.9mm > \frac{12000}{360} = 33.3mm \ \textit{(Not OK)}$$

Loading acting on this primary beam is relatively low, and the beam section is adequate to resist all bending moment and shear force. However, the design of beam is inadequate in deflection in serviceability limit state. Therefore, another section is chosen for deflection check. Try 457×152×60 UB.

SECTION PROPERTIES

$D = 454.6mm$, $B = 152.9mm$, $t = 8.1mm$, $T = 13.3mm$, $d = 407.6mm$, $I_x = 25500cm^4$,

$Z_x = 1120cm^3$, $S_x = 1290cm^3$

DEFLECTION

From the above result, maximum deflection due to imposed load,

$$\delta = 36.9 \times \frac{21400}{25500} \leq \frac{L}{360} \qquad \textit{(Table 5.1)}$$

$$= 31.0mm \leq \frac{12000}{360} = 33.33mm \ \textit{(OK)}$$

6.5 Design of unrestrained beams

When a beam member is under lateral load or moment without full lateral restraints along its length, it is then considered as a not fully restrained or simply an unrestrained beam. It is necessary to check the beam resistance with allowance for buckling effects. The lateral-torsional buckling behaviour of unrestrained beam can be viewed as the compression flange of the beam deflects out-of-plane due to compression induced from the bending moment. This phenomenon leads to the compression flange to buckle like a column with restraints provided by the lateral and twisting stiffness of the member. The buckling mode of a beam can be seen in Figure 6.7. As can be seen in the figure the failure of the beam is due to a combined action of twisting and lateral bending.

The design of an unrestrained beam member considers the segment of a beam as the portion lying between two lateral restraints for the compression flanges, so the length of beam can be considered as equal to distance between lateral restraints. The strength required for the lateral restraints is minimum 2.5% of the maximum force in the compression flanges.

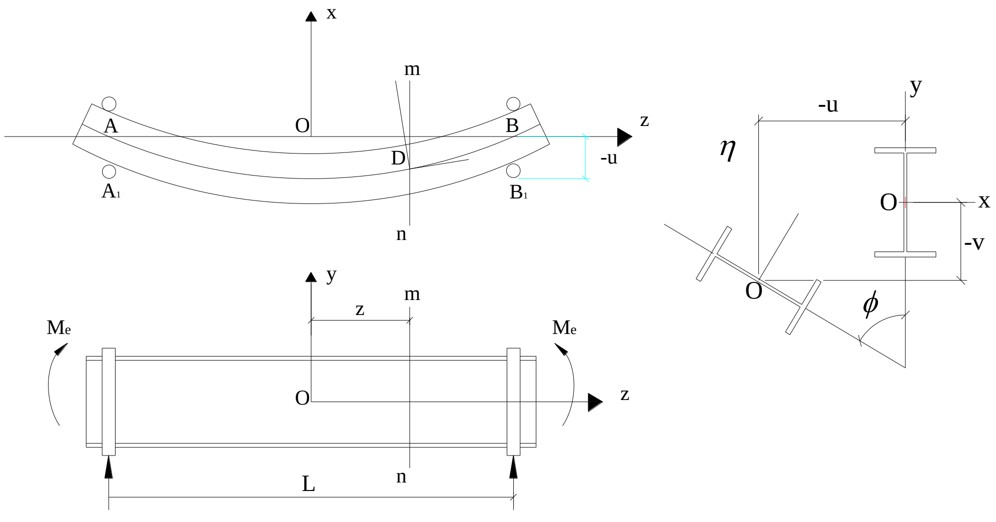


Figure 6.7 Lateral-torsional buckling of a beam member

The basic requirement for ensuring the structural adequacy of an unrestrained beam is to satisfy the following Equation (6.28) as,

$$m_{LT} M_x \leq M_b \tag{6.28}$$

in which m_{LT} is the equivalent uniform moment factor to account for moment variation along a beam or a beam segment, M_x is the maximum bending moment about major x-axis and M_b is the buckling resistance moment accounting for the effects of initial imperfection, material strength and lateral-torsional buckling effect.

6.5.1 Elastic Lateral-Torsional buckling of beams

In Section 6.3, the restrained beam is assumed to deform in the loading plane until it fails. In this Section, there are the cases of unrestrained beam in which a member buckles either by twisting or by a combination of bending laterally and twisting. This buckling mode is therefore named as lateral-torsional or flexural-torsional buckling. The member resistance of an unrestrained beam can be substantially less than its in-plane load carrying capacity. This lateral-torsional buckling is of importance in the design of beams without full lateral restraints along its member length.

For a simply supported elastic beam under uniform moment, the buckling moment can be obtained numerically by the finite element method or analytically by solving differential equation obtained as,

$$M_{cr} = \sqrt{\frac{\pi^2 EI_y}{L^2}} \sqrt{GJ + \frac{\pi^2 EI_w}{L^2}} \tag{6.29}$$

in which M_{cr} is the elastic lateral-torsional buckling moment, I_y is the second moment of area about the minor axis, J is the torsional constant, I_w is the warping constant, and L is the span of the simply supported beam. For beams under other boundary condition, the effective length L_E should be used in place of L.

It can be seen from Equation (6.29) that the buckling resistance of a beam depends on the following factors.

1 The effective length factor from the boundary condition and the span L,
2 The sectional properties as torsional constant J and second moment of area about the minor axis I_y,
3 The load height above shear center which affects the buckling resistance and its effect is considered by increasing the effective length in the HK Code,
4 The varying pattern of bending moment of the beam under consideration and,
5 The material design strength which is not included in Equation (6.29) for elastic flexural-torsional buckling moment.

The effects from point 1 to 3 are considered in a single term as the equivalent slenderness λ_{LT} and the influence of point 4 above is allowed for in the equivalent uniform moment factor m_{LT}. Detailing at connection affects the effective length and Figure 6.8 shows typical connections in beams.

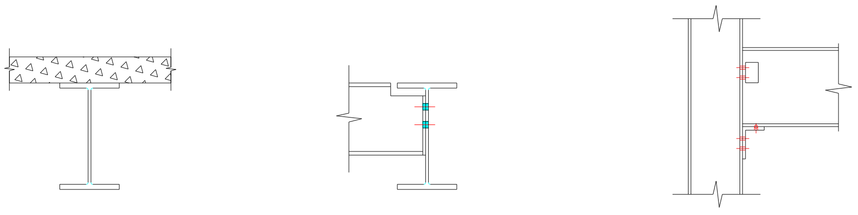


Figure 6.8 Different kinds of restraints for unrestrained beam member

6.5.2 Buckling resistance moment

The determination of buckling resistance moment, M_b in Equation (6.28), can be carried out with the information in the section tables. The equivalent slenderness λ_{LT} can be obtained as,

$$\lambda_{LT} = uv\lambda\sqrt{\beta_W} \tag{6.30}$$

in which u can be obtained from section design tables or taken as 0.9 conservatively, v is the slenderness factor obtained as,

$$v = \frac{1}{\left(1 + 0.05(\lambda/x)^2\right)^{0.25}} \tag{6.31}$$

where x is a torsional constant available from the section table or taken conservatively as D/T for I-beams with equal flanges, λ is the slenderness ratio equal to L_E/r_y, L_E is the effective length, r_y is the radius of gyration about minor axis of a section and β_w is section modulus ratio given by the followings.

$\beta_w = 1.0$ for Class 1 plastic section and Class 2 compact section,

$$\beta_w = \frac{Z_x}{S_x} \text{ or } \frac{S_{x,eff}}{S_x} \text{ for Class 3 semi-compact sections and,} \tag{6.32}$$

$$\beta_w = \frac{Z_{x,eff}}{S_x} \text{ for Class 4 slender sections} \tag{6.33}$$

With the use of λ_{LT} and the design strength p_y, the buckling strength p_b can be obtained from *Table 8.3* of HK Code. When the effective stress method is used, β_w can be taken as 1.0 as the local buckling effect has been accounted for in the use of reduced design strength p_{yr}. The buckling resistance moment, M_b, is then equal to $M_b = p_b S_x$ for Classes 1 and 2 plastic and compact sections, $M_b = p_b Z_x$ or $p_b S_{x,eff}$ for Class 3 semi-compact sections and $M_b = p_b Z_{x,eff}$ or $p_b \frac{p_{yr}}{p_y} Z_x$ for Class 4 slender sections.

6.5.3 Normal and Destabilizing loads

The formulae in the HK Code are based on the case of loads applied at the level of shear centre of the beam. If the load is applied above the shear centre of the beam and when the beam deflects laterally as shown in Figure 6.9, an additional torsional moment will be generated and makes the beam to buckle at a lower load than when the load is at the shear centre. The effect of destabilizing loading condition is considered in the HK Code by using a larger effective length factor as indicated in *Clause 8.3.4.1(d)* of HK Code.

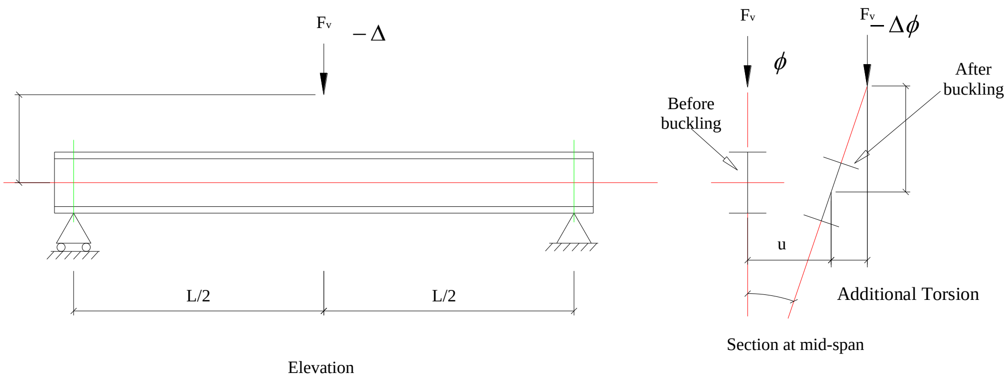


Figure 6.9 Destabilizing load in a beam

6.5.4 Effective length in an unrestrained beam

A stocky beam fails by action of moment attaining its plastic moment. On the other hand, a slender beam is failed by the action of moment closer to its elastic buckling moment. For beams of intermediate slenderness, the moment resistance is due to an elasto-plastic buckling by having part of the material in a section yields, leading to a reduction in effective sectional properties and also flexural-torsional buckling. Figure 6.10 shows the effect of buckling over a range of beam slenderness.

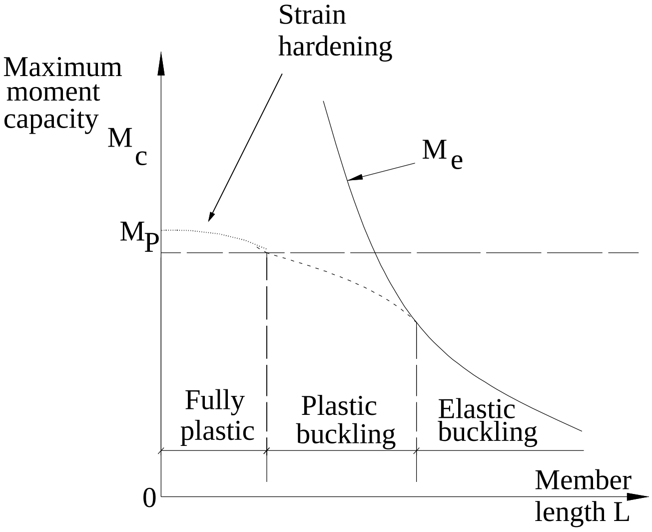


Figure 6.10 Moment capacity of beam with various member lengths

In Figure 6.10, the solid line is plotted by using Equation (6.29) for elastic lateral-torsional buckling of beams and the dotted line indicates the inelastic buckling moment of the beam with varying slenderness. It can be seen that both buckling and material yielding can affect the moment resistance of an unrestrained beam. A reduced bending buckling strength p_b is used to account for the lateral buckling effect of a rolled and a welded sections. For a very short beam, the moment capacity can be higher than plastic moment M_p due to the effect of strain-hardening and it is indicated in line ⑤ in Figure 6.2.

The typical values of effective length factor for beams without intermediate restraints, or the effective length ratio L_E/L_{LT} are given in Table 6.2 below where L_{LT} is the distance between supports. For unrestrained beams with intermediate restraints, the effective length L_E can be taken as the member length between the lateral restraints, which should be increased to $1.2L_E$ for destabilizing loading condition. The minimum resistance of the intermediate restraints should be taken as 2.5% of the maximum force in compression flanges which can be reduced by a factor $k_r = \sqrt{0.2 + \frac{1}{N_r}}$ with N_r equal to the number of restraining members sharing a common restraint.

Support condition	Normal load	Destabilising load
i. Lateral restraint, ii. Free to rotate on plan, iii. Torsional restraint.	$L_E=L_{LT}$	$L_E=1.2L_{LT}$
i. Lateral restraint, ii. Not free to rotate on plan, iii. Torsional restraint.	$L_E=0.8L_{LT}$	$L_E=1.2\times(0.8L_{LT})$
i. No lateral restraint, ii. Free to rotate on plan, iii. Partial torsional restraint.	$L_E=1.2L_{LT}+2D$	$L_E=1.2\times(1.2L_{LT}+2D)$

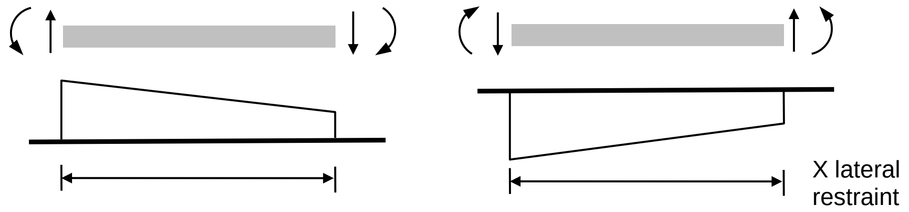
Table 6.2 Effective length of typical unrestrained beams

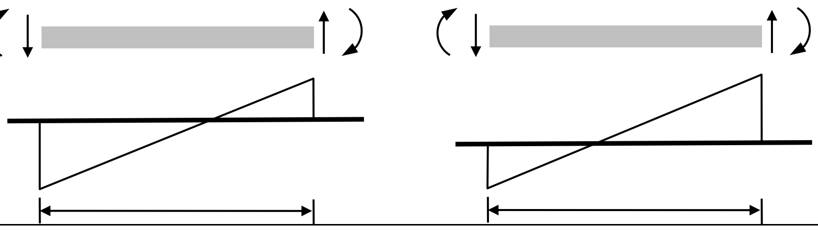
In most practical applications, it is generally considered adequate that the effective length of a cantilever is taken as twice the member length and of beam the same as member length. When the load is destabilizing, a factor of 1.2 should be multiplied to the effective length.

6.5.5 Equivalent uniform moment factor m_{LT}

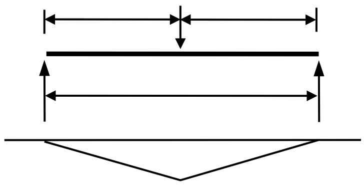
The formulae in the beam buckling check in the HK Code are based on the assumption of uniform moment causing the beam or beam segment under uniform compression on a flange. When a simply supported elastic beam is subjected to unequal end moments M and βM as shown in Table 6.3, the moment resistance of the unrestrained beam can be increased. The buckling resistance of unrestrained beam is determined by the formulae under uniform bending case causing uniform stress on compression flange. When the moment is non-uniform, the buckling resistance in terms of maximum bending moment can be increased, as part of the beams is not under the maximum moment. This effect can be considered by the use of the equivalent uniform moment factor m_{LT} which accounts for the effect of the non-uniform moment distribution along the major axis. The end moment ratio β varies from 1 (single curvature bending) to -1 (double curvature bending). The ratio for the end moments β and the equivalent uniform moment factor m_{LT} can be approximated and shown in *Table 8.4* of the HK Code reproduced below.

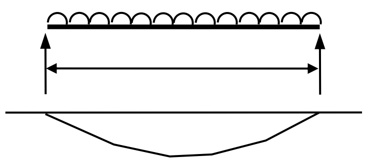
Segment with end moment only (values of m_{LT} from the formula for the general case)	β	m_{LT}
β positive	1.0	1.00
	0.9	0.96
	0.8	0.92
	0.7	0.88
	0.6	0.84
	0.5	0.80
	0.4	0.76
	0.3	0.72
	0.2	0.68
	0.1	0.64
β negative	0.0	0.60
	-0.1	0.56
	-0.2	0.52
	-0.3	0.48
	-0.4	0.46
	-0.5	0.44
	-0.6	0.44
	-0.7	0.44
	-0.8	0.44
	-0.9	0.44
	-1.0	0.44

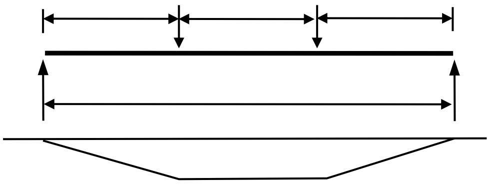




Special case (no intermediate lateral restraint)







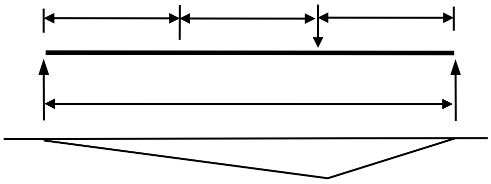


Table 6.3 Equivalent uniform moment factor m_{LT} for unrestrained beams under end moments and typical loads

For general case and suitable for use in computer program, the following formula can be used for evaluation of equivalent uniform moment factor.as given in Equation (6.34).

$$m_{LT} = 0.2 + \frac{0.15M_2 + 0.5M_3 + 0.15M_4}{M_{max}} \geq 0.44 \qquad (6.34)$$

in which M_2 and M_4 are moments at quarter points of the beam or the segment of a beam, M_3 is the moment at mid-span and M_{max} is the maximum bending moment

The equivalent uniform moment factor, m_{LT}, is taken as 1 for cantilever and the HKCode allows the same m_{LT} for normal and destabilizing loads. It is non-similar to the BS5950(2000) which assumes m_{LT} as 1 for destabilizing load. This use of unity m_{LT} implies a non-uniform factor of safety for beams under uniform and non-uniform moments and also an inconsistency with the case for beams under normal loads. The buckling design of unsymmetrical beams can be referred a non-linear finite element package for buckling analysis to determine the buckling moment.

6.6 Design procedures of unrestrained beams

The procedure for design of unrestrained beams can be summarized as follows.

I SECTION CLASSIFICATION AND DETERMINATION OF DESIGN STRENGTH p_y

The basic design strength is determined according to the steel grade and thickness of plates in the section. A selected section is classified to Class 1, 2, 3 or 4. The design strength or section properties for Class 4 sections are revised as necessary.

II EFFECTIVE LENGTH AND SLENDERNESS RATIO

The boundary conditions of end supports or intermediate lateral restraints are assessed. The effective length L_E and slenderness ratio $\lambda = \frac{L_E}{r_y}$ are then calculated.

III EQUIVALENT SLENDERNESS RATIO

The equivalent slenderness ratio λ_{LT} is calculated as $\lambda_{LT} = uv\lambda\sqrt{\beta_w}$. In conjunction with the design strength p_y, the buckling strength p_b is determined and moment resistance is calculated as $M_b = p_b S_x$ for Classes 1 and 2 sections, $M_b = p_b Z_x$ or $p_b S_{x,eff}$ for Classes 3 sections and $M_b = p_b \frac{p_{yr}}{p_y} Z_x$ for Class 4 sections ($p_b Z_{x,eff}$ may be used alternatively when the effective width method is used).

IV BUCKLING STRENGTH CHECK

The moment resistance M_b should be checked to be not less than $m_{LT}M_x$ and the moment capacity M_c should also be not less than than M_x.

V WEB BEARING AND BUCKLING

Checking for web against bearing, shear buckling and compression buckling should be carried out as for restrained beams. The discussion should be referred to Chapter 9.

VI OTHER LIMIT STATES

Other limit states such as the deflection and vibration limit states shall also be checked.

6.7 Worked examples

6.7.1 Moment resistance of hot-rolled and welded sections

Determine the factored design uniform bending capacity of a 457×152×60 UB in S275 steel, simply supported a span of 3.0 m. Assume both the cases for hot-rolled and welded sections and under normal load.

Solution

SECTION PROPERTIES

$D = 454.6mm$, $B = 152.9mm$, $t = 8.1mm$, $T = 13.3mm$, $d = 407.6mm$, $r_y = 3.23cm$, $Z_x = 1120cm^3$, $S_x = 1290cm^3$, $x = 37.5$

SECTION CLASSIFICATION

Design strength, $p_y = 275N/mm^2$ for $T \le 16mm$ *(Table 3.2)*

$$\varepsilon = \sqrt{\frac{275}{275}} = 1$$ *(Table 7.1 Note b)*

Plastic limiting value of b/T for outstand flange of an I-section is 8ε

$$\frac{b}{T} = \frac{152.9}{2 \times 13.3} = 5.75 \le 8 \times 1 = 8$$ *(Table 7.1)*

$\therefore$ flange is plastic

Plastic limiting value of d/t for web of an I-section with neutral axis at mid-depth is 80ε

$$\frac{d}{t} = \frac{407.6}{8.1} = 50.3 \le 80 \times 1 = 80$$ *(Table 7.1)*

$\therefore$ web is plastic

$\therefore$ the section is class 1 plastic

MOMENT CAPACITY

It is low shear condition *(Clause 8.2.2.1)*

Moment capacity, $M_{cx} = p_y S_x \le 1.2 p_y Z_x$ *(8.2)*

$$= 275 \times 1290 \times 10^3 \le 1.2 \times 275 \times 1120 \times 10^3$$
$$= 354.8kNm \le 369.6kNm$$

LATERAL-TORSIONAL BUCKLING

Effective length, $L_E = L_{LT} = 3.0m$ for normal load *(Clause 8.3.4.1(a))*

Slenderness ratio, $\lambda = \frac{L_E}{r_y} = \frac{3000}{32.3} = 92.9$ *(8.26)*

$$v = \frac{1}{\left[1 + 0.05(\lambda/x)^2\right]^{0.25}} = \frac{1}{\left[1 + 0.05(92.9/37.5)^2\right]^{0.25}} = 0.935$$ *(8.27)*

$\beta_w = 1.0$ for class 1 plastic section *(Clause 8.3.5.3)*

For hot-rolled section

$u = 0.9$ for conservative approach *(Clause 8.3.5.3)*

Equivalent slenderness, $\lambda_{LT} = uv\lambda\sqrt{\beta_w} = 0.9 \times 0.935 \times 92.9 \times 1 = 78.2$ *(8.25)*

Buckling strength, $p_b = 169.0 N/mm^2$ *(Table 8.3a)*

Buckling resistance moment, $M_b = p_b S_x = 169 \times 1290 \times 10^3 = 218.0 kNm$ *(8.20)*

Equivalent uniform moment factor, $m_{LT} = 1.0$ for uniform moment *(Table 8.4a)*

$m_{LT} M_x \le M_b$ *(8.18)*

$\therefore M_x = \frac{M_b}{m_{LT}} = 218.0 kNm \le M_{cx}$ *(8.19)*

Therefore, the factored design uniform bending capacity for hot-rolled section is 218.0kNm.

For welded section

$u = 1.0$ *(Clause 8.3.5.3)*

Equivalent slenderness, $\lambda_{LT} = uv\lambda\sqrt{\beta_w} = 1 \times 0.935 \times 92.9 \times 1 = 86.9$ *(8.25)*

Buckling strength, $p_b = 138.1 N/mm^2$ *(Table 8.3b)*

Buckling resistance moment, $M_b = p_b S_x = 138.1 \times 1290 \times 10^3 = 178.1 kNm$ *(8.20)*

Equivalent uniform moment factor, $m_{LT} = 1.0$ for uniform moment *(Table 8.4a)*

$m_{LT} M_x \le M_b$ *(8.18)*

$\therefore M_x = \frac{M_b}{m_{LT}} = 178.1 kNm \le M_{cx}$ *(8.19)*

Therefore, the factored design uniform bending capacity for welded section is 178.1kNm.

6.7.2 Beam under double curvature

A simply supported 203×203×60 UC section beam of S355 steel has a span of 3.5 m and end moments M and 0.4 M which cause double curvature bending. Determine the maximum design value of M.

Solution

SECTION PROPERTIES

$D = 209.6mm$, $B = 205.8mm$, $t = 9.4mm$, $T = 14.2mm$, $d = 160.8mm$, $r_y = 5.20cm$, $Z_x = 584cm^3$, $S_x = 656cm^3$, $u = 0.846$, $x = 14.1$

SECTION CLASSIFICATION

Design strength, $p_y = 355N/mm^2$ for $T \leq 16mm$ *(Table 3.2)*

$\varepsilon = \sqrt{\frac{275}{355}} = 0.88$ *(Table 7.1 Note b)*

Plastic limiting value of b/T for outstand flange of an H-section is 9ε

$\frac{b}{T} = \frac{205.8}{2 \times 14.2} = 7.25 \leq 9 \times 0.88 = 7.92$ *(Table 7.1)*

$\therefore$ flange is plastic

Plastic limiting value of d/t for web of an H-seciton with neutral axis at mid-depth is 80ε

$\frac{d}{t} = \frac{160.8}{9.4} = 17.1 \leq 80 \times 0.88 = 70.4$ *(Table 7.1)*

$\therefore$ web is plastic

$\therefore$ the section is Class 1 plastic

MOMENT CAPACITY

Low shear condition is assumed *(Clause 8.2.2.1)*

Moment capacity, $M_{cx} = p_y S_x \leq 1.2 p_y Z_x$ *(8.2)*

$= 355 \times 656 \times 10^3 \leq 1.2 \times 355 \times 584 \times 10^3$

$= 232.9kNm \leq 248.8kNm$

LATERAL-TORSIONAL BUCKLING

It is normal loading condition *(Clause 8.3.3)*

Effective length, $L_E = L_{LT} = 3.5m$ for normal load *(Clause 8.3.4.1(a))*

Slenderness ratio, $\lambda = \frac{L_E}{r_y} = \frac{3500}{52} = 67.3$ *(8.26)*

$u = 0.9$ conservatively for hot-rolled section or $u = 0.846$ from section table used below *(Clause 8.3.5.3)*

$v = \frac{1}{\left[1 + 0.05(\lambda/x)^2\right]^{0.25}} = \frac{1}{\left[1 + 0.05(67.3/14.1)^2\right]^{0.25}} = 0.827$ *(8.27)*

$\beta_w = 1$ for Class 1 plastic section *(Clause 8.3.5.3)*

Equivalent slenderness, $\lambda_{LT} = uv\lambda\sqrt{\beta_w} = 0.846 \times 0.827 \times 67.3 \times 1 = 47.1$ *(8.25)*

Buckling strength, $p_b = 301.9N/mm^2$ *(Table 8.3a)*

Buckling resistance moment, $M_b = p_b S_x = 301.9 \times 656 \times 10^3 = 198.0kNm$ *(8.20)*

Equivalent uniform moment factor, $m_{LT} = 0.46$ for $\beta = -\frac{0.4M}{M} = -0.4$ *(Table 8.4a)*

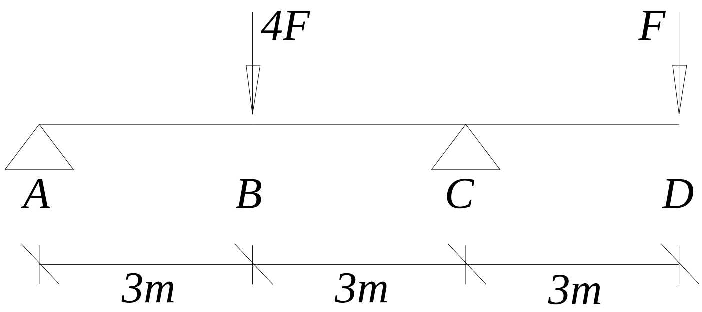
$m_{LT} M_x \leq M_b$ *(8.18)*

$\therefore\ M_x = \frac{M_b}{m_{LT}} = \frac{198.0}{0.46} = 430.4kNm$ but $> M_{cx}$ *(8.19)*

Therefore, the maximum design moment is 232.9kNm.

6.7.3 Over-hung Beam

A 203×203×60 UC section beam of S355 steel is simply supported at A, continuous over a support at C and free at D. AB = BC = CD = 3.0 m. The beam is torsionally restrained with and its compression flange restrained against lateral movement but free to rotate on plan at A and torsionally and laterally restrained at C. Downwards loads of 4F at B and F at D act at the top flange and are free to deflect laterally. Determine the maximum design value of F.



Solution

DESIGN LOAD

By moment equilibrium at C, $R_A \times 6 - 4F \times 3 + F \times 3 = 0$, $\therefore R_A = 1.5F$

By force equilibrium, $R_A + R_C = 4F + F$, $\therefore R_C = 3.5F$

Maximum shear at B, $V_B = 1.5F - 4F = -2.5F$

Maximum shear at C, $V_C = F$

Maximum sagging moment at B, $M_B = R_A \times 3 = 4.5F$

Maximum hogging moment at C, $M_C = -F \times 3 = 3F$

SECTION PROPERTIES

$D = 209.6mm$, $B = 205.8mm$, $t = 9.4mm$, $T = 14.2mm$, $d = 160.8mm$, $r_y = 5.20cm$, $Z_x = 584cm^3$, $S_x = 656cm^3$, $u = 0.846$, $x = 14.1$

SECTION CLASSIFICATION

Design strength, $p_y = 355N/mm^2$ for $T \le 16mm$ *(Table 3.2)*

$$\varepsilon = \sqrt{\frac{275}{355}} = 0.88$$ *(Table 7.1 Note b)*

Plastic limiting value of b/T for outstand flange of an I-section is 9ε

$$\frac{b}{T} = \frac{205.8}{2 \times 14.2} = 7.25 \le 9 \times 0.88 = 7.92$$ *(Table 7.1)*

$\therefore$ flange is plastic

Limiting value of d/t for web of an I-section with neutral axis at mid-depth is 80ε

$$\frac{d}{t} = \frac{160.8}{9.4} = 17.1 \le 80 \times 0.88 = 70.4$$ *(Table 7.1)*

$\therefore$ web is plastic

$\therefore$ the section is Class 1 plastic

SHEAR CAPACITY

Shear area, $A_v = tD = 9.4 \times 209.6 = 1970mm^2$ *(Clause 8.2.1)*

Shear capacity, $V_c = \frac{p_y A_v}{\sqrt{3}} = \frac{355 \times 1970}{\sqrt{3}} = 403.8kN$ *(8.1)*

MOMENT CAPACITY

Low shear condition is assumed *(Clause 8.2.2.1)*

Moment capacity, $M_{cx} = p_y S_x \le 1.2 p_y Z_x$ *(8.2)*

$$= 355 \times 656 \times 10^{-3} \le 1.2 \times 355 \times 584$$
$$= 232.9kNm \le 248.8kNm$$

LATERAL-TORSIONAL BUCKLING

For segment AC,

Destabilizing loading condition is assumed *(Clause 8.3.3)*

Effective length, $L_E = 1.2 L_{LT} = 1.2 \times 6 = 7.2m$ *(Clause 8.3.4.1(d))*

Slenderness ratio, $\lambda = \frac{L_E}{r_y} = \frac{7200}{52} = 138.5$ *(8.26)*

$u = 0.9$ conservatively for hot-rolled section *(Clause 8.3.5.3)*

$$v = \frac{1}{\left[1 + 0.05(\lambda/x)^2\right]^{0.25}} = \frac{1}{\left[1 + 0.05 \times (138.5/14.1)^2\right]^{0.25}} = 0.644$$ *(8.27)*

$\beta_w = 1$ for Class 1 plastic section *(Clause 8.3.5.3)*

Equivalent slenderness, $\lambda_{LT} = uv\lambda\sqrt{\beta_w} = 0.9 \times 0.644 \times 138.5 \times 1 = 80.3$ *(8.25)*

Buckling strength, $p_b = 189.1 N/mm^2$ *(Table 8.3a)*

Buckling resistance moment, $M_b = p_b S_x = 189.1 \times 656 \times 10^3 = 124.0kNm$ *(8.20)*

Equivalent uniform moment factor is given by

$$m_{LT} = 0.2 + \frac{0.15M_2 + 0.5M_3 + 0.15M_4}{M_{max}}$$ *(Table 8.4b)*

$M_2 = 2.25F$, $M_3 = M_{max} = 4.5F$, $M_4 = 0.75F$

$$m_{LT} = 0.2 + \frac{0.15 \times 2.25 + 0.5 \times 4.5 + 0.15 \times 0.75}{4.5} = 0.8 \ge 0.44$$

$m_{LT} M_x \le M_b$ *(8.18)*

$0.8 \times 4.5F \le 124$

$\therefore F \le 34.4kN$

For segment CD,

Destabilizing loading condition is assumed *(Clause 8.3.3)*

Effective length, $L_E = 2.5 L_{LT} = 2.5 \times 3 = 7.5m$ *(Table 8.1)*

Slenderness ratio, $\lambda = \frac{L_E}{r_y} = \frac{7500}{52} = 144.2$ *(8.26)*

$u = 0.9$ conservatively for hot-rolled section *(Clause 8.3.5.3)*

$$v = \frac{1}{\left[1 + 0.05(\lambda/x)^2\right]^{0.25}} = \frac{1}{\left[1 + 0.05 \times (144.2/14.1)^2\right]^{0.25}} = 0.633$$ *(8.27)*

$\beta_w = 1$ for Class 1 plastic section *(Clause 8.3.5.3)*

Equivalent slenderness, $\lambda_{LT} = uv\lambda\sqrt{\beta_w} = 0.9 \times 0.633 \times 144.2 \times 1 = 82.2$ *(8.25)*

Buckling strength, $p_b = 183.4 N/mm^2$ *(Table 8.3a)*

Buckling resistance moment, $M_b = p_b S_x = 183.4 \times 656 \times 10^3 = 120.3kNm$ *(8.20)*

Equivalent uniform moment factor, $m_{LT} = 1.0$ for cantilever *(Clause 8.3.5.2)*

$m_{LT} M_x \le M_b$ *(8.18)*

$1 \times 3F \le 120.3$

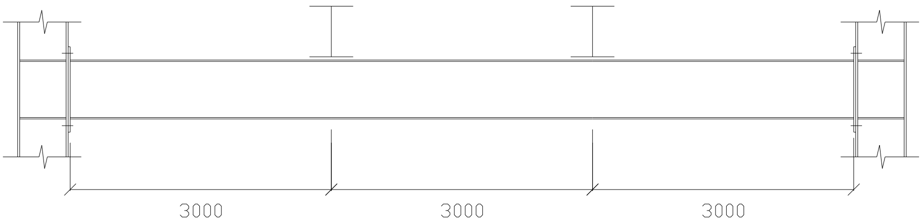
$\therefore F \le 40.1kN$

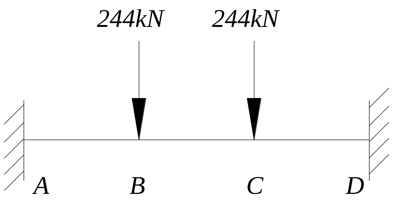
Therefore, the maximum design value of F is $34.4kN$.

Maximum shear at B is given by $V_B = -2.5F = 86.0kN$, which is smaller than $0.6V_c = 242.3kN$. Therefore, it is low shear condition. *(Clause 8.2.2.1)*

6.7.4 I-section beam with intermediate restraints

An I-section beam of 457×191×89 UB in S275 steel of span 9m and with rigid connection to columns (not free to rotate on plan). The beam further supports two secondary beams as shown. The end supports of the beam provide adequate lateral and torsional restraint with compression flange fully restrained against rotation on plan, whereas intermediate lateral restraint prevents the compression flange of the beam from lateral movement but rotations are free at these intermediate restraint locations. The unfactored dead load of 60kN and imposed load of 100kN are transferred from the secondary beams to the I-section beam. Check the member capacities of the steel beam.





Solution

DESIGN LOAD

Factored point load, $P = 1.4P_Q + 1.6P_G = 1.4 \times 60 + 1.6 \times 100 = 244kN$

Maximum shear, $V = 244kN$

Maximum hogging moment, $M_{hog} = -\frac{2PL}{9} = -488kNm$

Maximum sagging moment, $M_{sag} = \frac{PL}{9} = 244kNm$

SECTION PROPERTIES

$D = 463.4mm$, $B = 191.9mm$, $t = 10.5mm$, $T = 17.7mm$, $d = 407.6mm$, $r_y = 4.29cm$,

$Z_x = 1770cm^3$, $S_x = 2010cm^3$, $u = 0.880$, $x = 28.3$

SECTION CLASSIFICATION

Design strength, $p_y = 265N/mm^2$ for $16mm < T \leq 40mm$ *(Table 3.2)*

$\varepsilon = \sqrt{\frac{275}{265}} = 1.02$ *(Table 7.1 Note b)*

Plastic limiting value of b/T for outstand flange of an I-section is 9ε

$$\frac{b}{T}=\frac{191.9}{2\times17.7}=5.42\le9\times1.02=9.18 \qquad \text{(Table 7.1)}$$

$\therefore$ flange is plastic

Plastic limiting value of d/t for web of an I-section with neutral axis at mid-depth is 80ε

$$\frac{d}{t}=\frac{407.6}{10.5}=38.8\le80\times1.02=81.6 \qquad \text{(Table 7.1)}$$

$\therefore$ web is plastic

$\therefore$ the section is Class 1 plastic

SHEAR CAPACITY

Shear area, $A_v=tD=10.5\times463.4=4866mm^2$ *(Clause 8.2.1)*

Shear capacity, $V_c=\frac{p_yA_v}{\sqrt{3}}=\frac{265\times4866}{\sqrt{3}}=744.5kN>V$ *(OK)* *(8.1)*

MOMENT CAPACITY

$V=244kN\le0.6V_c=446.7kN$ *(Clause 8.2.2.1)*

$\therefore$ It is low shear condition

Moment capacity, $M_c=p_yS_x\le1.2p_yZ_x$ *(8.2)*

$$=265\times2010\times10^3\le1.2\times265\times1770\times10^3$$
$$=532.7kNm\le562.9kNm$$
$$>M_{hog}\ \ (OK)$$

LATERAL-TORSIONAL BUCKLING

Destabilizing loading condition is assumed as the load is at top flange *(Clause 8.3.3)*

Segment AB

Effective length, $L_E=1.2\times0.9\times L_{LT}=3.24m$ *(Clauses 8.3.4.1 & 8.3.4.2)*

Slenderness ratio, $\lambda=\frac{L_E}{r_y}=\frac{3240}{42.9}=75.5$ *(8.26)*

$$v=\frac{1}{\left[1+0.05(\lambda/x)^2\right]^{0.25}}=\frac{1}{\left(1+0.05(75.5/28.3)^2\right)^{0.25}}=0.927 \qquad (8.27)$$

$\beta_w=1$ for Class 1 plastic section *(Clause 8.3.5.3)*

Equivalent slenderness, $\lambda_{LT}=uv\lambda\sqrt{\beta_w}=0.88\times0.927\times75.5\times1=61.6$ *(8.25)*

Buckling strength, $p_b=203.5N/mm^2$ *(Table 8.3a)*

Bucling resistance moment, $M_b=p_bS_x=203.5\times2010\times10^3=409.0kNm$ *(8.20)*

Equivalent uniform moment factor, $m_{LT}=0.44$ for $\beta=-\frac{244}{488}=-0.5$ *(Table 8.4a)*

$m_{LT}M_x=0.44\times488=214.7kNm\le M_b$ *(OK)* *(8.18)*

Segment BC

Effective length, $L_E = 1.2L_{LT} = 3.6m$ *(Clauses 8.3.4.2)*

Slenderness ratio, $\lambda = \frac{L_E}{r_y} = \frac{3600}{42.9} = 83.9$ *(8.26)*

$$v = \frac{1}{\left[1 + 0.05(\lambda/x)^2\right]^{0.25}} = \frac{1}{\left(1 + 0.05(83.9/28.3)^2\right)^{0.25}} = 0.913$$ *(8.27)*

$\beta_w = 1$ for Class 1 plastic section *(Clause 8.3.5.3)*

Equivalent slenderness, $\lambda_{LT} = uv\lambda\sqrt{\beta_w} = 0.88 \times 0.913 \times 83.9 \times 1 = 67.4$ *(8.25)*

Buckling strength, $p_b = 190.2\,N/mm^2$ *(Table 8.3a)*

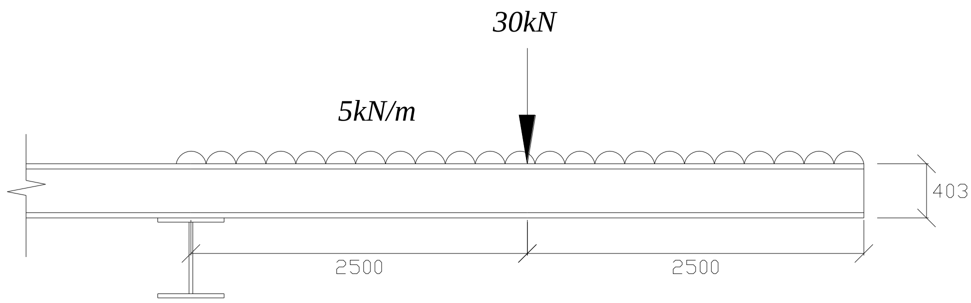
Bucling resistance moment, $M_b = p_b S_x = 190.2 \times 2010 \times 10^3 = 382.3kNm$ *(8.20)*

Equivalent uniform moment factor, $m_{LT} = 1$ for $\beta = 1$ *(Table 8.4a)*

$m_{LT} M_x = 244kNm \le M_b$ *(OK)* *(8.18)*

6.7.5 Cantilever without intermediate restraint

A cantilever of span 5m supports an imposed point load of 30kN at 2.5m from a beam. The beam is of section 406×178×54 UB in S275 steel. As shown in the figure below, the cantilever beam is restrained with partial torsional restraint at the end support and is free at the tip. It is further subjected to a downward uniformly distributed wind load of 5kN/m. The load is applied as normal load condition.



<u>Solution</u>

DESIGN LOAD

Maximum shear, $V = 1.2 \times 30 + 1.2 \times 5 \times 5 = 66kN$

Maximum moment, $M_x = 1.2 \times 30 \times 2.5 + 1.2 \times 5 \times \frac{5^2}{2} = 165kNm$

SECTION PROPERTIES

$D = 402.6mm$, $B = 177.7mm$, $t = 7.7mm$, $T = 10.9mm$, $d = 360.4mm$, $r_y = 3.85cm$, $Z_x = 930cm^3$, $S_x = 1050m^3$, $u = 0.871$, $x = 38.3$

SECTION CLASSIFICATION

Design strength, $p_y = 275N/mm^2$ for $T \le 16mm$ *(Table 3.2)*

$\varepsilon = \sqrt{\frac{275}{275}} = 1$ *(Table 7.1 Note b)*

Plastic limiting value of b/T for outstand flange of an I-section is 9ε

$\frac{b}{T} = \frac{177.1}{2 \times 10.9} = 8.15 \le 9 \times 1 = 9$ *(Table 3.2)*

$\therefore$ flange is plastic

Plastic limiting value of d/t for web of an I-section with neutral axis at mid-depth is 80ε

$\frac{d}{t} = \frac{360.4}{7.7} = 46.8 \le 80 \times 1 = 80$ *(Table 3.2)*

$\therefore$ web is plastic

$\therefore$ the section is Class 1 Plastic

SHEAR CAPACITY

Shear area, $A_v = tD = 7.7 \times 402.6 = 3100 mm^2$ *(Clause 8.2.1)*

Shear capacity, $V_c = \dfrac{p_y A_v}{\sqrt{3}} = \dfrac{275 \times 3100}{\sqrt{3}} = 492.2kN > V$ *(OK)* *(8.1)*

MOMENT CAPACITY

$V = 66kN \le 0.6V_c = 295.3kN$

$\therefore$ it is low shear condition *(Clause 8.2.2.1)*

Moment capacity, $M_c = p_y S_x \le 1.2 p_y Z_x$ *(8.2)*

$$= 275 \times 1050 \times 10^3 \le 1.2 \times 275 \times 930 \times 10^3$$
$$= 288.8kNm \le 306.9kNm$$
$$> M_x \ (OK)$$

LATERAL-TORSIONAL BUCKLING

Normal loading condition is assumed *(Clause 8.3.3)*

Effective length, $L_E = 2L_{LT} = 2 \times 5 = 10m$ *(Table 8.1)*

Slenderness ratio, $\lambda = \dfrac{L_E}{r_y} = \dfrac{10000}{38.5} = 259.7$ *(8.26)*

$v = \dfrac{1}{\left[1 + 0.05(\lambda/x)^2\right]^{0.25}} = \dfrac{1}{\left[1 + 0.05(259.7/38.3)^2\right]^{0.25}} = 0.742$ *(8.27)*

$\beta_w = 1$ for Class 1 plastic section *(Clause 8.3.5.3)*

Equivalent slenderness, $\lambda_{LT} = uv\lambda\sqrt{\beta_w} = 0.871 \times 0.742 \times 259.7 \times 1 = 167.8$ *(8.25)*

Buckling strength, $p_b = 55.3N/mm^2$ *(Table 8.3a)*

Buckling resistance moment, $M_b = p_b S_x = 55.3 \times 1050 \times 10^3 = 58.1kNm$ *(8.20)*

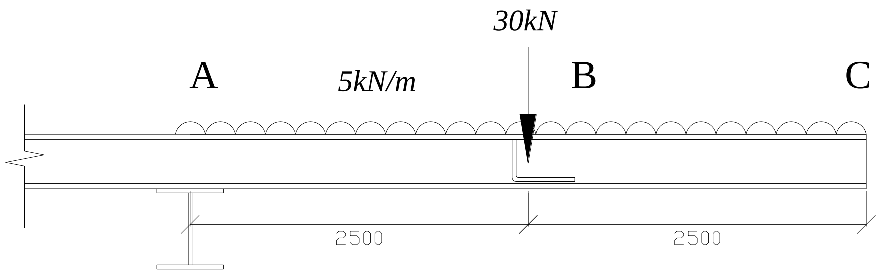
Equivalent uniform moment factor, $m_{LT} = 1$ for cantilever *(Clause 8.3.5.2)*

$m_{LT} M_x = 165kNm > M_b$ *(Not OK)* *(8.18)*

A subsequent worked example repeats the same design procedure again by considering a tie beam as intermediate restraint to the cantilever.

6.7.6 Cantilever with intermediate restraint

For the cantilever under the same load in Example 6.7.5, the imposed load is applied to the cantilever beam at the same location as the steel angle in the transverse direction as shown in the figure below. This steel angle can be treated as an internal tie and provides an intermediate lateral and torsional restraint to the cantilever beam such that the segment length is equal to 2.5m.



Solution

For the same cantilever beam, the shear and moment capacities of this beam member are sufficient to resist applied shear forces and bending moments. However, the lateral-torsional buckling resistance of the cantilever with the same section size in the previous example is inadequate. The effective length about minor axis in this case is reduced, when the angle connects to the cantilever as lateral and torsional restraint.

LATERAL-TORSIONAL BUCKLING

Segment AB

Normal loading condition is assumed *(Clause 8.3.3)*

Effective length, $L_E = 1.4L_{LT} = 1.4 \times 2.5 = 3.5m$ *(Table 8.1)*

Slenderness ratio, $\lambda = \frac{L_E}{r_y} = \frac{3500}{38.5} = 90.9$ *(8.26)*

$$v = \frac{1}{\left[1 + 0.05(\lambda/x)^2\right]^{0.25}} = \frac{1}{\left[1 + 0.05(90.9/38.3)^2\right]^{0.25}} = 0.940$$ *(8.27)*

$\beta_w = 1$ for Class 1 plastic section *(Clause 8.3.5.3)*

Equivalent slenderness, $\lambda_{LT} = uv\lambda\sqrt{\beta_w} = 0.871 \times 0.94 \times 90.9 \times 1 = 74.4$ *(8.25)*

Buckling strength, $p_b = 177.4N/mm^2$ *(Table 8.3a)*

Buckling resistance moment, $M_b = p_b S_x = 177.4 \times 1050 \times 10^3 = 186.3kNm$ *(8.20)*

Equivalent uniform moment factor, $m_{LT} = 1$ for cantilever *(Clause 8.3.5.2)*

$m_{LT}M_x = 165kNm \le M_b$ *(OK)* *(8.18)*

6.7.7 Simply supported I-beam

A simply supported beam of 8m length is under a uniformly distributed destabilizing live load of 30kN/m and dead load of 15kN/m. At end supports, the compression flanges of the beam are laterally restrained and fully restrained against torsion with both flanges free to rotate on plan such that the effective length of the beam against flexural-torsional buckling is the same as its physical length. Check the adequacy of the hot-rolled beam 610×305×149 UB of grade S355.

Solution

DESIGN LOAD

Factored distributed load, $\omega = 1.4 \times 15 + 1.6 \times 30 = 69kN/m$

Maximum shear, $V = \dfrac{69 \times 8}{2} = 276kN$

Maximum moment, $M = \dfrac{69 \times 8^2}{8} = 552kNm$

SECTION PROPERTIES

$D = 612.4mm$, $B = 304.8mm$, $t = 11.8mm$, $T = 19.7mm$, $d = 540mm$, $I_x = 126000cm^4$,

$r_y = 7.00cm$, $Z_x = 4110cm^3$, $S_x = 4590cm^3$, $u = 0.886$, $x = 32.7$

SECTION CLASSIFICATION

Design strength, $p_y = 345N/mm^2$ for $16mm < T \le 40mm$ *(Table 3.2)*

$\varepsilon = \sqrt{\dfrac{275}{345}} = 0.89$ *(Table 7.1 Note b)*

Plastic limiting value of b/T for outstand flange of an I-section is 9ε

$\dfrac{b}{T} = \dfrac{304.8}{2 \times 19.7} = 7.74 \le 9 \times 0.89 = 8.01$ *(Table 7.1)*

$\therefore$ flange is plastic

Plastic limiting value of d/t for web of an I-section with neutral axis at mid-depth is 80ε

$\dfrac{d}{t} = \dfrac{540}{11.8} = 45.8 \le 80 \times 0.89 = 71.2$ *(Table 7.1)*

$\therefore$ web is plastic

$\therefore$ the section is Class 1 plastic

SHEAR CAPACITY

Shear area, $A_v = tD = 11.8 \times 612.4 = 7226mm^2$ *(Clause 8.2.1)*

Shear capacity, $V_c = \dfrac{p_y A_v}{\sqrt{3}} = \dfrac{345 \times 7226}{\sqrt{3}} = 1439.3.kN > V$ *(OK)* *(8.1)*

MOMENT CAPACITY

$V \le 0.6V_c = 863.6kN$ *(Clause 8.2.2.1)*

$\therefore$ It is low shear condition

Moment capacity, $M_c = p_y S_x \le 1.2 p_y Z_x$ *(8.2)*

$$= 345 \times 4590 \times 10^3 \le 1.2 \times 345 \times 4110 \times 10^3$$
$$= 1583.6kNm \le 1701.5kNm$$
$$> M \quad (OK)$$

LATERAL-TORSIONAL BUCKLING

For destabilizing loading condition:

Effective length, $L_E = 1.2L_{LT} = 1.2 \times 8 = 9.6m$ *(Clauses 8.3.4.1 & 8.3.4.2)*

Slenderness ratio, $\lambda = \dfrac{L_E}{r_y} = \dfrac{9600}{70} = 137.1$ *(8.26)*

$$v = \frac{1}{\left[1 + 0.05(\lambda/x)^2\right]^{0.25}} = \frac{1}{\left(1 + 0.05(137.1/32.7)^2\right)^{0.25}} = 0.854$$ *(8.27)*

$\beta_w = 1$ for Class 1 plastic section *(Clause 8.3.5.3)*

Equivalent slenderness, $\lambda_{LT} = uv\lambda\sqrt{\beta_w} = 0.886 \times 0.854 \times 137.1 \times 1 = 103.7$ *(8.25)*

Buckling strength, $p_b = 130.3N/mm^2$ *(Table 8.3a)*

Bucling resistance moment, $M_b = p_b S_x = 130.3 \times 4590 \times 10^3 = 598.1kNm$ *(8.20)*

Equivalent uniform moment factor, $m_{LT} = 0.93$ *(Table 8.4a)*

$m_{LT} M_x = 0.93 \times 552 = 513.4kNm \le M_b \quad (OK)$ *(8.18)*

DEFLECTION

Unfactored uniform imposed load, $\omega = 30kN/m$

Maximum deflection due to imposed load,

$$\delta = \frac{5}{384}\frac{\omega L^4}{EI_x} \le \frac{L}{360}$$ *(Table 5.1)*

$$= \frac{5}{384} \times \frac{30 \times 8000^4}{205000 \times 126000 \times 10^4} \le \frac{8000}{360}$$
$$= 6.2mm \le 22.2mm \quad (OK)$$

Chapter 7 Compression Members

7.1 Introduction and uses of compression member

Compression member refers to structural element taking loads principally by its axial resistance, which includes column, strut and stanchion. The compression resistance of compression member is governed either by the material strength, the buckling resistance or their combination. The buckling resistance of compression members is generally controlled by class of section and its properties, material strength and member slenderness. Columns are normally vertical structural elements to transfer the loads from superstructure to foundation and they are commonly used to support horizontal members, such as beams, trusses and lattice girders. Common forms of compression member in the building structures are shown in Figure 7.1. Rolled or welded H-section and box section shown in Figure 7.2(a) are commonly used for columns of which the large second moments of area are employed to increase their resistance against buckling. Some other lighter duty compression members used as struts are made of the rolled angle or channel section as shown in Figure 7.2(b). These light duty compression members are widely used as bracings of framed structures and as structural members in lattice structures and transmission line towers.

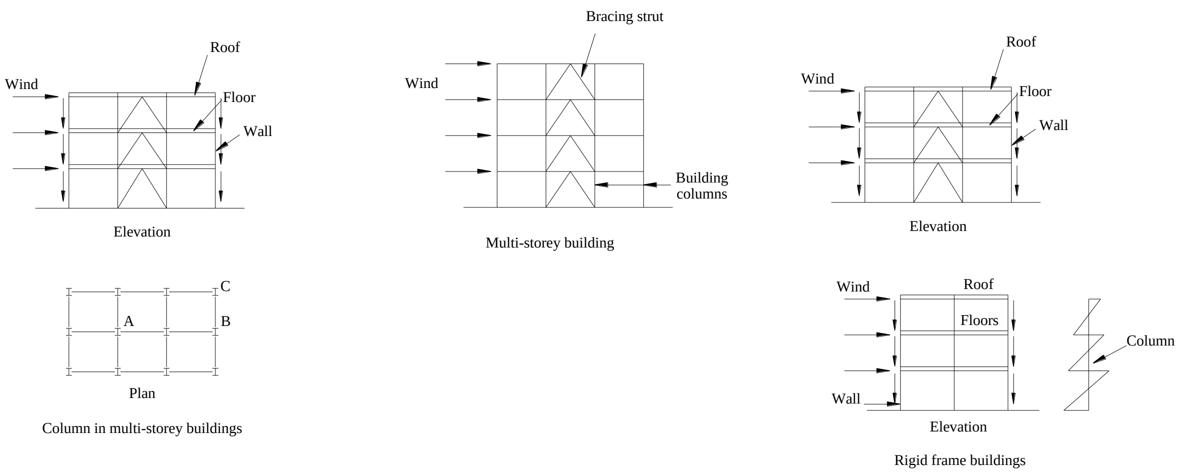


Figure 7.1 Compression members in different types of building structures

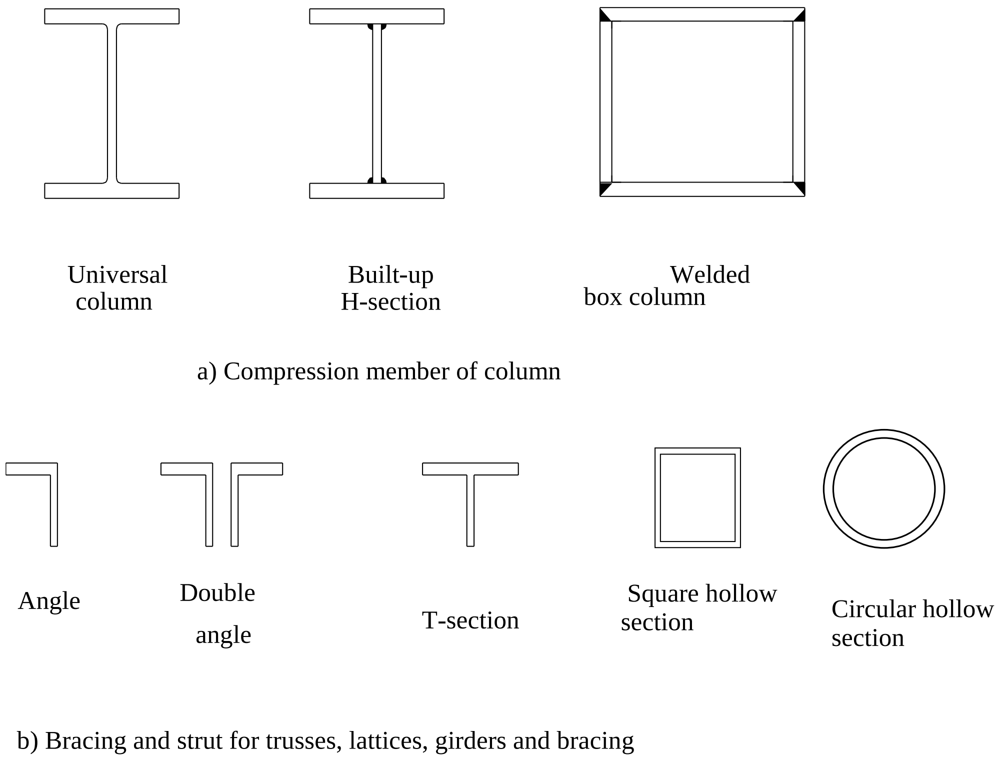


Figure 7.2 Different sections for compression member

The load capacity of a short compression member can be obtained as the section strength allowing for local buckling effects, as depicted in Figure 7.3. For long compression member, the member is susceptible to compression or column buckling. The buckling resistance of slender columns is controlled by the cross sectional properties and slenderness detailed in Section 7.3. As shown in Figure 7.3, the buckling resistance of a column is normally a fraction of its tension capacity which is not affected by the member buckling. Therefore it is common but not always possible to derive a framing system to allow vertical members in tension to avoid their strength being controlled by buckling. Nevertheless, it is sometimes unavoidable to have all axially loaded columns in compression.

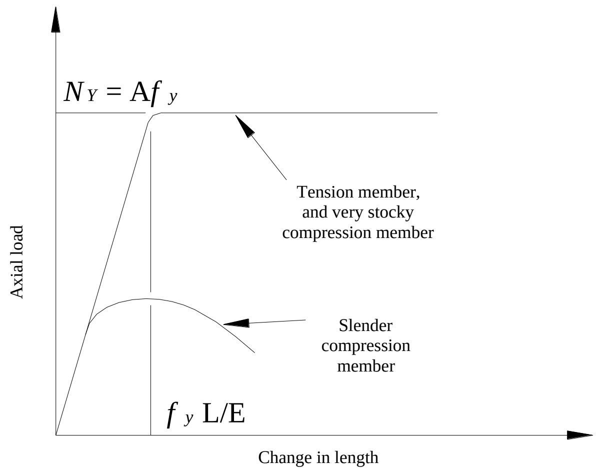


Figure 7.3 Strength of tension member and compression member

The effective length method is adopted as an alternative method in the HK Code for designing compression members with allowance for buckling effects. The method of effective length is based on an imaginary buckling length of a member to estimate its compression resistance. The accuracy, economy and reliability of the method very much depend on the effective length factor (L_E/L). An undesirable scenario will be that one can fine tune his derived compression resistance of a member by assuming an incorrect value of effective length factor. In spite of its shortcoming, the method is still widely used in design of steel structures due to its simplicity.

7.2 Behaviour of compression members

7.2.1 Introduction

Except for some very stocky members where the member load capacity is unaffected by the boundary condition and effective length, most compression members resist external forces under the influence of buckling. It was first recognized in the end of 18th century that the ultimate compression capacity of the compression member depends on its geometry, such as member length. When the member length increases, the geometric second-order effects, namely the P-Δ and P-δ effects, exaggerate rapidly and decrease the ultimate compression resistance of the column. The P-δ effect is referred to as the second-order effect due to deflection or bowing along a member, whereas the P-Δ effect is caused by the lateral movement or displacement at member ends to create an additional moment, which depends on displacement or sway of frame. This sway-dependent moment is termed as the P-Δ moment and can deteriorate the stiffness of the framed structures. Consideration of these two effects (i.e. P-Δ-δ effects) together with their initial imperfections will lead to a design output superior to the effective length method as the former eliminates the need to classify a frame. This method is termed as second-order direct elastic and plastic analysis and it is called the direct analysis in North America. Since the P-Δ and P-δ effects cannot be accurately and automatically taken into account by effective length method, hand calculation is therefore required to check the buckling strength of every member under different load cases of which the procedure is tedious and inconvenient. Unfortunately, most structural analysis or design programs have not been programmed to consider both these two effects and, of equal importance, their initial imperfections and therefore the method is not widely used in the industry at the time of writing this book. Note that most software considers only the P-Δ effect which is much easier to program by adding displacements to the nodal coordinates. Chapter 10 gives a more detailed account on the use of the new and advanced analysis and design method.

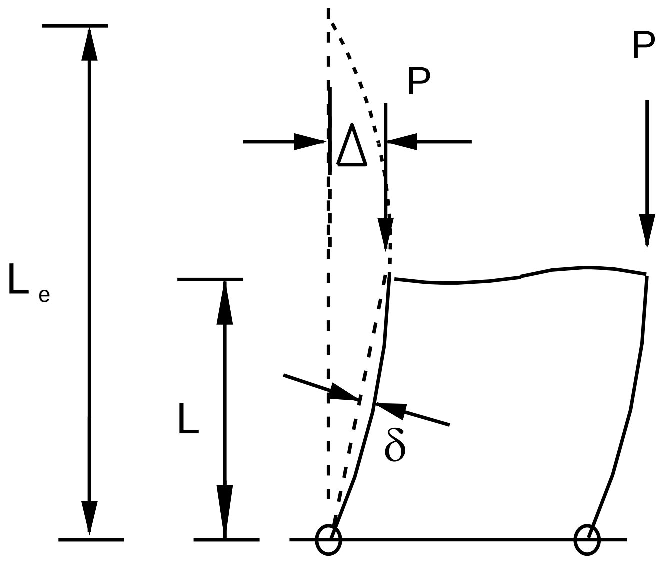


Figure 7.4 P-Δ and P-δ effect on compression members

The phenomenon of buckling of a column can be viewed as the equilibrium states shown in Figure 7.5 below. When the axial force acting on a simply supported elastic slender column is small and less than the elastic buckling load (P_{cr}) of the column equal to $P_{cr} = \frac{\pi^2 EI}{L_E^2}$ in which EI is the flexural constant and L_E is the effective length, the column is in a state of stable equilibrium such that the column deflects more only when the axial force is increased. The behaviour of compression member is no longer in a state of stable equilibrium with a further increase in applied force. The column will eventually become unstable with increase in displacement under a constant axial force. When the column is further loaded, it comes to an unstable equilibrium state that the displacement increases even when the applied force is infinitesimal.

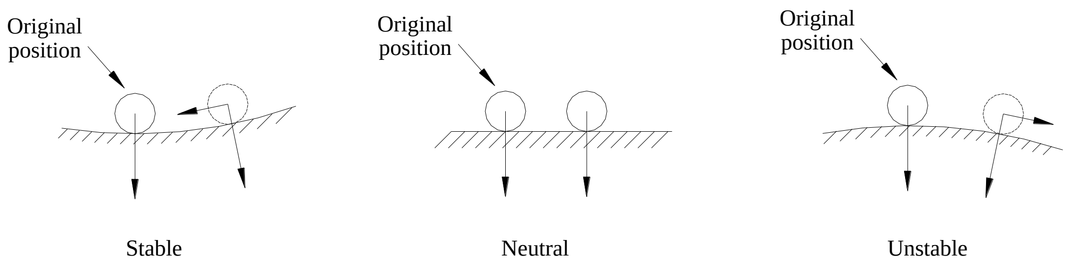


Figure 7.5 Different types of equilibrium

The resistance of a compression member depends on the member length or more precisely the effective length taking into account the boundary condition of the member. The method based on the effective length of a compression column for determination of the buckling resistance of a column is a simple but approximate method for estimation of buckling resistance, as the correct value of effective length of compression member may not be obvious in many real structures. In view of this, most codes including the HK Code do not allow the use of the method when a structure is susceptible to sway or when a slender structure is designed. The buckling problem is quantified by the elastic critical load factor being equal to the ratio of elastic buckling load to the design load ($\lambda_{cr} = \frac{P_{cr}}{F_c}$). The expression of P_{cr} was first derived by Euler (1759) and has been used as a reference and upper bound buckling load for centuries. When calculating the elastic critical load factor λ_{cr}, one needs not consider the deflection of a member and therefore member imperfection, frame imperfection and load eccentricity are not required in the computation. The method of eigen-buckling analysis cannot calculate deflection as it assumes no deflection perpendicular to the load until the deflection become infinite and the structure becomes unstable. The eigen-buckling analysis relies on the condition of vanishing of the tangent stiffness of a structure which implies the attainment of the condition of elastic buckling.

In the HK Code, the effective length method should not be used when λ_{cr} is less than 5 (i.e. $\lambda_{cr} < 5$), and a more accurate second-order direct analysis should be employed for the design. Under this condition, the frame is termed as sway ultra-sensitive frame. When λ_{cr} is between 5 and 10 (i.e. $5 \leq \lambda_{cr} < 10$), the frame is classified as sway and the P-Δ effect is important and must be considered in design. When λ_{cr} is greater than 10 (i.e. $\lambda_{cr} \geq 10$), the frame is considered as non-sway and sway P-Δ moment can be ignored but P-δ effect should always be considered unless the slenderness ratio $\frac{L_E}{r}$ is less than around 15. Determination of λ_{cr} can be carried out by an option of eigen-buckling analysis as in software like NIDA Ver. 9 (2015) or by the following empirical displacement method. A more detailed discussion of frame stability will be given in the Chapter 10.

$$\lambda_{cr} = \frac{F_N}{F_V}\frac{h}{\delta_N} \tag{7.1}$$

in which

F_V	is the factored dead plus live loads on the floor considered.
F_N	is the notional horizontal force taken typically as 0.5% of F_V for building frames;
h	is the storey height and
δ_N	is the notional horizontal deflection of the upper storey relative to the lower storey due to the notional horizontal force F_N .

7.2.2 Buckling of imperfection columns

When a practical member is under compression load, it deflects laterally as shown in Figure 7.6. It deflects in pace with the applied load because of the unavoidable member imperfection and load eccentricity.

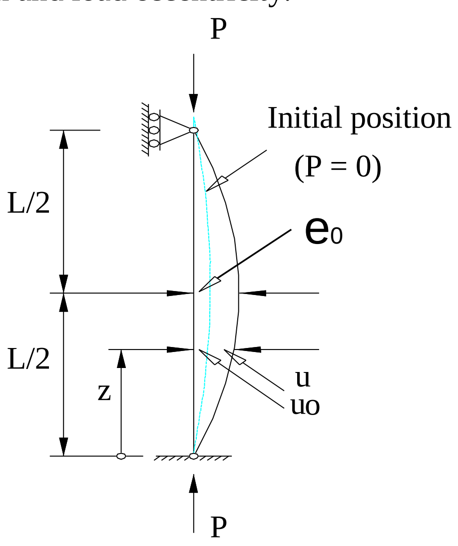


Figure 7.6 Deflection of compression member
(e_0 and u_0 are initial imperfections)

Based on the Timoshenko's beam-column theory with negligible shear deformation, the force equilibrium equation for a compression member is given in Equation (7.2). In this case, the applied bending moment *M* is replaced with *Pu* as shown in Equation (7.2) in which *P* is axially compressive force and *u* is lateral deflection. When using the eigen-buckling analysis, a perfectly straight and elastic compression member is considered and the boundary conditions of the compression member are regarded as frictionless pinned.

$$EI\frac{d^2u}{dz^2} = -Pu \tag{7.2}$$

where E and I are respectively the Young's modulus and second moment of inertia. Pu is an additional moment due to deflection, namely as the P-δ moment, induced by the lateral deflection and the axial force as shown in Figure 7.6. Using the simply supported boundary conditions, the force equilibrium equation of Equation (7.2) is solved and the buckling load for this simply supported case or the Euler buckling load P_E is obtained as Equation (7.3) and shown in Figure 7.7(a).

$$P_E = \frac{\pi^2 EI}{L^2} \leq p_y A \tag{7.3}$$

in which L is the member length and A is cross-section area of the member. The Euler buckling load P_E can therefore be seen as depending on member length. The Euler buckling load P_E and squash load ($P_y = p_y A$) of a column represent two upper bound failure loads of a member in all range of member slenderness.

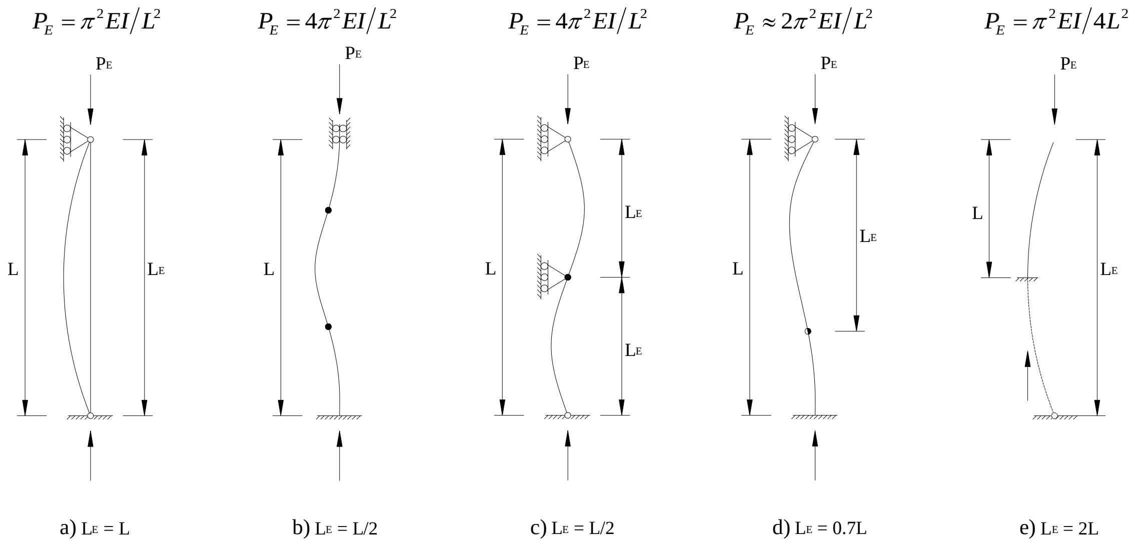


Figure 7.7 Effective lengths of compression members

The Euler buckling load P_E varies with different boundary conditions. For example, when both ends are fixed, the Euler buckling load P_E is increased by 4 times with an equivalent use of effective length L_E half of the actual length of the column as shown in Figure 7.7(b). For a cantilever, the effective length is equal to $2L$ as shown in Figure 7.7(e) and the elastic buckling load depends on the square of the effective length such that an error in approximating an effective length will lead to a quadratic increase in the error in the computed elastic buckling load. The effective length factors for other support conditions are indicated in Figure 7.7(c) and (d) which requires a careful determination of effective length to correctly approximate the buckling resistance of a practical column which unfortunately seldom has such an idealized boundary condition as in Figure 7.7.

Rearranging the terms for Euler buckling load as a function of slenderness ratio λ, we have,

$$P_E = \frac{\pi^2 EA}{(L_E/r)^2} = \frac{\pi^2 EA}{\lambda^2} \tag{7.4}$$

in which L_E is the effective length of the compression member, L_E/r is the slenderness ratio λ, r is radius of gyration equal to $\sqrt{\frac{I}{A}}$. Using Equation (7.4), the strength of a compression member depends on the member slenderness ratio λ and axial constant EA. When the non-dimensional slenderness ratio $\bar{\lambda} = \sqrt{\frac{P_y}{P_E}} = \frac{L_E}{r}\sqrt{\frac{p_y}{\pi^2 E}}$ is less than 1, failure of the column is more controlled by material yielding or the squash load as indicated in Figure 7.3. Conversely, the member with a non-dimensional slenderness ratio greater than 1, it is then considered as a slender member with its strength more controlled by buckling. The solid line in Figure 7.8 indicates inelastic compression buckling curve for imperfect columns in practice.

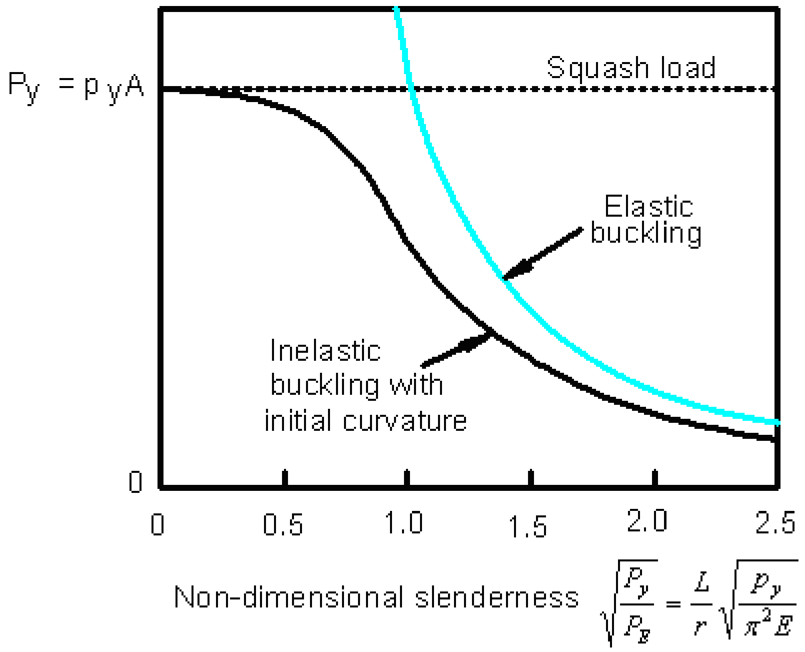


Figure 7.8 Elastic and inelastic buckling of compression member

7.2.3 Perry-Robertson formula for column buckling

Perfectly straight steel columns or columns free from residual stress are not available in practice. Member initial crookedness or curvature and residual stresses are present in all practical steel members and frames. Consequently, a realistic buckling resistance of a column must take into account these imperfections.

For the compression members with both ends pinned and with an initial imperfection u_0 where $u_0 = \delta_0 \sin\left(\frac{\pi \cdot z}{L}\right)$ as illustrated in Figure 7.6, the equilibrium equation is similar to Equation (7.2) but with an additional term for initial curvature $\frac{d^2u_0}{dz^2}$ given by Equation (7.5) as,

$$EI\frac{d^2u}{dz^2} = -Pu + EI\frac{d^2u_0}{dz^2} \tag{7.5}$$

Solving Equation (7.5) using the simply supported boundary condition, the maximum lateral deflection is obtained as,

$$u_{max} = \frac{\delta_0 P_E}{(P_E - P)} \tag{7.6}$$

in which u_{max} and δ_0 are respectively the maximum and initial imperfections at mid-span.

Therefore, the critical P-δ moment induced by initial imperfection on the compression member is given by,

$$M = Pu_{max} = \frac{\delta_0 P P_E}{(P_E - P)} \tag{7.7}$$

Based on the simple beam bending theory, the maximum compressive stress due to combined bending and compressive load can be expressed in Equation (7.8). Setting the maximum compressive stress equal to the material design strength p_y of the member in Equation (7.8), the compression load P_c causing the cross section of a member to yield is given by,

$$p_y = \frac{\delta_0 P_E P_c}{(P_E - P_c)} \times \frac{y}{I} + \frac{P_c}{A} = \frac{\eta p_E p_c}{p_E - p_c} + p_c \tag{7.8}$$

in which p_y is the maximum stress attained on the member or yield stress, p_c and p_E are respectively the compressive design strength on the member and Euler buckling stress as $\frac{\pi^2 E}{(L_E / r)^2}$, η is the Perry factor equal to $\frac{\delta_0 y}{r^2}$, and y is the distance from the centroid to the extreme location in the cross-section. Rearranging Equation (7.8), the Perry-Robertson formula is obtained as follows.

$$(p_E - p_c)(p_y - p_c) = \eta p_E p_c \tag{7.9}$$

The different buckling curves of the compression member with various initial imperfections are plotted in Figure 7.9. The elastic buckling load with initial imperfection is determined from the smaller root of Equation (7.9). The buckling loads of imperfect members can be seen to be lower than the Euler load or elastic buckling curve. When the initial imperfection λ_0 is near zero, compressive stress p_c is very close either to Euler buckling design strength p_E or to the yield stress p_y. Thus, the formula is useful in computing the buckling strength allowing for buckling and material yielding.

The discrepancy in buckling resistance of members under the 4 buckling curves depicted in Figure 7.9 are due to the difference in use of the Perry factor η in Equation (7.9) as,

$$\eta = \alpha\left(\lambda - \lambda_0\right)/1000 \geq 0 \tag{7.10}$$

in which α is the Robertson constant, which can be adjusted for types of sections, λ is the slenderness ratio as $\frac{L_E}{r}$, λ_0 is limit slenderness ratio and equal to $0.2\sqrt{\frac{\pi^2 E}{p_y}}$. When the slenderness ratio λ of the compression member is less than the limiting slenderness ratio λ_0, compression buckling is considered not to occur in the member. For a general section not belonging to one of the typical sectional types, Equation (7.9) can be adjusted by matching the buckling curve from the test results which include the effects of initial imperfection and residual stress due to welding or other manufacturing processes.

7.3 Compression strength and buckling curves

The safety check of a compression member requires information on its bucking strength, p_c in the HK Code. Totally five buckling curves are given in the Code with each of them used to represent a particular section type and manufacturing process. Also the effects of geometrical imperfection, which is normally in the order of 0.1% of the member length, as well as the effect of residual stress are considered as the equivalent initial imperfection of a column. Typical plots of residual stress across a hot-rolled and a welded section are shown in Figure 2.2 and Figure 2.3 of this book. *Table 6.1* of the HK Code gives the imperfections for various types of sections and *Appendix 8.4* provides details on the use of Perry constant in constructing the buckling curves of a section. In line with these parameters, software NIDA contains the set of imperfections and a minimum default imperfection of 0.1% in order to disallow use of imperfection less than the geometrical imperfection.

The buckling curves are initially prepared for hot-rolled sections as shown in Figure 7.9. For use by welded columns and struts, their design strength is required to be reduced by 20N/mm^2 for the more serious effect of residual stress. A buckling curve new in 2011 version of the HK Code for top quality annealed sections, the a_0 curve, is not shown in Figure below.

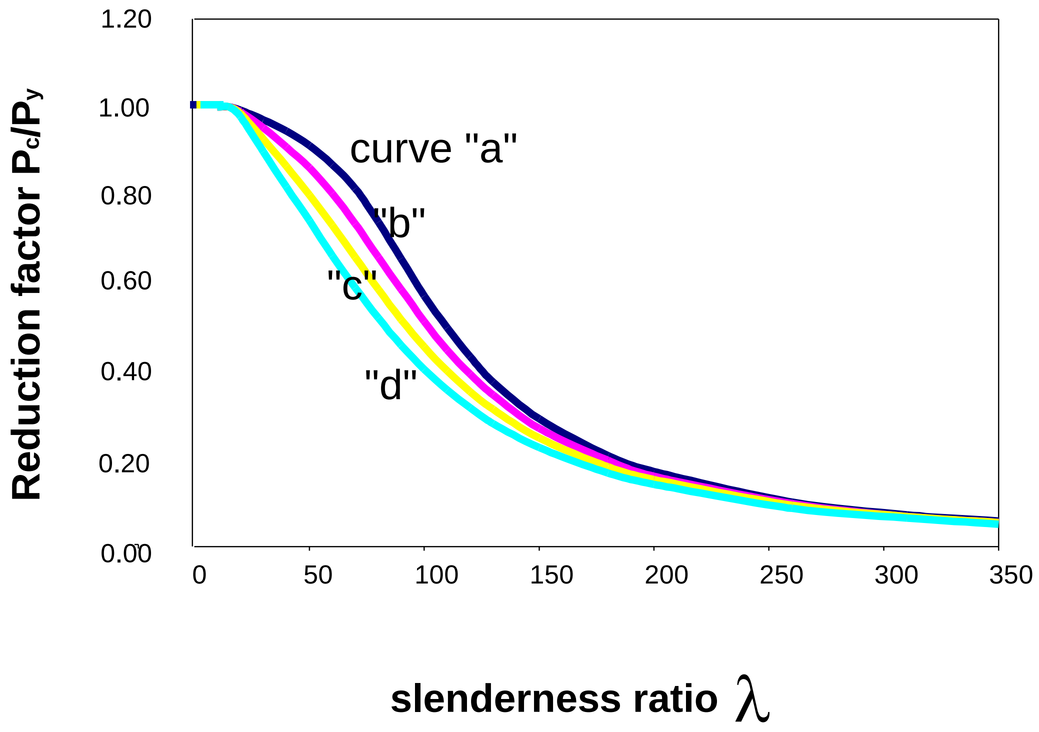


Figure 7.9 The 4 buckling curves

7.3.1 Effective length

As illustrated above, the effective length is an important parameter for determination of buckling resistance of an axial compression member. In the HK Code, there are several approaches for dealing with the problem of buckling strength check with different levels of accuracy.

7.3.1.1 Column in a simple or single storey frame

For simple one-storey structures or a compression member designed with idealized support conditions, the effective length factor in *Table 8.6* in the HK Code can be referred as follows.

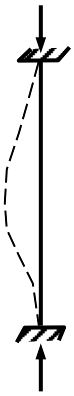
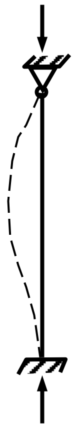
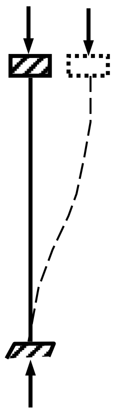
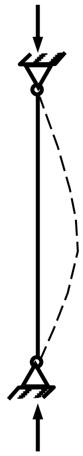
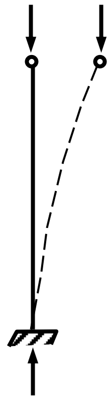
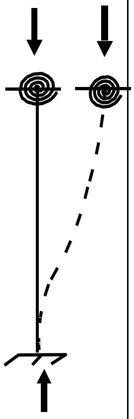
Flexural Buckling						
Buckled shape of column shown by dashed line						
Theoretical K value	0.5	0.7	1.0	1.0	2.0	/
Recommended K value when ideal conditions are approximated	0.70	0.85	1.20	1.00	2.10	1.5
End condition code		Rotation fixed. Transition fixed.				
		Rotation free. Transition fixed.				
		Rotation fixed. Transition free.				
		Rotation free. Transition free.				
		Rotation partially restrained. Transition free.				

Table 7.1 Effective length of idealised columns

For restraining supports, the minimum force required in the restraining members should not be less than 1% of the force in the member being restrained.

7.3.1.2 Column in a multi-storey frame

For columns in the sub-frame of a multi-storey frames shown in Figure 7.10, a more rigorous method considering the boundary conditions and the interaction with the restraining beams is available. *Figure 6.4* in HK Code is used to approximate the effective length factor (L_E/L) of a column in a multi-storey frame. For frames with elastic critical load factor not less than 10 (i.e. $\lambda_{cr} \geq 10$), the frame is considered as non-sway or braced, and *Figure 6.5a* in HK Code should be used for finding the effective length factor. For frames with elastic critical load factor less than 10 (i.e. $\lambda_{cr} < 10$), the frame is sway or unbraced, and *Figure 6.5b* in HK Code should be used for finding of the effective length factor. The distinction between these two classes of frames is essential as we can see the range of effective length factor (L_E/L) is from 0.5 to 1.0 for non-sway frames and 1.0 to infinity for sway frames. λ_{cr} can be obtained from Equation (7.1) of this chapter or from *Clause 6.3.2.2* in HK Code.

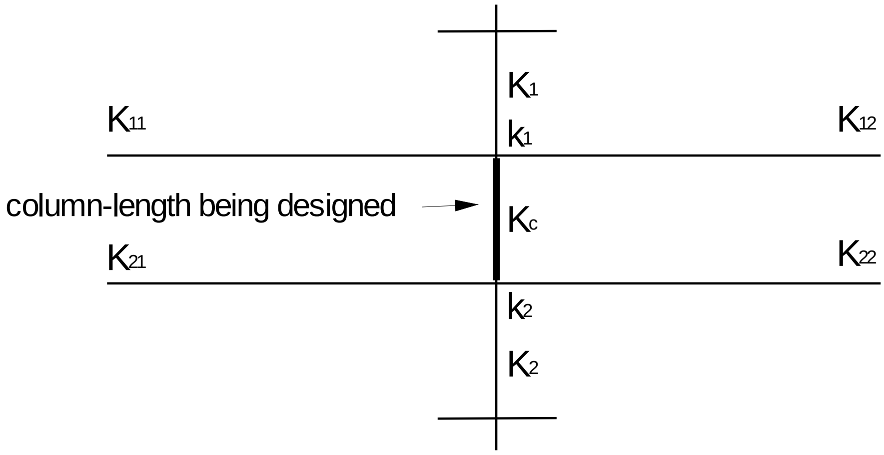


Figure 7.10 Column in a sub-frame

The non-dimensional parameters k_1 and k_2 can be found by finding the ratio of the column under consideration and the connecting beam stiffness as,

$$k_1 = \frac{K_c + K_1}{K_c + K_1 + K_{11} + K_{12}} \quad (7.11)$$

$$k_2 = \frac{K_c + K_2}{K_c + K_2 + K_{21} + K_{22}} \quad (7.12)$$

where
K_1 and K_2 are the values of stiffness for the adjacent column lengths;
K_{11}, K_{12}, K_{21} and K_{22} are the values of stiffness for the adjacent beams.

In order to cater for the restraining effect of beam stiffness under sway and non-sway frames and frames supporting concrete slab, the factors K_{11}, K_{12}, K_{21} and K_{22} are required to be modified to respectively 1.5, 0.5 and 1.0 time flexural constant of the beam (i.e. $\frac{EI}{L}$ for these three cases and when the beams are principally under end moments. For case where the beam is principally under load along its span, the modification factors should be changed to 1.0, 0.75 and 1.0 respectively for the three cases.

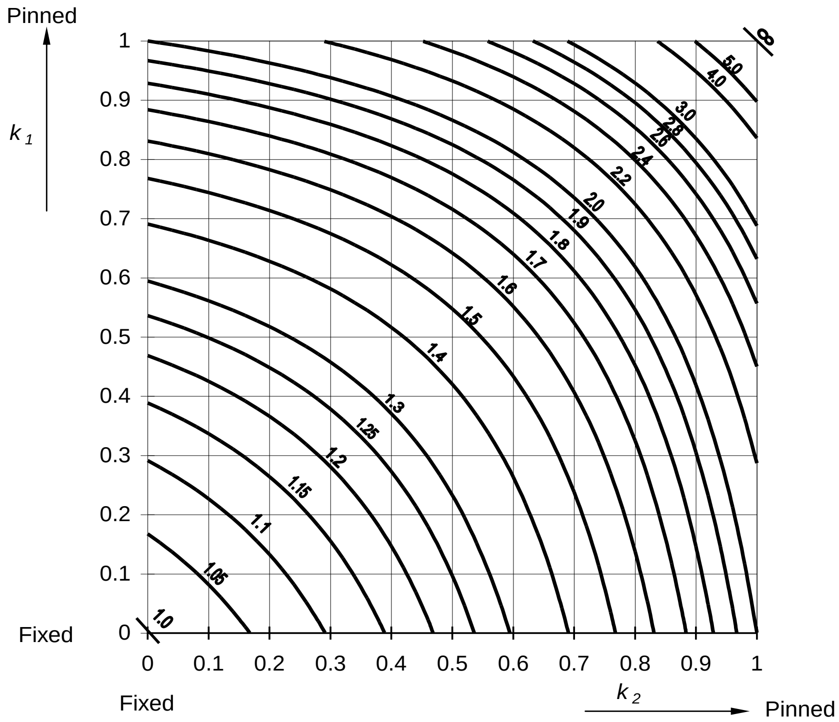


Figure 7.11(a) Effective length factor of sway frames

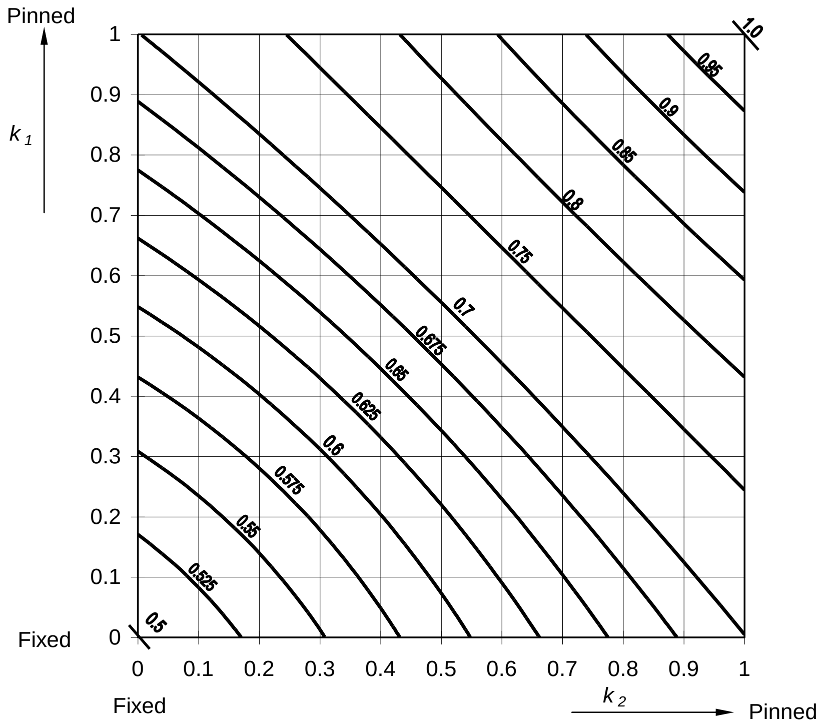


Figure 7.11(b) Effective length factor for non-sway frames

7.3.1.3 Compression members in general

Effective length of members in other types of structures like triangulated trusses and frames is mainly empirical with the recommended effective length given in *Clause 8.7.9* in the HK Code. A safe, economical and more reliable design for this type of structures should take into account the effect of eccentric connections via the eccentric moment in the conventional design. For complex and advanced structural systems like domes as shown in Figure 7.12, the effective length is more difficult to assess and resort must be made to second-order elastic or inelastic (advanced) analysis. Great care should be taken here for checking of effective length under snap-through buckling or other modes in these special structural systems.

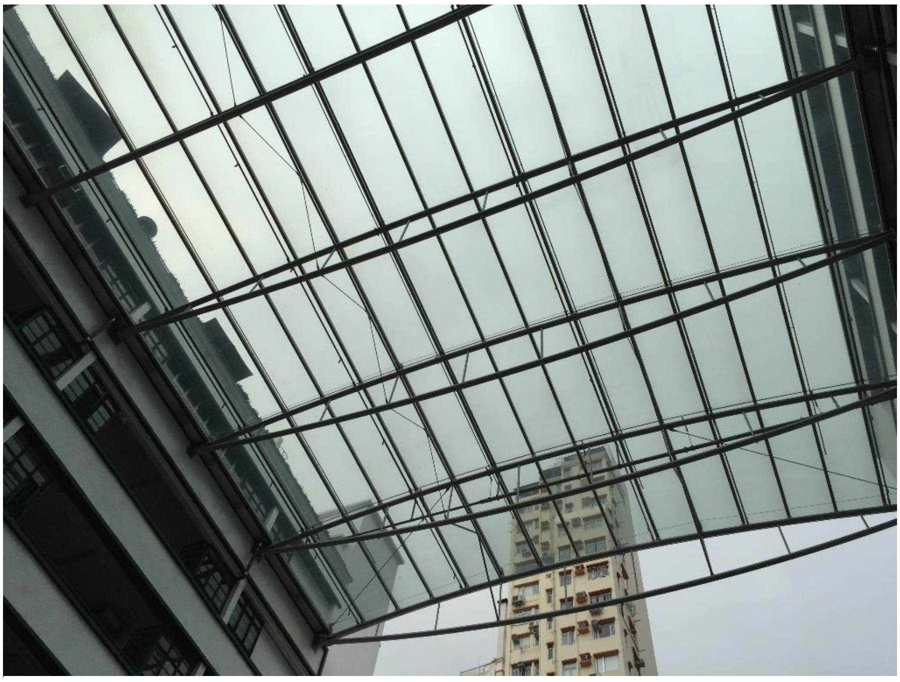


Figure 7.12 Inapplicability of the effective length method in design of the chords which have a slenderness ratio greater than 250

7.3.2 Slenderness ratio

With a value of effective length (L_E) for a member, the slenderness ratio can be determined as,

$$\lambda = \frac{L_E}{r} \quad \text{for Classes 1, 2 and 3 non-slender sections and} \qquad (7.13)$$

$$\lambda = \frac{L_E}{r}\sqrt{\frac{A_{eff}}{A_g}} \quad \text{for Class 4 slender sections} \qquad (7.14)$$

in which A_{eff} is the effective area of the section and A_g is the gross sectional area.

7.3.3 Buckling strength p_c and buckling resistance Pc

The buckling strength p_c can be found from *Table 8.8* of the HK Code and the buckling resistance of the column can be determined as follows.

For non-slender section including Class 1 plastic, Class 2 compact and Class 3 semi-compact cross sections,

$$P_c = A_g\, p_c \tag{7.15}$$

For Class 4 slender cross-sections,

$$P_c = A_{eff}\, p_{cs} \tag{7.16}$$

in which:

A_{eff} is the effective cross-sectional area in *Clause 7.6*;

A_g is the sum of gross sectional area in *Clause 9.3.4.1*;

p_c is the compressive strength in *Clause 8.7.6*;

p_{cs} is the value of p_c obtained using a reduced slenderness of $\lambda\sqrt{\dfrac{A_{eff}}{A_g}}$ where λ is the slenderness ratio calculated from the radius of gyration of the gross sectional area and member length.

For welded I, H or box section, design strength p_y should be reduced by 20N/mm^2 and p_c should then be determined from this reduced value of p_y.

In design of simple structures, all beams are assumed simply supported on columns. The bending behaviour on the compression member in a simple steel building frame should account for load eccentricity as indicated in *Clause 8.7.8* in HK Code. The equivalent moment factor m_{LT} in columns should be taken as unity with effective length for column buckling taken in *Clause 6.6.3* and the equivalent slenderness ratio for lateral-torsional buckling is calculated as follows.

$$\lambda_{LT} = \frac{0.5L}{r_y} \tag{7.17}$$

7.4 Design procedures of compression member

The procedure for design of a compression member can be summarized a follows.

1. Section and steel grade selection
2. Selection of design strength from *Tables 3.2 to 3.6*, HK Code
3. Section classification
4. If the section is Class 4 slender, reduce the design strength p_y by the effective stress method or calculate the effective area by the effective width method. For welded sections, reduce the design strength by 20 N/mm^2.
5. Classify the frame as non-sway, sway or sway ultra-sensitive frame by finding

of λ_{cr} by formula as $\lambda_{cr} = \frac{F_N}{F_V}\frac{h}{\delta_N}$ or by computer software.

6. Determine the effective length assuming the frame is simple one storey or typical multi-storey frame.
7. Calculate the slenderness ratio about the two principal axes as,

$\lambda = \frac{L_E}{r}$ for Classes 1, 2 and 3 non-slender sections and

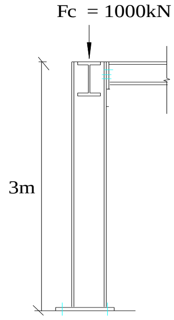
$\lambda = \frac{L_E}{r}\sqrt{\frac{A_{eff}}{A_g}}$ for Class 4 slender sections

8. Use appropriate clauses for other sections like channel, angle and T sections
9. Select a suitable buckling curve from a_0, a, b, c and d curves for the section
10. Determine the buckling strength p_c from *Table 8.8* in HK Code.
11. Calculate the compression resistance P_c as $P_c = p_c A_g$ for Classes 1, 2 and 3 sections or $P_c = p_{cs} A_{eff}$ or Class 4 section.

7.5 Worked Examples

7.5.1 Compression resistance of restrained column

A 3m high H-section steel column of section 254×254×73 UC in S275 steel material, supports a factored compression load of 1000kN. The column is assumed to be pinned at the base support and at top of column as shown in figure below. Check the adequacy of compression resistance of the column.



Solution

SECTION PROPERTIES

$D = 254.1mm,\ B = 254.6mm,\ t = 8.6mm,\ T = 14.2mm,\ d = 200.3mm,$

$r_y = 6.48cm,\ A = 93.1cm^2$

SECTION CLASSIFICATION

Design strength, $p_y = 275N/mm^2$ for $T \le 16mm$ *(Table 3.2)*

$$\varepsilon = \sqrt{\frac{275}{275}} = 1$$ *(Table 7.1 Note b)*

Limiting value of *b/T* for outstand flange of an H-section is 13ε

$$\frac{b}{T} = \frac{254.6}{2\times 14.2} = 8.96 \le 13\times 1 = 13$$ *(Table 7.1)*

$\therefore$ flange is non-slender

Limiting value of *d/t* for web of an H-section under axial compression is $120\varepsilon/(1+2r_2)$

Stress ratio, $r_2 = \dfrac{F_c}{A_g p_{yw}} = \dfrac{1000\times 10^3}{9310\times 275} = 0.391$ *(7.2)*

$$\frac{d}{t} = \frac{200.3}{8.6} = 23.3 \le \frac{120\varepsilon}{1+2r_2} = \frac{120}{1+2\times 0.391} = 67.3 \ge 40\times 1 = 40$$ *(Table 7.1)*

$\therefore$ web is non-slender

$\therefore$ the section is non-slender

COMPRESSION RESISTANCE

Buckling about minor axis is more critical

Effective length, $L_E = 1.0L = 3m$ *(Table 8.6)*

Slenderness ratio, $\lambda = \dfrac{L_E}{r_y} = \dfrac{3000}{64.8} = 46.3$ *(Clause 8.7.4)*

Compressive strength for rolled H-section with maximum thickness ≤40mm bending about *y-y* axis should be obtained from buckling curve (c) *(Table 8.7)*

Compressive strength, $p_c = 227.4N/mm^2$ *(Table 8.8(c))*

Compression resistance, $P_c = A_g p_c = 9310\times 227.4 = 2117.1kN > F_c$ *(OK)* *(8.73)*

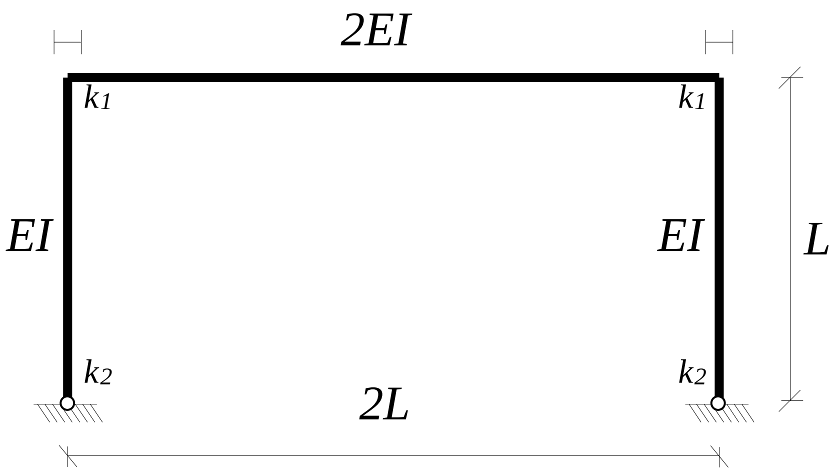
7.5.2 Compression resistance of column in the portal frame

(1) Determine the design compression resistance of 203×203×60 UC of Grade S355 steel with effective length L_E 3.5m.

(2) Determine the design compression resistance of the columns of the rigid-jointed frame as shown in the following figure if L is equal to 3.5m and the beam load is zero by assuming that

(a) frame is fully braced against sidesway and no out-of-plane buckling, and

(b) frame is unbraced against sidesway and no out-of-plane buckling.



<u>Solution</u>

SECTION PROPERTIES

$D = 209.6mm$, $B = 205.8mm$, $t = 9.4mm$, $T = 14.2mm$, $d = 160.8mm$, $r_x = 8.96cm$, $r_y = 5.20cm$, $A = 76.4cm^2$

SECTION CLASSIFICATION

Design strength, $p_y = 355N/mm^2$ for $T \le 16mm$ *(Table 3.2)*

$$\varepsilon = \sqrt{\frac{275}{355}} = 0.88$$ *(Table 7.1 Note b)*

Limiting value of b/T for outstand flange of an H-section is 13ε

$$\frac{b}{T} = \frac{205.8}{2 \times 14.2} = 7.25 \le 13 \times 0.88 = 11.4$$ *(Table 7.1)*

$\therefore$ flange is non-slender

Limiting value of d/t for web of an H-section under axial compression is 40ε

$$\frac{d}{t} = \frac{160.8}{9.4} = 17.1 \le 40 \times 0.88 = 35.2$$ *(Table 7.1)*

$\therefore$ web is non-slender

$\therefore$ the section is non-slender

(1) COMPRESSION RESISTANCE

Buckling about minor axis is more critical in this case

Effective length, $L_E = 3.5m$

Slenderness ratio, $\lambda = \frac{L_E}{r_y} = \frac{3500}{52} = 67.3$ *(Clause 8.7.4)*

Compressive strength for rolled H-section with maximum thickness less than 40mm bending about *y-y* axis should be obtained from buckling curve c *(Table 8.7)*

Compressive strength, $p_c = 225.1 N/mm^2$ *(Table 8.8(c))*

Compressive resistance, $P_c = p_c A_g = 225.1 \times 7640 = 1719.8kN$ *(8.73)*

(2) COMPRESSION RESISTANCE

a) When the frame is classified as non-sway, beam stiffness should be taken as $0.5I/L$ *(Table 6.2)*

$$k_1 = \frac{K_c + K_1}{K_c + K_1 + K_{11} + K_{12}} = \frac{I/L}{I/L + 0.5 \times 2I/2L} = 0.67$$ *(Figure 6.4)*

$k_2 = 1$ for pinned end

Effective length, $L_E = 0.88L = 0.88 \times 3.5 = 3.08m$ *(Figure 6.5b)*

Slenderness ratio, $\lambda = \frac{L_E}{r_x} = \frac{3080}{89.6} = 34.4$ *(Clause 8.7.4)*

Compressive strength for rolled H-section with maximum thickness less than 40mm bending about *x-x* axis should be obtained from buckling curve b *(Table 8.7)*

Compressive strength, $p_c = 328.0 N/mm^2$ *(Table 8.8(b))*

Compressive resistance, $P_c = p_c A_g = 328.0 \times 7640 = 2505.9kN$ *(8.73)*

b) When the frame is classified as sway, beam stiffness should be taken as $1.5I/L$ *(Table 6.2)*

$$k_1 = \frac{K_c + K_1}{K_c + K_1 + K_{11} + K_{12}} = \frac{I/L}{I/L + 1.5 \times 2I/2L} = 0.4$$ *(Figure 6.4)*

$k_2 = 1$ for pinned end

Effective length, $L_E = 2.34L = 2.34 \times 3.5 = 8.19m$ *(Figure 6.5a)*

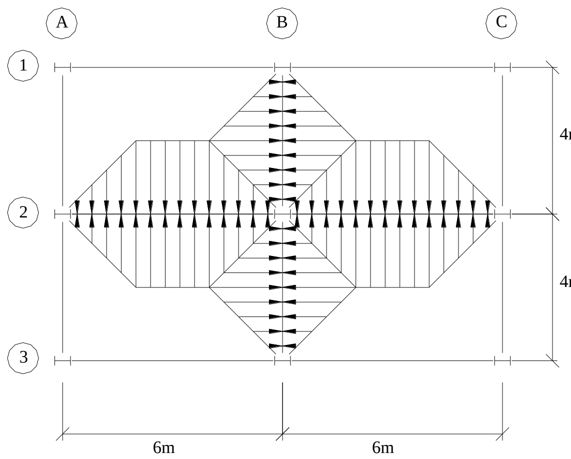
Slenderness ratio, $\lambda = \frac{L_E}{r_x} = \frac{8190}{89.6} = 91.4$ *(Clause 8.7.4)*

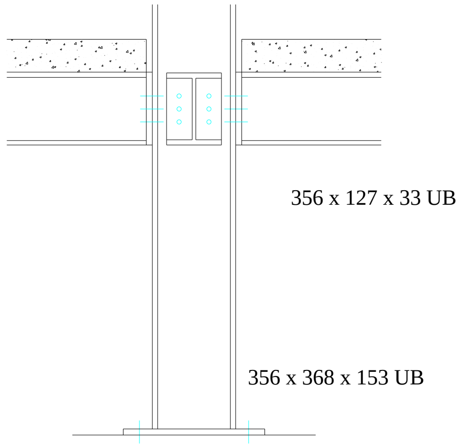
Compressive strength, $p_c = 177.5 N/mm^2$ *(Table 8.8(b))*

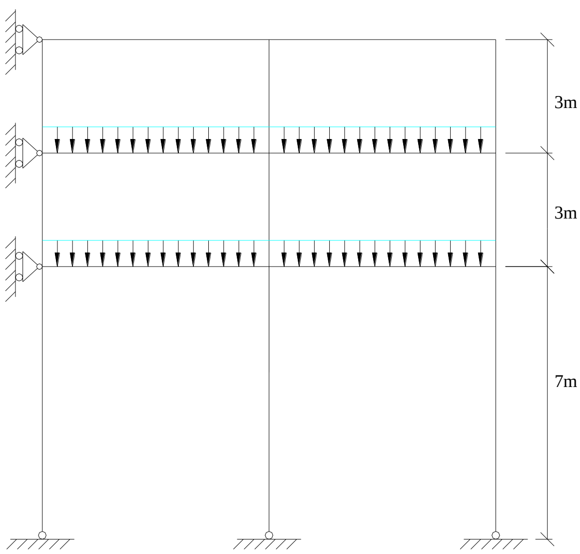
Compressive resistance, $P_c = p_c A_g = 177.5 \times 7640 = 1356.1kN$ *(8.73)*

7.5.3 Compression member in the braced multi-storey frame

A 3-storey composite frame shown in the figure below has been classified as non-sway. A plan view of the frame is shown with a floor system. A 7m high column support the I-beams, which carry concrete slab floor. The connection detail between column and beams is also shown in the figure below. Thus the top end condition of column is rigidly held in position without rotational restraints as indicated and the column base is designed as pinned end. The dead load on the concrete floor slab is $4kN/m^2$ (including the self-weight of floor slab and finishes) and the imposed load is $4.5kN/m^2$. The column section is 356×368×153 UC in S275 steel material. The loading on the roof level is $4kN/m^2$ dead load. Check the structural adequacy of the H-column at gridlines ② & Ⓑ.







Solution

DESIGN LOAD

Factored distributed load on floor, $\omega_1 = 1.4 \times 4 + 1.6 \times 4.5 = 12.8 kN/m^2$ *(Table 4.2)*

Factored distributed load on roof, $\omega_2 = 1.4 \times 4 = 5.6 kN/m^2$

Compression force on column, $F_c = (12.8 \times 2 + 5.6) \times 24 = 748.8 kN$

SECTION PROPERTIES

$D = 362.0mm$, $B = 370.5mm$, $t = 12.3mm$, $T = 20.7mm$, $d = 290.2mm$, $I_x = 48600cm^4$,

$I_y = 17600cm^4$, $r_x = 15.8cm$, $r_y = 9.49cm$, $A = 195cm^2$

SECTION CLASSIFICATION

Design strength, $p_y = 265 N/mm^2$ for $16mm < T \le 40mm$ *(Table 3.2)*

$\varepsilon = \sqrt{\frac{275}{265}} = 1.02$ *(Table 7.1 Note b)*

Limiting value of b/T for outstand flange of an H-section is 13ε

$\frac{b}{T} = \frac{370.5}{2 \times 20.7} = 8.95 \le 13 \times 1.02 = 13.3$ *(Table 7.1)*

$\therefore$ flange is non-slender

Limiting value of d/t for web of an H-section under axial compression is $120\varepsilon/(1 + 2r_2)$

Stress ratio, $r_2 = \frac{F_c}{A_g p_{yw}} = \frac{748.8 \times 10^3}{19500 \times 265} = 0.145$ *(7.2)*

$\frac{d}{t} = \frac{290.2}{12.3} = 23.6 \le \frac{120 \times 1.02}{1 + 2 \times 0.145} = 94.9$ *(Table 7.1)*

$\therefore$ web is non-slender

$\therefore$ the section is non-slender

COMPRESSION RESISTANCE

Buckling about minor axis is more critical

Effective length, $L_E = 1.0L = 7m$ *(Table 8.6)*

Slenderness ratio, $\lambda = \frac{L_E}{r_y} = \frac{7000}{94.9} = 73.8$ *(Clause 8.7.4)*

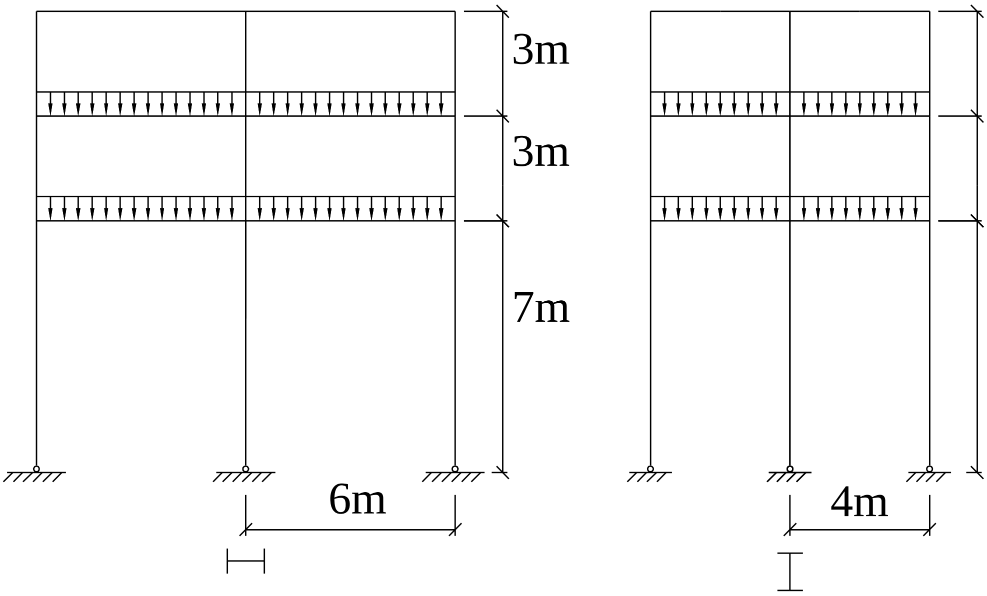
Compressive strength for rolled H-section with maximum thickness less than 40mm bending about *y-y*
axis should be obtained from buckling curve c *(Table 8.7)*

Compressive strength, $p_c = 169.3 N/mm^2$ *(Table 8.8(c))*

Compressive resistance, $P_c = A_g p_c = 19500 \times 169.3 = 3301.4 kN > F_c$ *(OK)* *(8.73)*

7.5.4 Compression member in unbraced multi-storey frame

The 3-storey frame of steel grade S275 and columns of 356×368×153 UC and beams of 356×127×33 UB is a moment frame with rigid member connections and pinned bases. Check the compression resistance of the column. Dead load is 2kN/m^2 and Live load is 4.5kN/m^2.



<u>Solution</u>

DESIGN LOAD

Factored distributed load on floor, $\omega = 1.4 \times 2 + 1.6 \times 4.5 = 10 kN/m^2$ *(Table 4.2)*

Compression force on column, $F_c = 10 \times 6 \times 4 \times 2 = 480kN$

SECTION PROPERTIES

$D = 362.0mm$, $B = 370.5mm$, $t = 12.3mm$, $T = 20.7mm$, $d = 290.2mm$, $I_x = 48600cm^4$,

$I_y = 17600cm^4$, $r_x = 15.8cm$, $r_y = 9.49cm$, $A = 195cm^2$

SECTION CLASSIFICATION

Design strength, $p_y = 265 N/mm^2$ for $16mm < T \le 40mm$ *(Table 3.2)*

$$\varepsilon = \sqrt{\frac{275}{265}} = 1.02$$ *(Table 7.1 Note b)*

Limiting value of b/T for outstand flange of an H-section is 13ε

$$\frac{b}{T} = \frac{370.5}{2 \times 20.7} = 8.95 \le 13 \times 1.02 = 13.3$$ *(Table 7.1)*

$\therefore$ flange is non-slender

Limiting value of d/t for web of an H-section under axial compression is $120\varepsilon/(1 + 2r_2)$

Stress ratio, $$r_2 = \frac{F_c}{A_g p_{yw}} = \frac{480 \times 10^3}{19500 \times 265} = 0.093$$ *(7.2)*

$$\frac{d}{t}=\frac{290.2}{12.3}=23.6\leq\frac{120\times1.02}{1+2\times0.093}=103.2$$ *(Table 7.1)*

$\therefore$ web is non-slender

$\therefore$ the section is non-slender

FRAME CLASSIFICATION

The notional horizontal force applied at the centre node due to factored total load is 10×12×4×0.5%= 2.4kN in both directions. The notional force applied at the end node is 2.4/2 = 1.2kN in both directions. From linear analysis by NIDA, the deflections about major and minor of the column under consideration are 3.24mm and 5.97mm respectively.

About major axis,

Elastic critical load factor, $\lambda_{cr}=\frac{F_N}{F_v}\frac{h}{\delta_N}=\frac{1}{200}\times\frac{7000}{3.24}=10.8$ *(6.1)*

$\because \lambda_{cr}\geq 10$ *(6.2)*

$\therefore$ it is a non-sway frame *(Clause 6.3.3)*

About minor axis,

Elastic critical load factor, $\lambda_{cr}=\frac{F_N}{F_v}\frac{h}{\delta_N}=\frac{1}{200}\times\frac{7000}{5.97}=5.86$ *(6.1)*

$\because 10>\lambda_{cr}\geq 5$ *(6.4)*

$\therefore$ it is a sway frame *(Clause 6.3.4)*

COMPRESSION RESISTANCE

The boundary condition of the interior column should be assessed by considering it as a column in a sub-frame, while the stiffness of the connected beam contributing to the effective length of interior column is taken into account in the frame. Thus the determination of effective length of the interior column allows for the effect of restraining beam stiffness here.

The second moment of inertia of connecting beam (356×127×33 UB) is 8250cm^4

About major axis,

Beam stiffness in non-sway mode should be taken as $0.75\frac{I}{L}$ *(Table 6.2)*

$$K_c=\frac{I_x}{L_c}=\frac{48600}{700}=69.43cm^3$$

$$K_1=\frac{I_x}{L_1}=\frac{48600}{300}=162cm^3$$

$$K_{11}=K_{12}=\frac{I_{bx}}{L_b}=\frac{8250}{600}=13.75cm^3$$

$$k_1=\frac{K_c+K_1}{K_c+K_1+K_{11}+K_{12}}=\frac{69.43+162}{69.43+162+0.75\times13.75\times2}=0.92$$ *(Figure 6.4)*

$k_2=1$ for pinned end

Effective length, $L_E=0.97L=0.97\times7=6.79m$ *(Figure 6.5b)*

Slenderness ratio, $\lambda=\frac{L_E}{r_x}=\frac{6790}{158}=43.0$ *(Clause 8.7.4)*

Compressive strength for rolled H-section with maximum thickness less than 40mm bending about *x-x*

axis should be obtained from buckling curve b *(Table 8.7)*

Compressive strength, $p_c=238.0\,N/mm^2$ *(Table 8.8(b))*

About minor axis,

Beam stiffness in sway mode should be taken as $1.0\dfrac{I}{L}$ *(Table 6.2)*

$$K_c = \frac{I_y}{L_c} = \frac{17600}{700} = 25.14cm^3$$

$$K_1 = \frac{I_y}{L_1} = \frac{17600}{300} = 58.67cm^3$$

$$K_{11} = K_{12} = \frac{I_{bx}}{L_b} = \frac{8250}{400} = 20.63cm^3$$

$$k_1 = \frac{K_c + K_1}{K_c + K_1 + K_{11} + K_{12}} = \frac{25.14 + 58.67}{25.14 + 58.67 + 20.63 \times 2} = 0.67$$ *(Figure 6.4)*

$k_2 = 1$ for pinned end

Effective length, $L_E = 2.9L = 2.9 \times 7 = 20.3m$ *(Figure 6.5a)*

Slenderness ratio, $\lambda = \dfrac{L_E}{r_y} = \dfrac{20300}{94.9} = 213.9$ *(Clause 8.7.4)*

Compressive strength for rolled H-section with maximum thickness less than 40mm bending about *y*-*y*
axis should be obtained from buckling curve c *(Table 8.8(c))*

Compressive strength, $p_c = 36.8 N/mm^2$

$\therefore$ buckling about minor axis is more critical

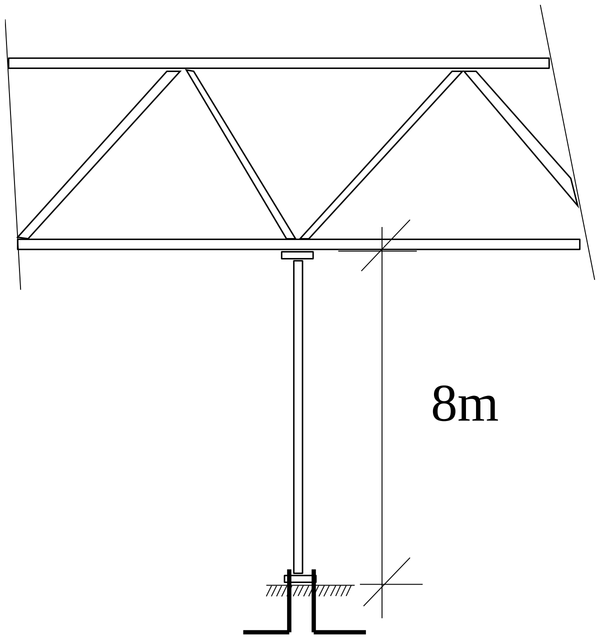
Compression resistance, $P_c = p_c A_g = 36.8 \times 19500 = 717.6kN > F_c$ *(OK)* *(8.73)*

The compression resistance of the column in the frame is greatly reduced when the frame is changed from the non-sway mode to the sway mode, because of the *P-Δ* effect.

The above represents one load case with maximum axial force and minimum moment since wind load and loads on alternative bays are not considered. Other load cases should also be considered.

7.5.5 Column with circular hollow section in Chinese steel

Part of a truss system with rigid lateral restraint is supported by a circular tube column of CHS 219×16 grade Q345 steel and 8m length as shown in the figure below. The column is under a factored load of 1000kN. And the base of circular tube column is assumed pinned to the ground and the upper column is pined to the truss. Check the buckling resistance of the tube column.



<u>Solution</u>

SECTION PROPERTIES

$D = 219.1mm$, $t = 16mm$, $A = 102cm^2$, $I = 5300cm^4$, $r = 7.20cm$, $Z = 483cm^3$, $S = 661cm^3$

SECTION CLASSIFIACTION

Design strength, $p_y = 310N/mm^2$ for $t \le 16mm$ *(Table 3.3)*

$$\varepsilon^2 = \frac{275}{310} = 0.89$$ *(Table 7.2 Note b)*

Limiting value of D/t for circular hollow section under axial compression is $80\varepsilon^2$

$$\frac{D}{t} = \frac{219.1}{16} = 13.7 \le 80 \times 0.89 = 71.2$$ *(Table 7.2)*

$\therefore$ the section is non-slender

COMPRESSION RESISTANCE

Effective length, $L_E = L = 8m$ *(Table 8.6)*

Slenderness ratio, $\lambda = \frac{L_E}{r} = \frac{8000}{72} = 111.1$ *(Clause 8.7.4)*

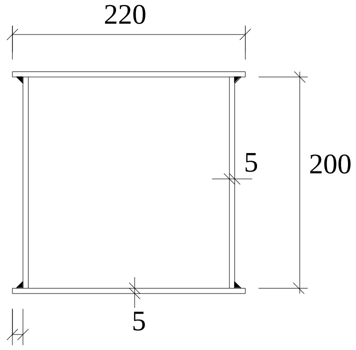
Compressive strength for hot-finihsed structural hollow section should be otained from buckling curve a *(Table 8.7)*

Compressive strength, $p_c = 138.3N/mm^2$ *(Table 8.8e)*

Compressive resistance, $P_c = A_g p_c = 10200 \times 138.3 = 1410.7kN > F_c$ *(OK)* *(8.73)*

7.5.6 Compression resistance of slender welded column

A 2.5m high column with both ends pinned and subjected to 1000kN factored load is to be designed. The section is welded box section with dimensions shown in the adjacent figure. All plate elements of the welded box section are S355 steel and of thickness 5mm.



Solution

SECTION CLASSIFICATION

Width of section, $b = 220 - 2\times10 - 2\times5 = 190mm$ *(Figure 7.1)*

Depth of section, $d = 200mm$

Design strength, $p_y = 355 N/mm^2$ *(Table 3.2)*

$\varepsilon = \sqrt{\dfrac{275}{355}} = 0.88$ *(Table 7.1 Note b)*

Limiting value of b_o/T for outstand flange of a box section under axial compression is 13ε

$$\frac{b_o}{T} = \frac{10}{5} = 2 \le 13\times0.88 = 11.4 \qquad \textit{(Table 7.1)}$$

$\therefore$ outstand flange is non-slender

Limiting value of b/T for internal flange of a box section under axial compression is 40ε

$$\frac{b}{T} = \frac{190}{5} = 38 \ge 40\times0.88 = 35.2 \qquad \textit{(Table 7.2)}$$

$\therefore$ internal flange is slender

Limiting value of d/t for web of a box section under axial compression is $120\varepsilon/(1+2r_2)$

Gross area, $A_g = (220+200)\times5\times2 = 4200mm^2$

$$\text{Stress ratio, } r_2 = \frac{F_c}{A_g p_{yw}} = \frac{1000\times10^3}{4200\times355} = 0.671 \qquad \textit{(7.6)}$$

$$\frac{d}{t} = \frac{200}{5} = 40 \le \frac{120\times0.88}{1+2\times0.671} = 45.1 \qquad \textit{(Table 7.1)}$$

$\therefore$ web is non-slender

$\therefore$ the section is slender

Assume the outstand flange is ignored, by the effective width method,

$A = (190+200)\times5\times2 = 3900mm^2$

$K = 4$ for conservative approach *(Clause 11.3.4.4.3)*

$$f_c = \frac{1000\times10^3}{3900} = 256.4 N/mm^2 \qquad \textit{(Clause 11.3.4.4.1)}$$

For flange,

$$p_{cr} = 0.904EK\left(\frac{t}{b}\right)^2 = 0.904\times205000\times4\times\left(\frac{5}{190}\right)^2 = 513.4 N/mm^2 \qquad \textit{(11.11)}$$

$$\rho = \frac{f_c}{p_{cr}} = \frac{256.4}{513.4} = 0.499 > 0.123 \qquad \textit{(11.10)}$$

$$\beta = \left\{1+14\left(\sqrt{\rho}-0.35\right)^4\right\}^{-0.2} = \left\{1+14\left(\sqrt{0.499}-0.35\right)^4\right\}^{-0.2} = 0.960 \qquad \textit{(11.9b)}$$

$$b_e = \beta b = 0.96\times190 = 182.4mm \qquad \textit{(11.8)}$$

For web,

$$p_{cr} = 0.904 \times 205000 \times 4 \times \left(\frac{5}{200}\right)^2 = 463.3\, N/mm^2$$

$$\rho = \frac{f_c}{p_{cr}} = \frac{256.4}{463.3} = 0.553 > 0.123 \qquad (11.10)$$

$$\beta = \left\{1 + 14\left(\sqrt{0.553} - 0.35\right)^4\right\}^{-0.2} = 0.944 \qquad (11.9b)$$

$$b_e = \beta b = 0.944 \times 200 = 188.8mm \qquad (11.8)$$

Effective Area, $A_{eff} = (182.4 + 188.8) \times 5 \times 2 = 3712mm^2$

For welded sections under axial compression buckling, p_y should be reduced by 20 N/mm^2

$$p_y = 355 - 20 = 335\, N/mm^2 \qquad (Clause\ 8.7.6)$$

Compressive strength for welded box section with thickness less than 40mm bending about both axes should be obtained from buckling curve b *(Table 8.7)*

$$I_x = \left(\frac{5 \times 200^3}{12} + \frac{220 \times 5^3}{12} + 220 \times 5 \times 102.5^2\right) \times 2 = 2.979 \times 10^7\, mm^4$$

$$I_y = \left(\frac{220 \times 5^3}{12} + \frac{200 \times 5^3}{12} + 200 \times 5 \times 97.5^2\right) \times 2 = 2.789 \times 10^7\, mm^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{2.789 \times 10^7}{4200}} = 81.5$$

Slenderness ratio, $\lambda = \frac{L_E}{r_y} = \frac{2500}{81.5} = 30.7$ *(Clause 8.7.4)*

Reduced slenderness, $\lambda\sqrt{\frac{A_{eff}}{A_g}} = 30.7\sqrt{\frac{3712}{4200}} = 28.9$ *(Clause 8.7.5)*

Compressive strength, $p_c = 317.5\, N/mm^2$ *(Table 8.8b)*

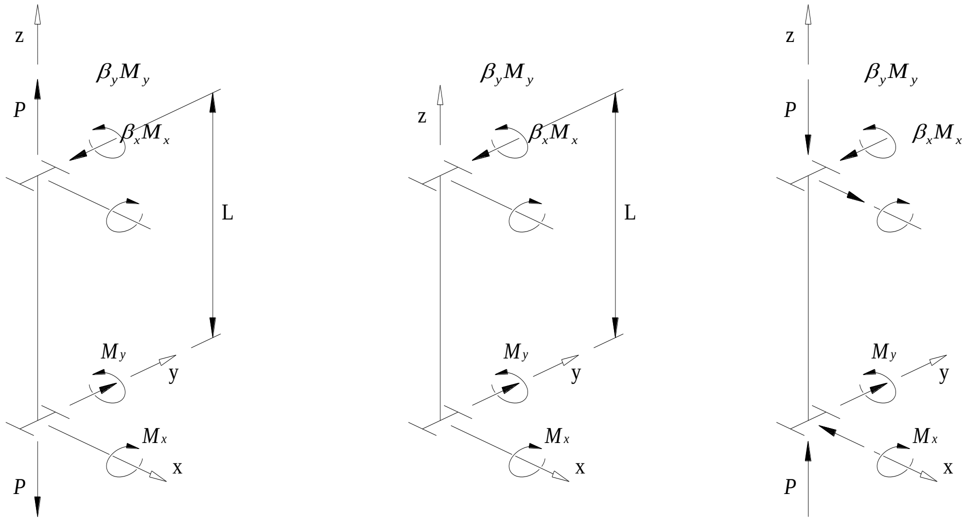
Compressive resistance, $P_c = A_{eff}\, p_{cs} = 3712 \times 317.5 = 1178.6kN > F_c$ *(OK)* *(8.74)*

Chapter 8 Beam-columns

8.1 Introduction to beam-columns

Columns and beams are strictly speaking idealized structural members under pure axial force or pure bending moment since all members are subjected to unavoidable small forces and moments. In the design context, we may consider beams or columns are structural elements with dominant axial force and bending moment. When a structural member is not under a single action of axial force or bending moment, the design should include their interaction behaviour. Columns are commonly under eccentric axial force that the ignorance of bending moment is on the non-conservative side. Many practical structural elements in steel building structures are under the simultaneous action of bending moment and axial force and they are termed as beam-columns. The design of beam-columns is based on both beam and column design and it is relatively complex. The structural behaviour on beam-columns is summarized in Figure 8.1, in which the first case is the member under tension and bending about two axes, the second case is under bending about two principal axes free from axial force and the third case is the column under a combined action of axial compression and bending about two principal axes.

For beam-columns in practical range, both the material nonlinear and geometric nonlinear effects play roles in the behaviour of beam-columns. For material nonlinear effects, the load carrying capacity of beam-column member can be based on the plastic design. Geometrically nonlinear effects on beam-column member comprise of local plate buckling, compression buckling, lateral-torsional buckling and axial-torsional buckling. According to the HK Code, the member resistance of beam-column member should be checked against cross-section capacity considering the material strength and member buckling resistance, which allows for both the geometric buckling and material yielding effects.



a) tension and biaxial bending b) biaxial bending c) compression and biaxial bending

Figure 8.1 Interaction effects on beam-column member

Nearly all sections can be used as beam-columns. The choice of a particular member section depends not only on the structural resistance and strength, but also the ease of fabrication which constitutes a high proportion of construction cost in steel structures.

In general, the structural check of a beam-column requires the checking on

- Local capacity and resistance check
- Cross section capacity check and
- Overall buckling check

8.2 Behaviour for combined tension and biaxial bending

8.2.1 Yield surface of tension members

Steel structures exist in three-dimensional space. For members under biaxial bending M_x and M_y, the normal stresses is equal to the bending stress σ_b about both x- and y-axes shown in Figure 8.2.

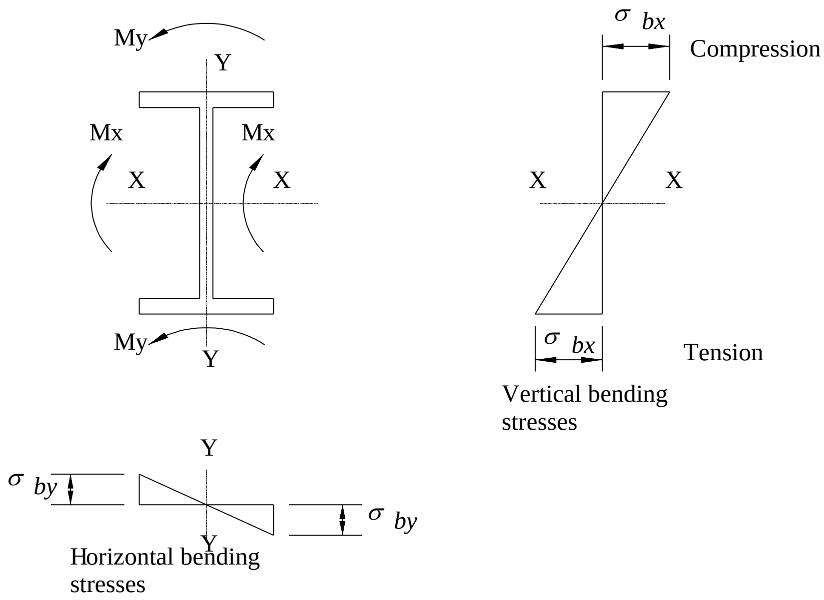


Figure 8.2 Bending stress distribution across section under biaxial bending

The combined stress formula under the elastic assumption is expressed in Equation (8.1). For more general elastic or plastic analysis, it can be re-written as Equation (8.2) for section capacity check. The conceptual considerations for the two equations can be significantly different, especially when the plastic moment capacity ($p_y S$) in place of the elastic moment capacity ($p_y Z$) of the section is used for the interactive function in Equation (8.2). Also, research is currently carried out on refining a more economical and yet safe yield function for governing the condition of forming a plastic hinge in a beam-column element in an elastic-plastic analysis (Chan and Chen 1995).

The member shown in Figure 8.1(a) resists tension load F_t and biaxial bending moments M_x and M_y. Unless the compressive stress created by bending moment is larger than the stress by tension force, the section is not controlled by compression buckling. The combined stress under tension load and bending moments can be determined by simple combined stress formula. However, in the HK Code, the case for having favourable tension force in a section should be checked by ignoring the presence of tension force unless the moment is directly due to eccentric tension force in which case the absence of moment implies the disappearance of axial force. For general cases, the favourable effect of tension force in a beam bent about its major principal axis should be ignored for beam lateral-torsional buckling check.

When a section is loaded under moments about two principal axes, the stresses induced on the beam-column member section is made up of normal tensile stress σ_t and bending stress σ_b about principal x- and y-axes. The resulting stress should be checked to be not greater than the material design strength p_y for structural adequacy when using the simplest linear interactive equation. Equation (8.1) is the elastic combined stress formula.

$$\sigma_t + \sigma_{bx} + \sigma_{by} = \frac{F_t}{A_g} + \frac{M_x}{Z_x} + \frac{M_y}{Z_y} \le p_y \qquad (8.1)$$

in which F_t is tension load, M_x and M_y are respectively bending moment about x- and y-axes, A_g is the gross section area and it should be replaced by the sectional net area (A_{net}) at the section with bolt opening, Z_x and Z_y are the elastic section modulus about the principal axes. Equation (8.1) can be normalized and refined as below.

$$\frac{F_t}{p_y A_g} + \frac{M_x}{p_y Z_x} + \frac{M_y}{p_y Z_y} = \frac{F_t}{P_t} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} \le 1 \qquad (8.2)$$

in which P_t is the tension load capacity, M_{cx} and M_{cy} are respectively the moment capacities about x- and y-axes. For plastic sections, the moment capacities M_{cx} and M_{cy} are revised to the plastic section capacity as $p_y S_x$ and $p_y S_y$, where S_x and S_y are the plastic section moduli about the respective axes. The use of plastic section modulus is based on a concept of section capacity strength rather than focusing on the stress in elastic stage.

The interaction between tension and bending is plotted in Figure 8.3. When a stress point lies outside the yield surface under the combined actions of force and moments, the beam-column member is considered as failed and the structure is considered to be inadequate when using the first plastic hinge design method or the member is not allowed to be further loaded. For simplicity and conservative design, the linear interaction surface represented by the dotted lines in Figure 8.3 is commonly adopted.

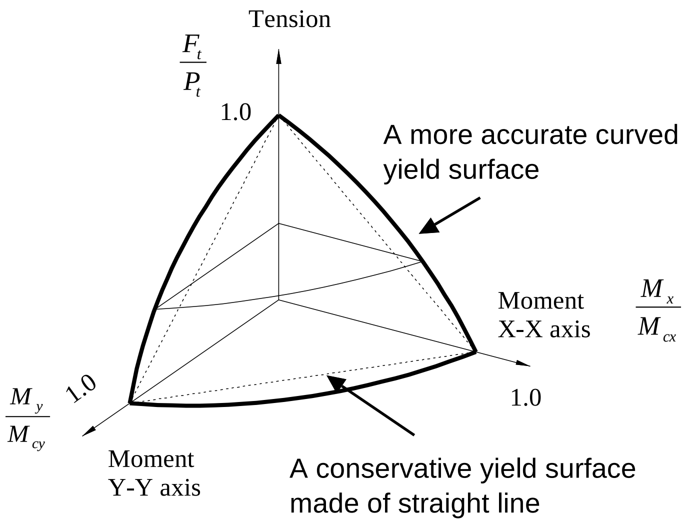


Figure 8.3 Actual and linearised yield surfaces of a section under tension and bending

The actual yield condition under bending moment and axial force involves shift of neutral axis. Its interaction with residual stress and local buckling makes the exact analysis or design complicated and unsuitable for practical uses. The simplified assumption of yield surface to control the combined axial force and moment makes the structural checking direct and convenient. For illustration, the two dimensional yield surfaces under the linear and non-linear interaction are shown in Figure 8.4. When a load point lies inside the linear failure surface at ‘B’, the summation of the tensile direct stress as ‘BC’ and bending stress as ‘AB’ is equal to unity in Figure 8.4 and indicated by Equation (8.2). Under such condition, the member is considered as structural adequate and safe. For a more accurate prediction of the section strength, the Von Mises-Hencky yield function can be used. *Clauses 8.8* and *8.9* in the HK Code allow the use of both the approaches of linear interaction and an assumption of axial force taken by central core area of the section around web with the bending moment taken by the remaining area. This approximation leads to a more economical design based on a curved yield surface with little additional computational effort in computer (Chan and Chui, 1997). The illustration can be seen in Figure 8.5 that the solid core at web is assigned to take the axial force.

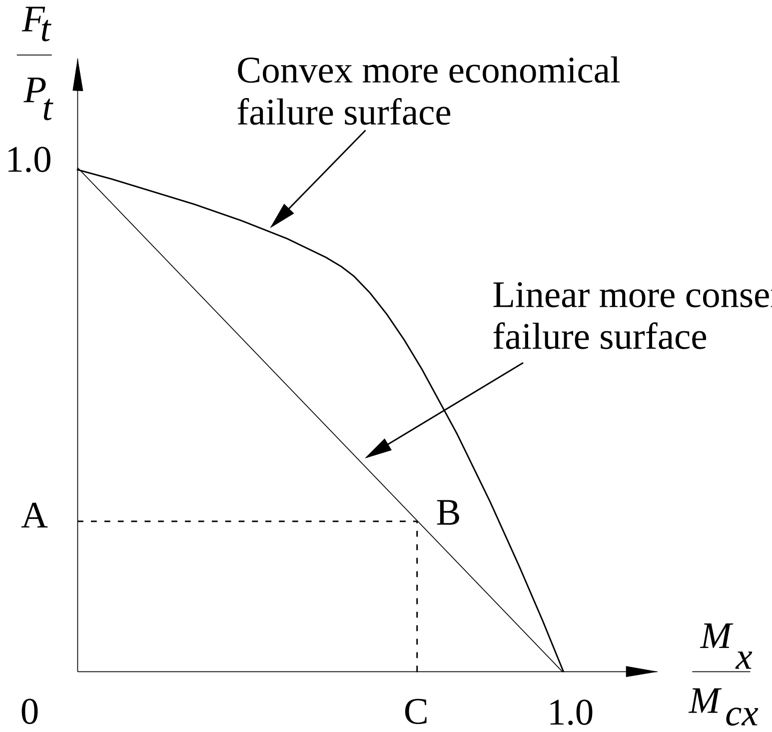


Figure 8.4 Interaction of tension force and bending moments

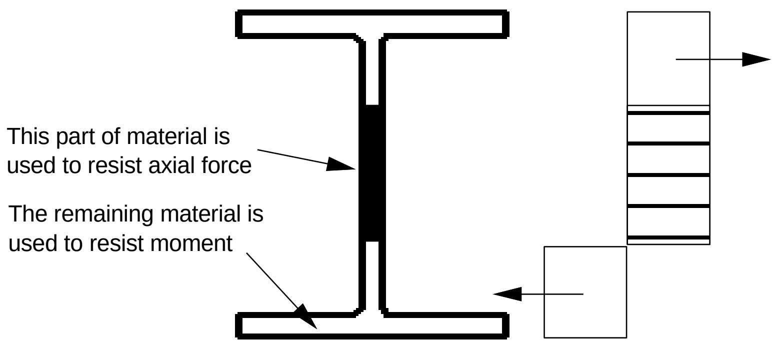


Figure 8.5 Sectional strength analysis under axial force and moment

8.2.2 Design procedures for stocky beam-columns

A section capable of resisting axial force and bending moment alone may become inadequate when the force and moment act simultaneously. The checking is covered in the HK Code.

Using the linear interaction between force and moments in Equation (8.2) with the complexity of shift of neutral axis ignored. The linearised interaction between the axial force and moments can be written as,

$$\frac{F_t}{P_t} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} \le 1 \tag{8.3}$$

in which F_t, M_x and M_y are the applied tension force and bending moment about the x- and y-axes, P_t is the tension capacity of the member under pure tension and it is equal to p_yA_e, where A_e is the effective area of the section to resist tension force and p_y is design strength, M_{cx} and M_{cy} are the moment capacity of beam member about x- and y-axes respectively and these moment capacities are discussed in Chapter 6 for beams and in *Clause 8.2.2* in HK Code.

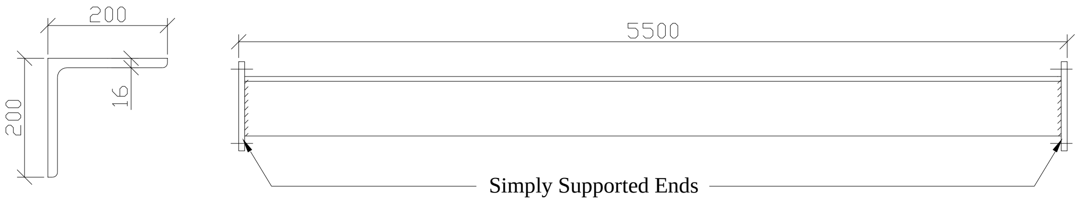
For Class 1 plastic and Class 2 compact sections, the moment capacity M_c is taken as the plastic moment of the section equal to p_yS. For Class 3 semi-compact sections, the moment capacity M_c is taken as the elastic moment capacity as p_yZ. Moment capacity M_c for Class 4 slender sections should be p_yZ_{eff} or $p_{yr}Z$ to allow for the effects of local plate buckling. The simplified linear failure surface in Equation (8.3) to account the interaction effect is a conservative design approach.

The reduction of bending resistance of member in the presence of tension load is considered by inclusion of the term of F_t/P_t in Equation (8.3). If this tension effect is considered directly in the computation of moment capacity of section (i.e. the moment capacity of the cross section is reduced in the presence of axial force shown in Figure 8.5), the term F_t/P_t tension should be omitted in the interaction Equation (8.3). This is a more economical and exact design approach using the convex failure surface in Figure 8.3.

8.3 Worked Examples

8.3.1 Combined tension and bending of angle beam

An angle tie with both ends pinned is subjected to a factored tension of 100kN and a factored distributed load of 8kN/m. The tie spans 5.5m and it is made of 200×200×16 equal angle in S275 steel. Check the adequacy of the tension tie under combined actions.



Solution

DESIGN LOAD

Maximum shear, $V = \dfrac{8 \times 5.5}{2} = 22.0kN$

Maximum moment, $M_x = \dfrac{8 \times 5.5^2}{8} = 30.3kNm$

SECTION PROPERTIES

$D = 200mm$, $B = 200mm$, $t = 16mm$, $A = 61.8cm^2$, $I_x = 2340cm^4$, $Z_x = 162cm^3$

SECTION CLASSIFICATION

Design strength, $p_y = 275N/mm^2$ for $t \leq 16mm$ *(Table 3.2)*

$\varepsilon = \sqrt{\dfrac{275}{275}} = 1$ *(Table 7.1 Note b)*

Semi-compact limiting value of b/t, d/t for angle in compression due to bending is 15ε

$\dfrac{b}{t} = \dfrac{d}{t} = \dfrac{200}{16} = 12.5 \leq 15 \times 1 = 15$ *(Table 7.1)*

$\therefore$ the section is Class 3 semi-compact

SHEAR CAPACITY

Shear area, $A_v = 0.9A_0 = 0.9 \times 200 \times 16 = 2880mm^2$ *(Clause 8.2.1)*

Shear capacity, $V_c = \dfrac{p_y A_v}{\sqrt{3}} = \dfrac{275 \times 2880}{\sqrt{3}} = 457.3kN > V$ *(OK)* *(8.1)*

TENSION CAPACITY

Tension capacity, $P_t = p_y A_e = 275 \times 6180 = 1699.5kN$ *(8.66)*

MOMENT CAPACITY

$V = 22kN \leq 0.6V_c = 274.4kN$ *(Clause 8.2.2.1)*

$\therefore$ It is low shear condition

Moment capacity,

$M_{cx} = p_y Z_x = 275 \times 162 \times 10^3 = 44.6Nm$ *(8.3)*

Buckling resistance moment,

$M_b = 0.8 p_y Z_x = 0.8 \times 275 \times 162 \times 10^3 = 35.6Nm > M_x$ *(OK)* *(8.64)*

CROSS-SECTION CAPACITY

$\frac{F_t}{P_t} + \frac{M_x}{M_{cx}} = \frac{100}{1699.5} + \frac{30.3}{44.6} = 0.74 \leq 1$ *(OK)* *(8.77)*

8.4 Beam-columns under tension and lateral-torsional buckling

When a slender beam-column in under tension force and bending moment about the major principal axis, it is required to be checked for resistance against lateral-torsional buckling. Similar to the case for biaxial bending of short beams, the slender beam has a failure yield surface under bending about two principal axes as shown in Figure 8.6.

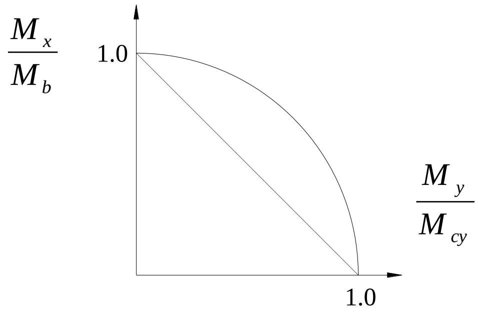


Figure 8.6 Buckling in beams under bending about two principal axes

Under this case where the lateral-torsional buckling governs, the member resistance about major axis is limited to the buckling resistance moment or the lateral torsional buckling moment M_b. As the tension force assists the member to resist buckling, the checking may ignore the favourable effect of tension force and also the equivalent uniform moment factor m_{LT} is allowed to consider the effect of the non-uniform bending moment distribution on beams in Chapter 6. The equation for checking of failure under biaxial bending allowing for lateral torsional buckling effect on this basis can be written as,

$$\frac{m_{LT} M_x}{M_b} + \frac{m_y M_y}{M_{cy}} \leq 1 \tag{8.4}$$

in which M_b is member resistance against lateral-torsional buckling based on *Clause 8.3.5.2* in HK Code for different sections, M_x is the maximum design moment about major principal x-axis governing M_b, m_{LT} is equivalent uniform moment factor in *Table 8.4* of HK Code, M_y is maximum design moment about the minor y-axis.

In addition to Equation (8.4) for buckling check, tension members should be checked against sectional strength in Equation (8.3).

8.5 Design procedures of unrestrained beam-column

The procedure for design of unrestrained beams can be summarized as follows.

I CHECK IF THE SECTION IS COMPLETELY IN TENSION AND SECTION CLASSIFICATION

Use the combined stress formula to check whether or not there is any compressive stress in the cross section. If compression is found in the cross section, section classification is required.

II DETERMINE THE MOMENT CAPACITY AND BUCKLING MOMENT RESISTANCE

If the beam-column is slender and susceptible to beam buckling, the procedure for finding the buckling resistance moment in Chapter 6 is applied. For stocky beams, the moment capacity is determined as elastic or plastic moment.

III INTERACTION EQUATION CHECK

Check the moment resistance of the member by the interactive equation as,

$$\frac{m_{LT}M_x}{M_b} + \frac{m_y M_y}{M_{cy}} \le 1 \quad \text{for beam buckling}$$

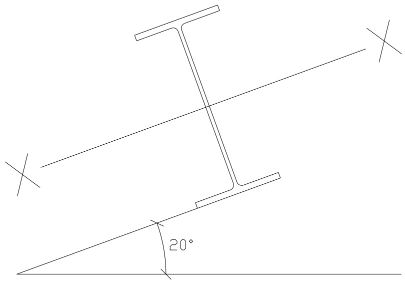
$$\frac{F_t}{P_t} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} \le 1 \quad \text{for sectional capacity}$$

A more refined and economical formula can be used for the checking by assuming the core area near web takes the tension axial force and the remaining area resists the moments about the two axes.

8.6 Worked Examples

8.6.1 Bending about two axes of an I beam

An I-section beam of span 5.5m and section 356×171×45 UB is simply supported on a sloping roof as shown. The factored design load is 10kN/m vertically and it is applied to the top flange of the I-beam as a destabilizing load. Check the adequacy of the I-section beam of Grade S275 steel material.



Solution

DESIGN LOAD

Maximum shear, $V = \dfrac{10 \times 5.5}{2} = 27.5kN$

Maximum moment, $M_{max} = \dfrac{10 \times 5.5^2}{8} = 37.8kNm$

Maximum shear along minor axis, $V_y = 27.5\cos 20° = 25.8kN$

Maximum shear along major axis, $V_x = 27.5\sin 20° = 9.4kN$

Maximum moment about major axis, $M_x = 37.8\cos 20° = 35.5kNm$

Maximum moment about minor axis, $M_y = 37.8\sin 20° = 12.9kNm$

SECTION PROPERTIES

$D = 351.4mm$, $B = 171.1mm$, $t = 7.0mm$, $T = 9.7mm$, $d = 311.6mm$, $Z_x = 687cm^3$, $Z_y = 94.8cm^3$, $S_x = 775cm^3$, $S_y = 147cm^3$, $r_y = 3.76cm$, $u = 0.874$, $x = 36.8$, $A = 57.3cm^2$

SECTION CLASSIFICATION

Design strength, $p_y = 275N/mm^2$ for $T \le 16mm$ *(Table 3.2)*

$\varepsilon = \sqrt{\dfrac{275}{275}} = 1$ *(Table 7.1 Note b)*

Plastic limiting value of b/T for outstand flange of an I-section is 9ε

$\dfrac{b}{T} = \dfrac{171.1}{2 \times 9.7} = 8.82 \le 9 \times 1 = 9$ *(Table 7.1)*

$\therefore$ flange is plastic

Plastic limiting value of d/t for web of an I-section with neutral axis at mid-depth is 80ε

$\dfrac{d}{t} = \dfrac{311.6}{7} = 44.5 \le 80 \times 1 = 80$ *(Table 7.1)*

$\therefore$ web is plastic

$\therefore$ the section is Class 1 plastic

SHEAR CAPACITY

Shear area, $A_{vy} = tD = 7 \times 351.4 = 2460mm^2$ *(Clause 8.2.1)*

$A_{vx} = 0.9 \times 2 \times BT = 0.9 \times 2 \times 171.1 \times 9.7 = 2987mm^2$

Shear capacity, $V_{cy} = \frac{p_y A_{vy}}{\sqrt{3}} = \frac{275 \times 2460}{\sqrt{3}} = 390.6kN > V_y$ *(OK)* *(8.1)*

$V_{cx} = \frac{p_y A_{vx}}{\sqrt{3}} = \frac{275 \times 2987}{\sqrt{3}} = 474.2kN > V_x$ *(OK)*

MOMENT CAPACITY

$V_y = 25.8kN \le 0.6V_{cy} = 234.4kN$ *(Clause 8.2.2.1)*

$V_x = 9.4kN \le 0.6V_{cx} = 284.5kN$

$\therefore$ it is low shear condition

Moment capacity, $M_{cx} = p_y S_x \le 1.2 p_y Z_x$ *(8.2)*

$= 275 \times 775 \times 10^3 \le 1.2 \times 275 \times 687 \times 10^3$

$= 213.1kNm \le 226.7kNm$

$> M_x$ *(OK)*

$M_{cy} = p_y S_y \le 1.2 p_y Z_y$ *(8.2)*

$= 275 \times 147 \times 10^3 = 40.4kNm$

but $> 1.2 \times 275 \times 94.8 \times 10^3 = 31.3kNm$

$\therefore M_{cy} = 31.3kNm > M_y$ *(OK)*

LATERAL-TORSIONAL BUCKLING

It is destabilizing loading condition as given *(Clause 8.3.3)*

Effective length, $L_E = 1.2L_{LT} = 1.2 \times 5.5 = 6.6m$ *(Clause 8.3.4.1(d))*

Slenderness ratio, $\lambda = \frac{L_E}{r_y} = \frac{6600}{37.6} = 175.5$ *(8.26)*

$$v = \frac{1}{\left[1 + 0.05(\lambda/x)^2\right]^{0.25}} = \frac{1}{\left[1 + 0.05 \times (175.5/36.8)^2\right]^{0.25}} = 0.827$$ *(8.27)*

$\beta_w = 1.0$ for Class 1 plastic section *(Clause 8.3.5.3)*

Equivalent slenderness, $\lambda_{LT} = uv\lambda\sqrt{\beta_w} = 0.874 \times 0.827 \times 175.5 \times 1 = 126.9$ *(8.25)*

Buckling strength, $p_b = 87.7\ N/mm^2$ *(Table 8.3a)*

Buckling resistance, $M_b = p_b S_x = 87.7 \times 775 \times 10^3 = 68.0kNm$ *(8.20)*

Equivalent uniform moment factor, $m_{LT} = 0.93$ *(Table 8.4a)*

$m_{LT} M_x = 0.93 \times 35.5 = 33.0kNm \le M_b$ *(OK)* *(8.18)*

CROSS-SECTION CAPACITY

$$\frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} = \frac{35.5}{213.1} + \frac{12.9}{31.3} = 0.58 \le 1 \ (OK)$$ *(8.78)*

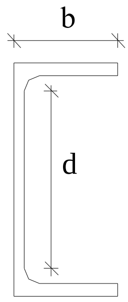
MEMBER BUCKLING RESISTANCE

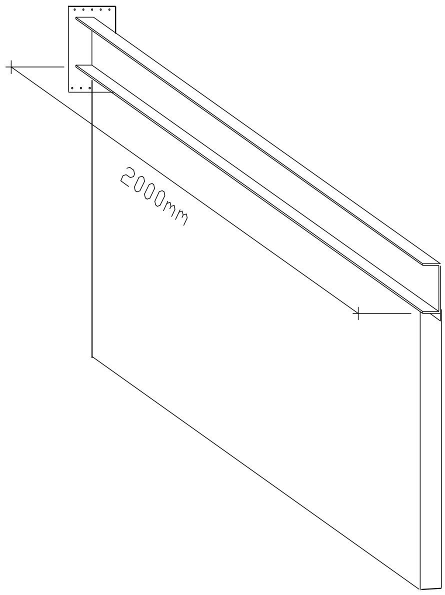
Equivalent moment factor, $m_y = 0.95$ *(Table 8.9)*

$$\frac{m_{LT} M_x}{M_b} + \frac{m_y M_y}{p_y Z_y} = \frac{0.93 \times 35.5}{68} + \frac{0.95 \times 12.9}{275 \times 94.8 \times 10^{-3}} = 0.96 \le 1 \ (OK)$$ *(8.81)*

8.6.2 Cantilever beam bent about two axes

A cantilever beam of channel section supports an advertisement board as shown in figure below. The loading from the advertisement board contains 2.5kN/m lateral wind load and 12kN/m vertical dead load. The channel section of the cantilever beam is [200×90×30 in S275 steel. Determine the adequacy of the member under combined interaction of bending about two axes.





Solution

DESIGN LOAD

Factored dead load, $\omega_G = 1.4 \times 12 = 16.8\,kN/m$ *(Table 4.2)*

Factored wind load, $\omega_w = 1.4 \times 2.5 = 3.5\,kN/m$

Maximum shear along minor axis, $V_y = 16.8 \times 2 = 33.6kN$

Maximum shear along major axis, $V_x = 3.5 \times 2 = 7.0kN$

Maximum moment about major axis, $M_x = \frac{1}{2} \times 16.8 \times 2^2 = 33.6kNm$

Maximum moment about minor axis, $M_y = \frac{1}{2} \times 3.5 \times 2^2 = 7.0kNm$

SECTION PROPERTIES

$D = 200mm$, $B = 90mm$, $t = 7.0mm$, $T = 14.0mm$, $d = 148mm$, $Z_x = 252cm^3$, $Z_y = 53.4cm^3$,

$S_x = 291cm^3$, $S_y = 94.5cm^3$, $r_y = 2.88cm$, $u = 0.952$, $x = 12.9$, $A = 37.9cm^2$

SECTION CLASSIFICATION

Design strength, $p_y = 275N/mm^2$ for $T \leq 16mm$ *(Table 3.2)*

$\varepsilon = \sqrt{\frac{275}{275}} = 1$ *(Table 7.1 Note b)*

Plastic limiting value of b/T for outstand flange of a channel is 9ε

$$\frac{b}{T}=\frac{90}{14}=6.43\le 9\times 1=9 \qquad \textit{(Table 7.1)}$$

$\therefore$ flange is plastic

Plastic limiting value of d/t for web of a channel with neutral axis at mid-depth is 40ε

$$\frac{d}{t}=\frac{148}{7}=21.1\le 40\times 1=40 \qquad \textit{(Table 7.1)}$$

$\therefore$ web is plastic

$\therefore$ the section is Class 1 plastic

SHEAR CAPACITY

Shear area, $A_{vy}=tD=7\times 200=1400mm^2$ *(Clause 8.2.1)*

$A_{vy}=0.9\times 2\times BT=0.9\times 2\times 90\times 14=2268mm^2$

Shear capacity, $V_{cy}=\dfrac{p_yA_{vy}}{\sqrt{3}}=\dfrac{275\times 1400}{\sqrt{3}}=222.3kN>V_y$ *(OK)* *(8.1)*

$V_{cx}=\dfrac{p_yA_{vx}}{\sqrt{3}}=\dfrac{275\times 2268}{\sqrt{3}}=360.1kN>V_x$ *(OK)*

MOMENT CAPACITY

$V_y\le 0.6V_{cy}=133.4kN$ *(Clause 8.2.2.1)*

$V_x\le 0.6V_{cx}=216.1kN$

$\therefore$ it is low shear condition

Moment capacity, $M_{cx}=p_yS_x\le 1.2p_yZ_x$ *(8.2)*

$=275\times 291\times 10^3\le 1.2\times 275\times 252\times 10^3$

$=80.0kNm\le 83.2kNm$

$>M_x$ *(OK)*

$M_{cy}=p_yS_y\le 1.2p_yZ_y$ *(8.2)*

$=275\times 94.5\times 10^3=26.0kNm$

but $>1.2\times 275\times 53.4\times 10^3=17.6kNm$

$\therefore M_{cy}=17.6kNm>M_y$ *(OK)*

LATERAL-TORSIONAL BUCKLING

It is normal loading condition *(Clause 8.3.3)*

Effective length, $L_E=0.8L_{LT}=0.8\times 2=1.6m$ *(Table 8.1)*

Slenderness ratio, $\lambda=\dfrac{L_E}{r_y}=\dfrac{1600}{28.8}=55.6$ *(8.26)*

$$v=\frac{1}{\left[1+0.05(\lambda/x)^2\right]^{0.25}}=\frac{1}{\left[1+0.05\times(55.6/12.9)^2\right]^{0.25}}=0.849 \qquad \textit{(8.27)}$$

$\beta_w=1.0$ for Class 1 plastic section *(Clause 8.3.5.3)*

Equivalent slenderness, $\lambda_{LT}=uv\lambda\sqrt{\beta_w}=0.952\times 0.849\times 55.6\times 1=44.9$ *(8.25)*

Buckling strength, $p_b=250.2\,N/mm^2$ *(Table 8.3a)*

Buckling resistance, $M_b=p_bS_x=250.2\times 291\times 10^3=72.8kNm$ *(8.20)*

Equivalent uniform moment factor, $m_{LT}=1$ for cantilever *(Table 8.4b)*

$m_{LT}M_x=1\times 33.6=33.6kNm\le M_b$ *(OK)* *(8.18)*

CROSS-SECTION CAPACITY

$$\frac{M_x}{M_{cx}}+\frac{M_y}{M_{cy}}=\frac{33.6}{80}+\frac{7}{17.6}=0.82\leq 1 \textit{ (OK)} \tag{8.78}$$

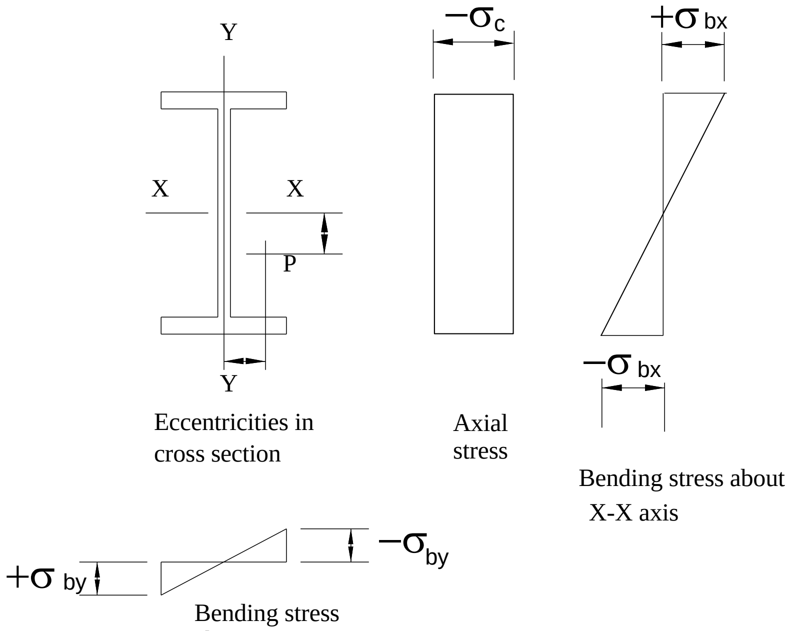
MEMBER BUCKLING RESISTANCE

$m_y = 1.0$

$$\frac{m_{LT}M_x}{M_b}+\frac{m_y M_y}{p_y Z_y}=\frac{1\times 33.6}{72.8}+\frac{1\times 7}{275\times 53.4\times 10^{-3}}=0.94\leq 1 \textit{ (OK)} \tag{8.81}$$

8.7 Sectional strength under compression and bending

For stocky members under compressive axial force and bending moments about two axes, both the capacity and buckling strength of the members are required for checking. When a member is under compression force F_c and biaxial bending moments M_x and M_y, the induced stresses on the member section will comprise compressive stress due to force and bending moments about *x*- and *y*-axes as shown in Figure 8.7.



$$p_y = \sigma_{\max} = -(\sigma_c + \sigma_{bx} + \sigma_{by})$$

Figure 8.7 Stress distribution across a section under the action of axial force and bending moments

For elastic design, the combined stress on the section should not be greater than the material design strength p_y. The strength check of the member should follow Equation (8.5), which is based on an elastic analysis superimposing the components of compression and bending stresses as,

$$\sigma_c + \sigma_{bx} + \sigma_{by} = \frac{F_c}{A_g} + \frac{M_x}{Z_x} + \frac{M_y}{Z_y} \le p_y \tag{8.5}$$

in which F_c, M_x and M_y are the axial force and bending moments for the critical section along a beam-column. The section capacity check for the interaction can be rewritten as Equation (8.6) in a section capacity format beyond the elastic limit as,

$$\frac{F_c}{p_y A_g} + \frac{M_x}{p_y Z_x} + \frac{M_y}{p_y Z_y} = \frac{F_c}{p_y A_g} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} \le 1 \tag{8.6}$$

where A_g is the gross area of member section. M_{cx} and M_{cy} are respectively the member capacity about x-axis and y-axis. For plastic analysis allowing for the entire section fully yielded, the moment capacities M_{cx} and M_{cy} can be taken as the plastic section capacity p_yS. The reduction of moment capacities due to compression load takes into account the three-dimensional failure surface in Figure 8.8.

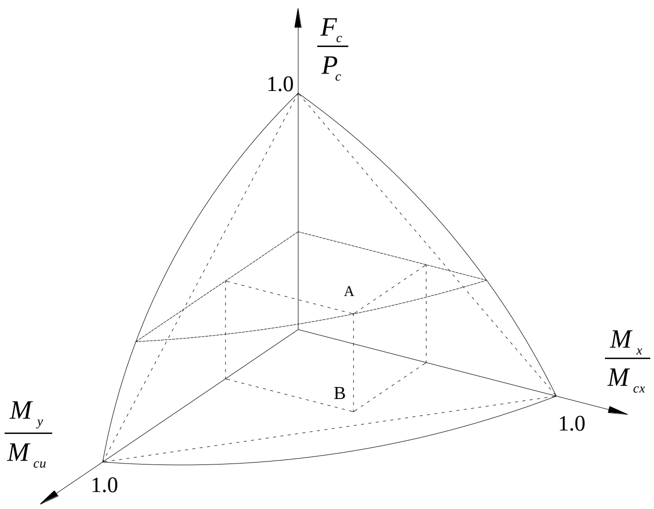


Figure 8.8 Failure surface of sectional strength under biaxial bending

The plastic failure surface is convex in space as shown by solid lines in Figure 8.8. The conservative linear failure surface under combined axial compression and biaxial bending are also plotted in dotted lines in Figure 8.8. For any loaded point lying inside or on the failure surface such as stress point "A" in Figure 8.8, the member is considered as structurally adequate and capable of resisting the loads and bending moments. When the axial force is released, the stress point drops to stress point "B" at the yield surface with zero axial force and the section is allowed to take greater moments because of this removal of axial force.

8.8 Buckling strength under biaxial bending

The structural check of compression member under axial force and bending moments should include the section capacity check and the overall buckling check. The sectional capacity check and the buckling resistance checks are detailed respectively in *Clauses 8.9.1 and 8.9.2* of the HK Code. The section capacity check is to ensure no section in the beam-column will be loaded beyond the failure yield surface of the section. The overall member buckling resistance check is to ascertain the member will not buckle under the combined action of axial force and moments.

8.8.1 Cross section capacity

The yield surface for a compression member is similar to the case for a tension member and the simple linear interaction equation is used in the HK Code as follows.

For non-slender sections,

$$\frac{F_c}{p_y A} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} \le 1 \qquad (8.7)$$

For slender sections,

$$\frac{F_c}{p_y A_{eff}} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} \le 1 \qquad (8.8)$$

in which A_{eff} is the effective cross sectional area allowing for local buckling. The external moments and force should be selected for the most critical section in a member. More exact method allowing the inclusion of axial force effect into the bending moment resistance is allowed in the HK Code. The method is based on the core area around web taking the axial force with the remaining areas taking the moments.

8.8.2 Overall buckling resistance

The buckling check in the HK Code involves (1) the checking for column buckling with prolonged effective buckling length, (2) the amplified moment due to sway and displacement in the frame and (3) the lateral-torsional buckling interacted with the axial force and flexural buckling about the minor axis. The interaction relation between the force and the moment is shown in Figure 8.9 below, which indicates that the failure surface is dependent not only on the cross sectional capacities, but also on the effective length of the member. In the figure, when the slenderness ratio λ of the member increases, the allowed axial compression resistance and moment resistance about major axis decrease. The moment resistance about the major axis reduces as the effective length increases. Generally speaking, the permissible axial force and moments reduce when the slenderness ratio increases.

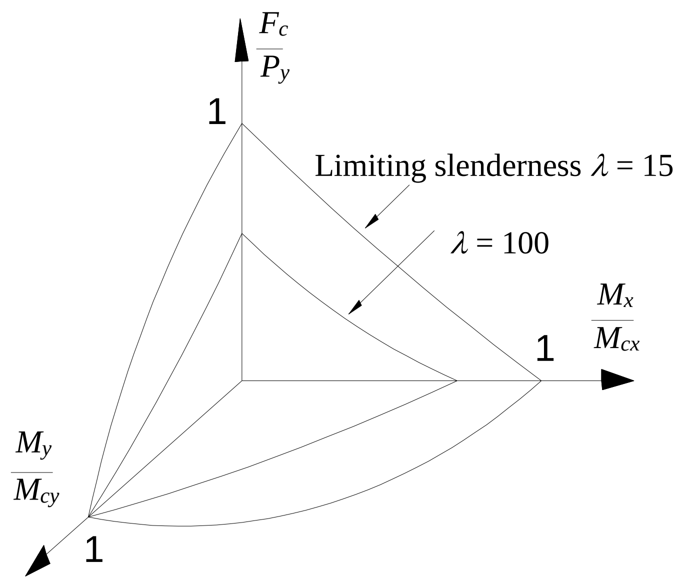


Figure 8.9 Failure surfaces of compression members under combined actions

The HK Code is different from most other design codes in the checking of beam-columns. In some steel design codes like Eurocode 3 (2005), either the moment amplification method or the buckling effective length method is needed for checking of buckling resistance of a beam-column. Below is an argument that the options are not always equivalent and they may produce considerably different design loads. Thus, it is one reason for not accepting the effective length method when the elastic critical load factor λ_{cr} is less than 5.

When the moment in a beam-column is small such as a concentrated load acting directly on top of a beam-column, the design of the beam-column is not controlled by the part for bending moment and the amplified moment remains small even the amplification factor is large. The failure of the beam-column here is dominated by column flexural buckling mode and *Equation (8.79)* of *Clause 8.9.2* of HK Code should be applied to check this mode of failure. On the other hand, when the moment on the beam-column is large such as the beam-column is under large end moment or moment along its length, the column flexural buckling is not in control of the member capacity and the amplified moment becomes dominant in member strength check. In this case *Equation (8.80)* of *Clause 8.9.2* of HK Code checks the resistance of the member against this scenario of failure. The remaining equation in the same clause obviously checks the case for the beam-column against lateral-torsional buckling. Thus, 3 equations are required to be checked when using the HK Code at least for consistency in design.

8.8.2.1 Member buckling check

The buckling of the member in the column flexural buckling mode is considered in this clause here. When the bending moment is small such that the amplified moment is moderate even the sway amplification factor is large, the flexural buckling is more critical and this checking is to prevent the failure case for column buckling mainly due to P-δ effect which is coupled with member buckling. The bending moment action reduces the buckling resistance of the beam column susceptible to column flexural buckling and the checking can be expressed as follows.

$$\frac{F_c}{P_c}+\frac{m_x\overline{M}_x}{M_{cx}}+\frac{m_y\overline{M}_y}{M_{cy}}\le 1 \quad (8.9)$$

in which P_c is the buckling resistance of the member using the effective length determined in Chapter 7, m_x and m_y are the uniform moment equivalent factors, $\overline{M}_x$ and $\overline{M}_y$ are the design moments which are not required for amplification as the effect may be doubly considered after using the prolonged effective length for finding the P_c. M_{cx} and M_{cy} are the resistance moment of the cross section. It can be seen great uncertainty exists in estimating P_c which further depends on the stiffness and sway amplitude of the whole frame and this is the disadvantage of the effective length method.

8.8.2.2 Sway amplified moment

When the beam-column in a frame sways considerably that the increase in effective length in the above section cannot cater for the amplified sway moment reliably, the checking in the following Equation (8.10) prevents this mode of failure. However, as the sway moment cannot be applied to the amplifying buckling effect of beam-column along its length, an effective length equal to the member length is still required to be assumed. The equation for the checking is given in the following equation.

$$\frac{F_c}{\overline{P}_c}+\frac{m_xM_x}{M_{cx}}+\frac{m_yM_y}{M_{cy}}\le 1 \quad (8.10)$$

in which $\overline{P}_c$ is the buckling resistance of the beam-column assuming the effective length equal to the member length, m_x and m_y are the equivalent uniform moment factors about the x- and y-axes. M_x and M_y are the amplified bending moments using the amplification factors in Equations (8.11) and (8.12) below.

$$M_x=\frac{\lambda_{cr}}{\lambda_{cr}-1}\overline{M}_x \quad (8.11)$$

$$M_y=\frac{\lambda_{cr}}{\lambda_{cr}-1}\overline{M}_y \quad (8.12)$$

where $\frac{\lambda_{cr}}{\lambda_{cr}-1}$ is the greater of following Equations (8.13) and (8.14).

$$\frac{\lambda_{cr}}{\lambda_{cr}-1}=\frac{1}{1-\frac{F_V\delta_N}{F_Nh}} \text{ or} \quad (8.13)$$

$$\frac{\lambda_{cr}}{\lambda_{cr}-1}=\frac{1}{1-\frac{F_cL_E^2}{\pi^2EI}} \quad (8.14)$$

in which λ_{cr} is the elastic critical load factor equal to the elastic buckling load divided by the design load of a perfect (no imperfection) structural system, $\overline{M}_x$ and $\overline{M}_y$ are the bending moment without amplification and obtained directly from a linearanalysis.

F_V and F_N are the vertical and notional horizontal forces acting on a floor. F_c is the axial force in the member being designed and L_E is the effective length. Further details of the concept of second-order nonlinear buckling analysis can be found in Chapter 10.

Equations (8.11) and (8.12) are to account for amplified effect from sway of frame and Equation (8.13) and (8.14) are for the amplification from column buckling. As only the P-Δ effect is required for consideration in sway frames, the use of Equations (8.13) is needed only for sway frames.

The amplification factor in Equation (8.13) is for the amplified moment effect when a frame sways, such that the induced moment becomes axial force times the lateral deflection. Equation (8.14) on the other hand is due to the effect of amplified moment resulted from deflection bowing and axial force.

8.8.2.3 Lateral-torisional buckling

When the beam column is bent about the major axis with a possibility of buckling about the minor axis, the following interaction equation is needed to ensure the member does not fail in this combined buckling mode for flexural and lateral-torsional buckling. Here the flexural buckling resistance P_{cy} about the minor axis is used as the concern is on the buckling about the minor axis, but not about the major principal axis.

$$\frac{F_c}{P_{cy}} + \frac{m_{LT} M_{LT}}{M_b} + \frac{m_y \overline{M}_y}{M_{cy}} \leq 1 \tag{8.15}$$

in which P_{cy} is the buckling resistance about the minor axis of the section, m_{LT} is the uniform equivalent moment factor for lateral-torsional buckling of beams, M_{LT} is the amplified bending moment about x-axis governing M_b.

In Equation (8.15), as P_{cy} is obtained from sway effective length, we need not amplified the M_y which is about the same y-axis. However, M_{LT} is about the major axis and the effective length about this major axis has not been considered and thus we need to amplify M_{LT}.

8.9 Design procedures of compression and bending

The procedure for design of an unrestrained beam column can be summarized as follows.

I SECTION CLASSIFICATION

Classify the sectional type into Class 1, 2, 3 or 4. The design strength or cross sectional area for Class 4 sections should be reduced.

II SECTIONAL CAPACITY CHECK

Determine the most critical section containing the greatest stress due to bending moments about the two axes. Apply Equations (8.7) and (8.8) for sectional strength check.

III MEMBER BUCKLING CHECK

Compute the buckling resistance moment of a beam-column to Chapter 6 and the buckling resistance against axial buckling in Chapter 7. Amplified factor $\frac{\lambda_{cr}}{\lambda_{cr}-1}$ should be applied. Check the member to Equations (8.9) to (8.15) which are adopted from *Clauses 8.9.2* in HK Code.

IV OTHER LOCAL EFFECTS AND WEB BEARING AND BUCKLING CHECK

As for beam and column design, checking for web against bearing, shear buckling and compression buckling should be carried out as for restrained beams. The discussion should be referred to Chapter 9.

V OTHER LIMIT STATES

Other limit states such as the deflection and vibration limit states shall also be checked.

8.10 Worked Examples

8.10.1 Column in simple frame

A steel stanchion of 203×203×100 UC of length 8m and grade S355 grade of steel in a multi-storey building frame is under a factored concentric axial force of 250 kN, a factored reaction from beam of 100 kN with nominal eccentricities of 100mm from the face of the web and another factored reaction from beam of 150 kN of 100mm nominal eccentricity from the face of the flange. Check the adequacy of the stanchion. The effective length for flexural column buckling is 1.0 of its physical length and the effective length for lateral-torsional beam buckling is 0.5 of its physical length.

<u>Solution</u>

SECTION PROPERTIES

$D = 228.6mm$, $B = 210.3mm$, $t = 14.5mm$, $T = 23.7mm$, $d = 160.8mm$, $I_x = 11300cm^4$,

$I_y = 3680cm^4$, $r_x = 9.44cm$, $r_y = 5.39cm$, $Z_x = 988cm^3$, $Z_y = 350cm^3$, $S_x = 1150cm^3$,

$S_y = 534cm^3$, $u = 0.852$, $x = 9.02$, $A = 127cm^2$

DESIGN LOAD

Axial load, $F_c = 250 + 100 + 150 = 500kN$

Moment about major axis, $M_x = 150 \times (228.6/2 + 100) \times 10^{-3} = 32.1kNm$

Moment about minor axis, $M_x = 100 \times (14.5/2 + 100) \times 10^{-3} = 10.7kNm$

SECTION CLASSIFICATION

Design strength, $p_y = 345N/mm^2$ for $16mm < T \le 40mm$ *(Table 3.2)*

$$\varepsilon = \sqrt{\frac{275}{345}} = 0.89$$ *(Table 7.1 Note b)*

Plastic limiting value of b/T for outstand flange of an H-section is 9ε

$$\frac{b}{T} = \frac{210.3}{2 \times 23.7} = 4.44 \le 9 \times 0.89 = 8.01$$ *(Table 7.1)*

$\therefore$ flange is plastic

Plastic limiting value of d/t for web of an H-section under both axial compression and bending is $80\varepsilon/(1 + r_1)$

Stress ratio, $$r_1 = \frac{F_c}{dtp_{yw}} = \frac{500 \times 10^3}{160.8 \times 14.5 \times 345} = 0.622 \le 1$$ *(7.1)*

$$\frac{d}{t} = \frac{160.8}{14.5} = 11.1 < \frac{80 \times 0.89}{1 + 0.622} = 43.9$$

$\therefore$ web is plastic

$\therefore$ the section is Class 1 plastic

MOMENT CAPACITY

Moment capacity, $M_{cx} = p_y S_x \le 1.2 p_y Z_x$

$= 345 \times 1150 \times 10^3 \le 1.2 \times 345 \times 988 \times 10^3$

$= 396.8kNm \le 409.0kNm$

$> M_x$ *(OK)*

$M_{cy} = p_y S_y \le 1.2 p_y Z_y$

$= 345 \times 534 \times 10^3 = 184.2kNm$

but $> 1.2 \times 345 \times 350 \times 10^3 = 144.9kNm$

$\therefore M_{cy} = 144.9kNm > M_y$ *(OK)*

LATERAL-TORSIONAL BUCKLING

Effective length, $L_E = 0.5 L_{LT} = 0.5 \times 8 = 4m$

Slenderness ratio, $\lambda = \dfrac{L_E}{r_y} = \dfrac{4000}{53.9} = 74.2$ *(8.26)*

$$v = \frac{1}{\left[1 + 0.05(\lambda/x)^2\right]^{0.25}} = \frac{1}{\left[1 + 0.05 \times (74.2/9.02)^2\right]^{0.25}} = 0.691$$ *(8.27)*

$\beta_w = 1.0$ for Class 1 plastic section *(Clause 8.3.5.3)*

Equivalent slenderness, $\lambda_{LT} = uv\lambda\sqrt{\beta_w} = 0.852 \times 0.691 \times 74.2 \times 1 = 43.7$ *(8.25)*

Buckling strength, $p_b = 305.9\, N/mm^2$ *(Table 8.3a)*

Buckling resistance, $M_b = p_b S_x = 305.9 \times 1150 \times 10^3 = 351.8kNm$ *(8.20)*

Equivalent uniform moment factor, $m_{LT} = 1.0$ *(Table 8.4a)*

$m_{LT} M_x = 1.0 \times 32.7 = 32.7kNm \le M_b$ *(OK)* *(8.18)*

COMPRESSION RESISTANCE

Bending about minor axis is more critical

Effective length, $L_E = L = 8m$

Slenderness ratio, $\lambda = \dfrac{L_E}{r_y} = \dfrac{8000}{53.9} = 148.4$ *(Clause 8.7.4)*

The compressive strength for rolled H-section bending about minor axis should be obtained from buckling curve c. *(Table 8.7)*

Compressive strength, $p_c = 73.6\, N/mm^2$ *(Table 8.8(c))*

Compressive resistance, $P_c = A_g p_c = 12700 \times 73.6 = 934.7kN > F_c$ *(OK)* *(8.73)*

CROSS-SECTION CAPACITY

$$\frac{F_c}{A_g p_y} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} = \frac{500 \times 10^3}{12700 \times 345} + \frac{32.1}{396.8} + \frac{10.7}{144.9} = 0.27 \le 1 \text{ (OK)}$$ *(8.78)*

MEMBER BUCKLING RESISTANCE

Equivalent moment factor, $m_x = m_y = m_{LT} = 1$ for simple construction *(Clause 8.7.8)*

Moment amplification factor about major axis,

$$\frac{\lambda_{cr}}{\lambda_{cr} - 1} = \frac{1}{1 - \dfrac{F_c L_E^2}{\pi^2 E I_x}} = \frac{1}{1 - \dfrac{500 \times 8^2}{\pi^2 \times 2.05 \times 11300}} = 1.163$$ *(8.82)*

Moment amplification factor about minor axis,

$$\frac{\lambda_{cr}}{\lambda_{cr}-1}=\frac{1}{1-\dfrac{F_c L_E^2}{\pi^2 EI_y}}=\frac{1}{1-\dfrac{500\times 8^2}{\pi^2\times 2.05\times 3680}}=1.754 \qquad (8.82)$$

$$\frac{F_c}{\overline{P}_c}+\frac{m_x M_x}{M_{cx}}+\frac{m_y M_y}{M_{cy}}=\frac{500}{934.7}+\frac{1.163\times 32.1\times 10^3}{345\times 988}+\frac{1.754\times 10.7\times 10^3}{345\times 350}=0.80\le 1 \qquad (8.80)$$

$$\frac{F_c}{P_{cy}}+\frac{m_{LT} M_{LT}}{M_b}+\frac{m_y \overline{M}_y}{M_{cy}}=\frac{500}{934.7}+\frac{1.163\times 32.1}{351.8}+\frac{10.7\times 10^3}{345\times 350}=0.73\le 1 \qquad (8.81)$$

Chapter 9 Connections

9.1 Introduction

Connections play an important role in the steel structure; they link individual members together and transfer loads from one member to another. The cost of connection can be 20 to 30% or more of the total construction cost of a steel structure. Details of connection can also affect the speed of construction, cost and safety of the complete structure. In some occasions, the appearance of connections is also controlled by architectural requirements.

The major consideration for connections is, apart from structural strength and safety, the ease of fabrication which indirectly relates to the cost. The design aspect requiring special consideration will be of the lack of ductility for most, especially the welded connections. It has been reported that failure of many steel structures occurs at connection and the strength and ductility of connections need to be considered and analysed

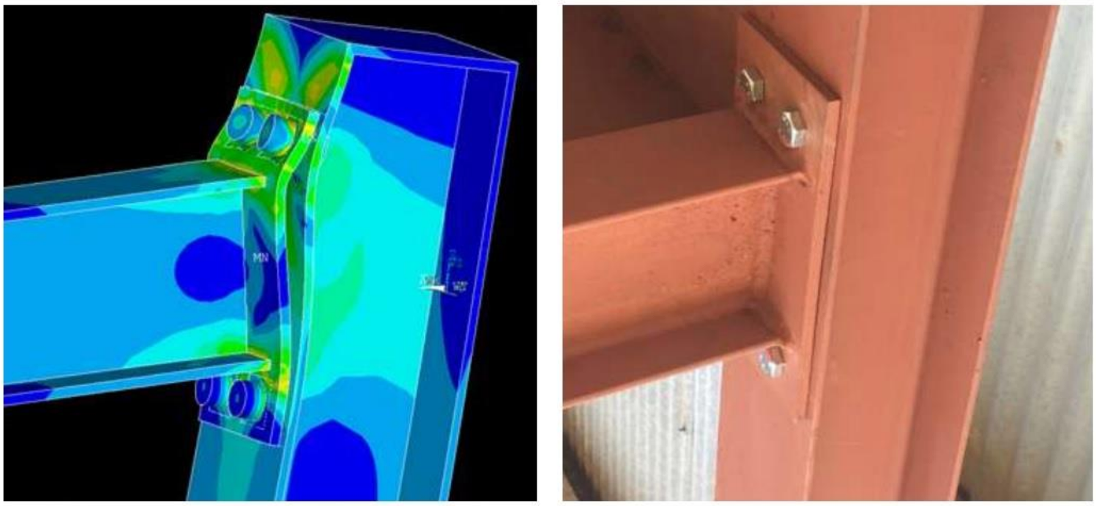


Figure 9.1 Semi-rigid bolted connection with computer simulation

In general, a connection is either designed as a pinned connection transferring only shear or a rigid connection transferring both shear and moment. The former connection type of pinned connections is more widely used in simple construction and the latter type of rigid connections is used in continuous construction. The newest development is the semi-rigid connections which allow transfer of shear with partial transfer of moment (see Figure 9.1 shows a finite element model of a semi-rigid bolted connection used in a green hoarding system designed by second-order direct plastic analysis in Hong Kong, Chan and Lo, 2019). When transferring moment through a connection, the strength is of paramount importance and when only shear is transferred with release of moment, the rotational capacity becomes the principal consideration. Designer details the pinned connection to be flexible in releasing moment and transferring shear only whereas the rigid connections are detailed to have sufficient strength to transfer moment as well as shear. Semi-rigid connections are designed to have both the capability of transferring moment and allowing rotations.

In structural design of steel structures, the assumption for connection characteristics should be consistent in detailing, strength calculation and in connection fixity used in computer modeling for structural frame analysis. As a general rule, bolt connections should be used on site and welded connections should be adopted in shop, because of ease of quality control. In some cold areas, site welding is not feasible during the winter or even not permitted. However, local practice in Hong Kong may prefer site welding for easy fabrication and greater tolerance for fixing of connections.

The aim of connections is to transmit the load from one member to another. Different forms of joining members should be referred to connections with different names, such as beam-to-beam connection, column splice, truss joints, beam-column connection and column base shown in Figure 9.2(a). These connections may perform different functions such as transfer of moment and shear, or shear only with adequate rotation capacity as shown in Figure 9.2(b). When load is transferred from one plate element to another, lap joints shown in Figure 9.2(c) can be used. Fabrication and erection procedures may be simplified by standardizing a number of common connection details and arrangements for common connections.

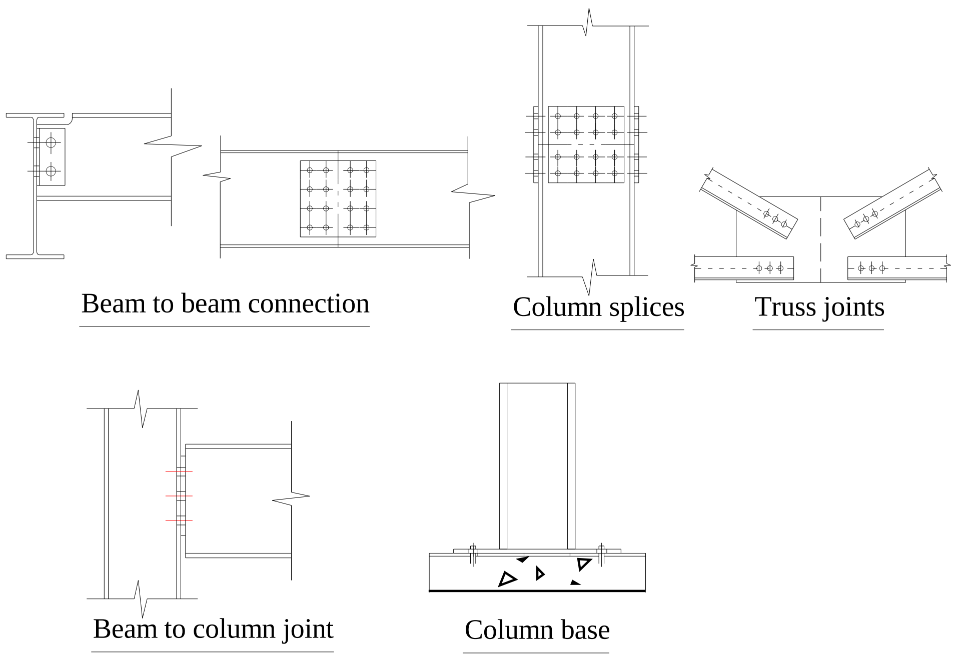
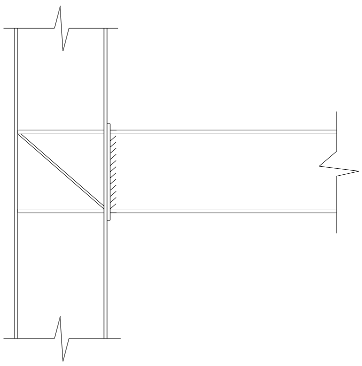
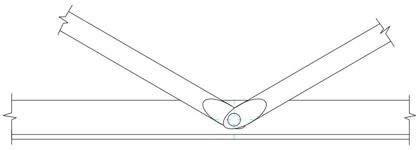


Figure 9.2(a) Types of member connections

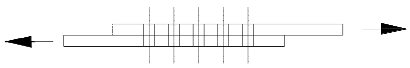


Moment connection

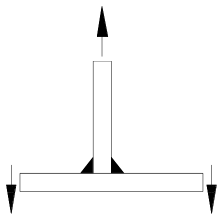


Pinned connection

Figure 9.2(b) Moment connection and pinned connection



Lap joint by bolts



Welded joint to form built-up element

Figure 9.2(c) Connections jointing the plate elements by bolts or welds

Figure 9.2 Typical connections

The common composing elements or components in different types of connection are of bolts, pins, welds, seats, cleats and end plates. Riveted connections are uncommon in modern construction and therefore not further discussed here. The key elements of component in connection for direct load transfer are bolts and welds.

Bolting contains two types. They are non-preloaded ordinary bolts in standard clearance or oversize holes and preloaded or high strength friction grip (HSFG) bolt. Welding includes mainly fillet and butt welds.

9.2 Connection behaviour in strength, stiffness and ductility

Connections are commonly assumed to be either perfectly rigid or ideally pinned. The assumption of joint stiffness should be sufficiently accurate to prevent unfavorable effect on frame behavior. A rigid frame or a frame with rigid connections assumes all connections to be rigid and the moment can be all transferred from one member to another. On the other hand, no moment is transferred through the pinned connection; the stiffness of pinned connection is equal to zero. However, pin connection must be able to transmit shear and to possess sufficient ductility for rotation as pinned connection. While all connections of a structure are idealized as pinned, the structure is called simple structure. It is convenient and simple for structural analysis and design of the structure. Strictly speaking, these two extreme cases can seldom exist in reality and the widely used assumption of perfectly rigid and frictionless pinned connection is practically unattainable. A more realistic and possibly more economical design is to allow for a certain degree of connection stiffness at the connection as shown in beam-column connection in Figure 9.3. The corresponding moment-rotation curves are illustrated in Figure 9.4. A certain degree of rotational deformation is allowed with part of the fixed end moment is transferred.

The behavior of semi-rigid connection influences the global structural behavior such as the classification of frame into sway or non-sway mode. And it also affects the connection design. However, this degree of connection stiffness caused by local behavior of the connection is not to be discussed here.

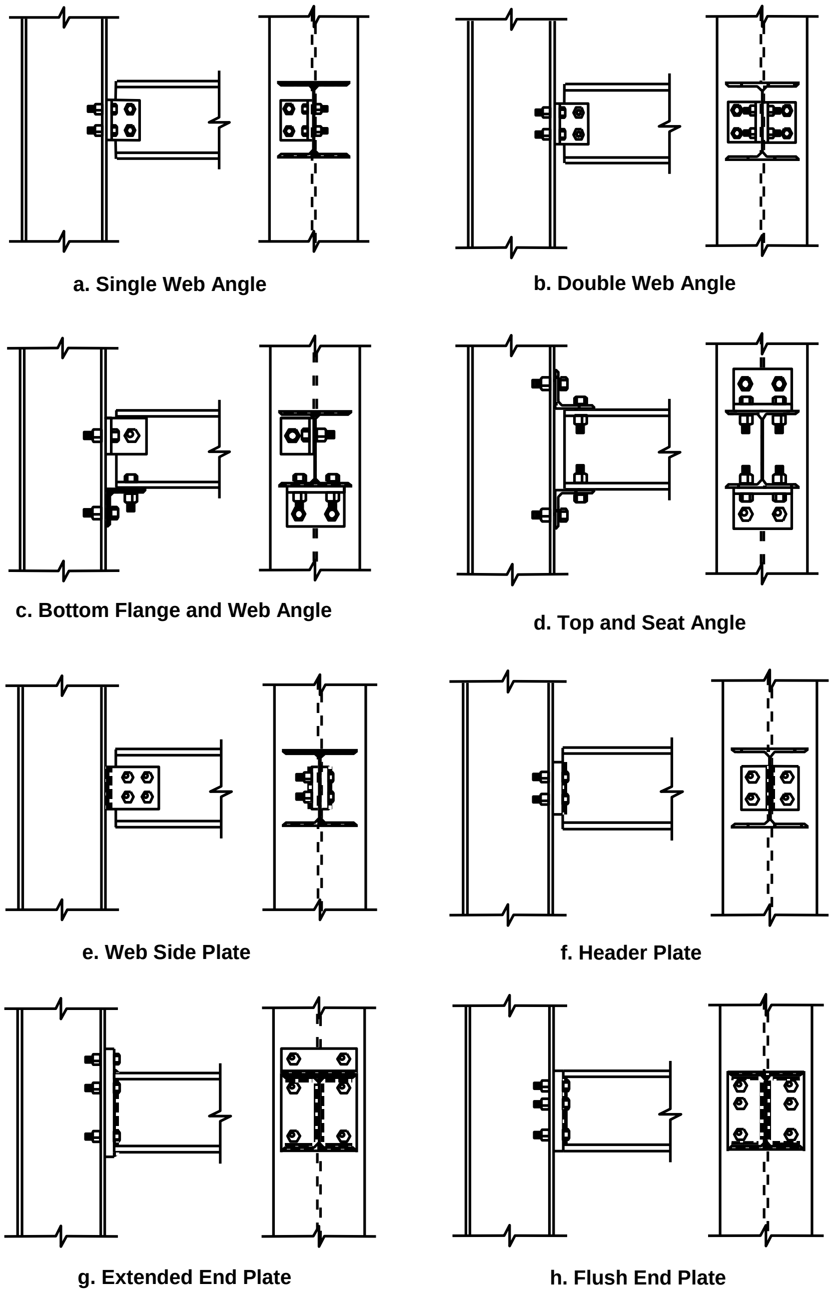


Figure 9.3 Different types of beam column connections

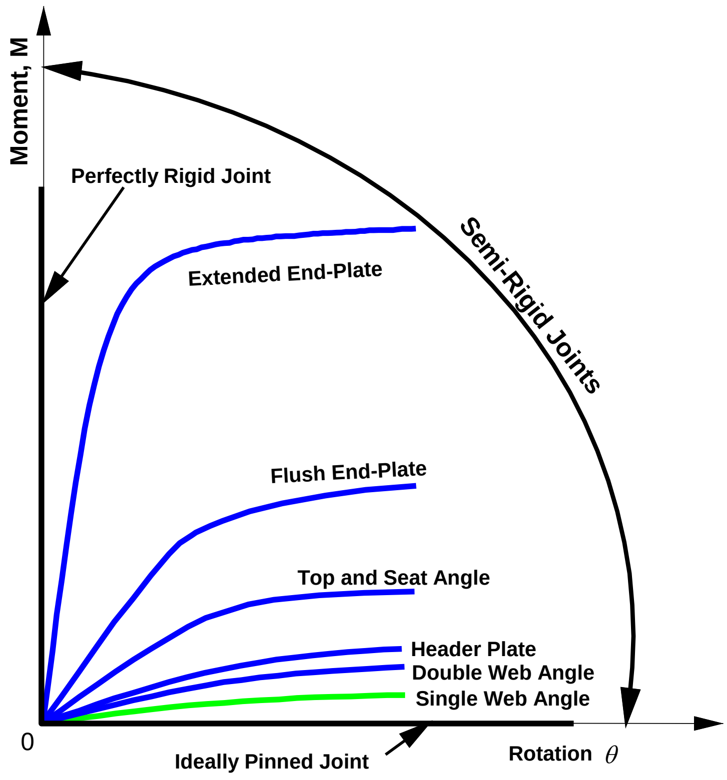


Figure 9.4 Moment-rotation curves for corresponding beam column connections

9.3 Welded connection

9.3.1 Weld process

Welding is a process of fusing two pieces of steel materials together through the use of heat. The required heat is produced by electric current or gas heat through electrode such that the metal in the electrode melts, fuses and cools to form a single piece of metal with the parent metals which refer to the pieces of metals intended for joining. Welded connections are widely used in steel construction to date and they provide great flexibility in connecting members. Lesser construction tolerance requirement is especially attractive on site construction. However, most welded connections are relatively less ductile than ordinary clearance bolts and they introduce high residual stress leading to cracking if the welding process is not carefully monitored.

There are two main types of welds, namely the fillet weld and the butt weld in Figures 9.5 and 9.6. Fillet weld is roughly triangular in its cross section formed at the re-entrant corners of a joint. The strength of weld is developed as the shear capacity of the weld across the size or throat of weld. The size of weld is defined as the width of throat which is called the leg length of weld discussed in Section 9.3.5.1.

Butt weld refers to the weld lying within the surface of the joining plates. The welding process commonly used is the metal arc welding and gas welding. Metal arc welding is carried out by fusion of metal accomplished by the heat of an electric arc. In gas welding, a filler material or bare electrode is used to supply material which is melted by high heat of gas like oxygen-acetylene flame. Shielding gas is used to shield the molten weld zone from the atmosphere. In metal inert-gas process (MIG), a bare wire electrode is used whereas in tungsten inert-gas process (TIG), tungsten electrode is used.

Welding may cause the region near to the weld to become relatively brittle. When a crack is formed due to brittle material in that region, it is easily propagated at a high stress concentration. This problem is particularly serious under lower temperature. Further, the welding induces residual stress and strain in the component of connection, which deteriorates the structural strength of connection in most cases.

9.3.2 Electrodes

The combined use of weldable steel, welding strength, welding condition and welding position requires the specification of electrode in terms on strength, welding position and supply of current to the electrode. In the HK Code, both the BS EN and the Chinese standards for electrodes are listed for fillet welds. Electrode classification of 35, 42 and 50 for BS EN standards and E43, E50 and E55 for Chinese GB standards are used and listed in *Table 9.2* of the HK Code.

9.3.3 Types of welds

The common welding types include fillet weld and butt weld. Fillet weld is that the weld metal is generally lying outside the profile of the connected elements as shown in Figure 9.5 while butt weld is that the weld metal is deposited with the profile of the connected elements as shown in Figure 9.6.

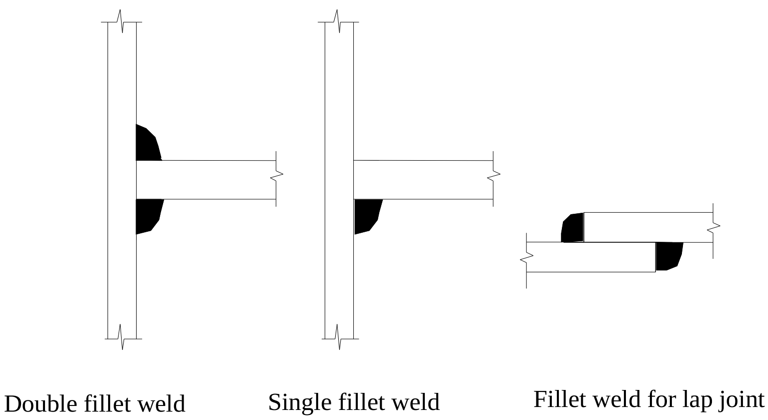


Figure 9.5 Different types of fillet welds

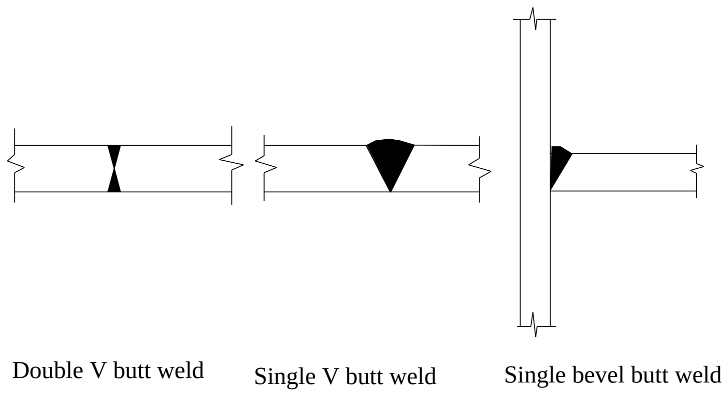


Figure 9.6 Different types of butt welds

9.3.3.1 Butt weld

Butt weld is classified into two catalogs of full penetration weld and partial penetration weld, which influence the strength of weld depending on the depth of penetration of weld. It normally requires 100% test and as the welding involves more number of welding passes, it is more expensive and less common unless it is strictly necessary to ensure the welded part will not fail earlier than the parent material.

9.3.3.2 Fillet weld

Fillet weld is relatively less expensive, when no preparation is required before welding process. It is more commonly used for fillet the corner reentrance of two pieces of metals. During welding, the electrode bisects its angle with the two pieces of metals. The size of fillet welds is measured as its leg length. The minimum size used in the HK Code is 3mm while the common size can be 6mm to 12mm or higher.

9.3.4 Welding symbols

In shop drawings and erection plan, the welds are shown on its type, size, length and locations on the connected parts. This information is indicated in form of symbols. Table 9.1 shows the common types of weld and the commonly used symbols are indicated in Table 9.2 below.

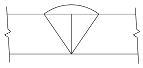
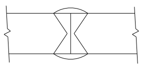
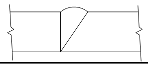
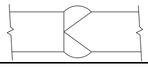
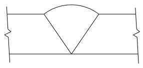
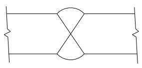
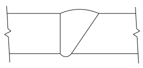
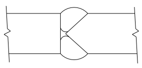
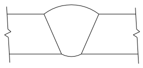
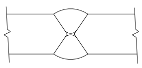
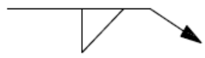
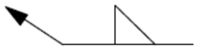
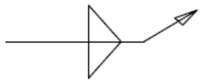
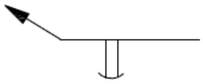
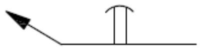
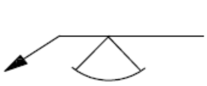
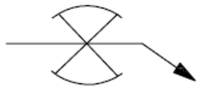
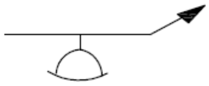
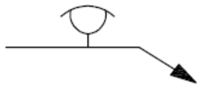
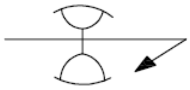
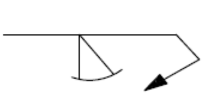
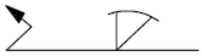
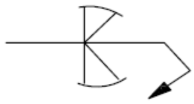
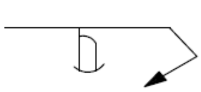
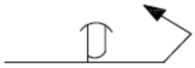
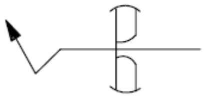
Weld type		Single	Double
Groove	Square		
	Bevel		
	Vee		
	J		
	U		

Table 9.1 Common types of butt weld

Form of weld	Arrow side	Other side	Both sides
FIllet			
Square Butt			
V Butt			
U Butt			
Bevel Butt			
J Butt			

Notes:
The side of the joint to which the arrow points is the arrow or near side and the opposite side of the joint is the other or far side.
All welds are continuous except otherwise stated. Arrow only points to the member grooved.
Dimensions of weld sizes, length and spacing are in millimeters.
`

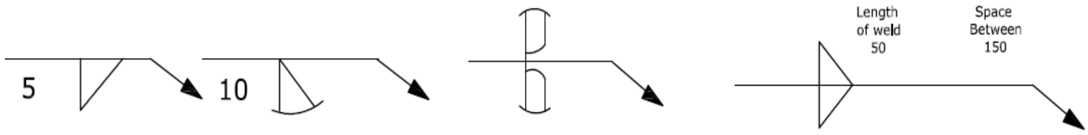


Table 9.2 Typical welding symbols

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9.3.5 Structural design of fillet welds

9.3.5.1 Strength of weld and leg length

The design strength of weld depends on the size of weld, such as leg length s, which is the size of fusion face on unprepared surface of parent metal as shown in Figure 9.7. Also the strength p_w of weld is based on the material used in the welding electrode and strength of parent metal. Further, the throat thickness a, which is the perpendicular distance from inclined surface of weld to root of weld illustrated in Figure 9.7, is determined from the leg length s.

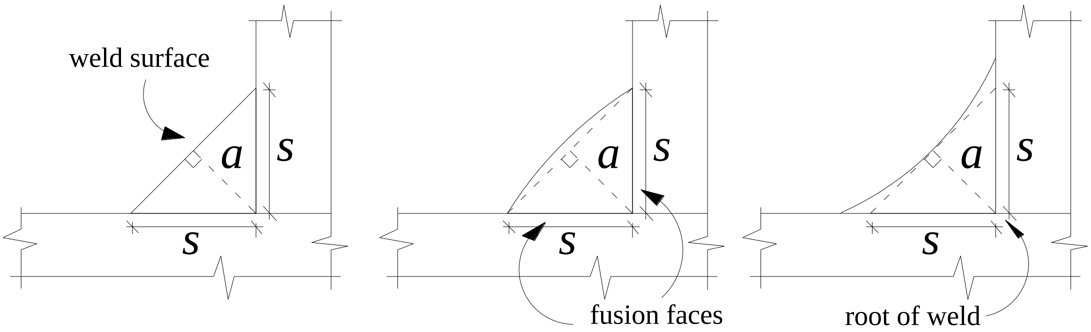


Figure 9.7 Equal leg length of typical fillet welds

For more complex connections, the throat size can be determined from engineering assessment and below are some of the examples for locating the throat size. In Figure 9.8(a), the throat thickness a is taken as the shortest distance from the root of weld to the fusion surface and s_1 and s_2 denote the leg lengths on both sides parallel to the parent metals. The throat thickness a for butt weld can be taken as perpendicular distance from root of weld as indicated in Figure 9.8(b). In the cases of deep fillet weld, throat thickness a is also taken shortest distance from root of weld as shown in Figure 9.8(c). For design calculation and drawing preparation, leg length with equal magnitude on two sides is normally specified.

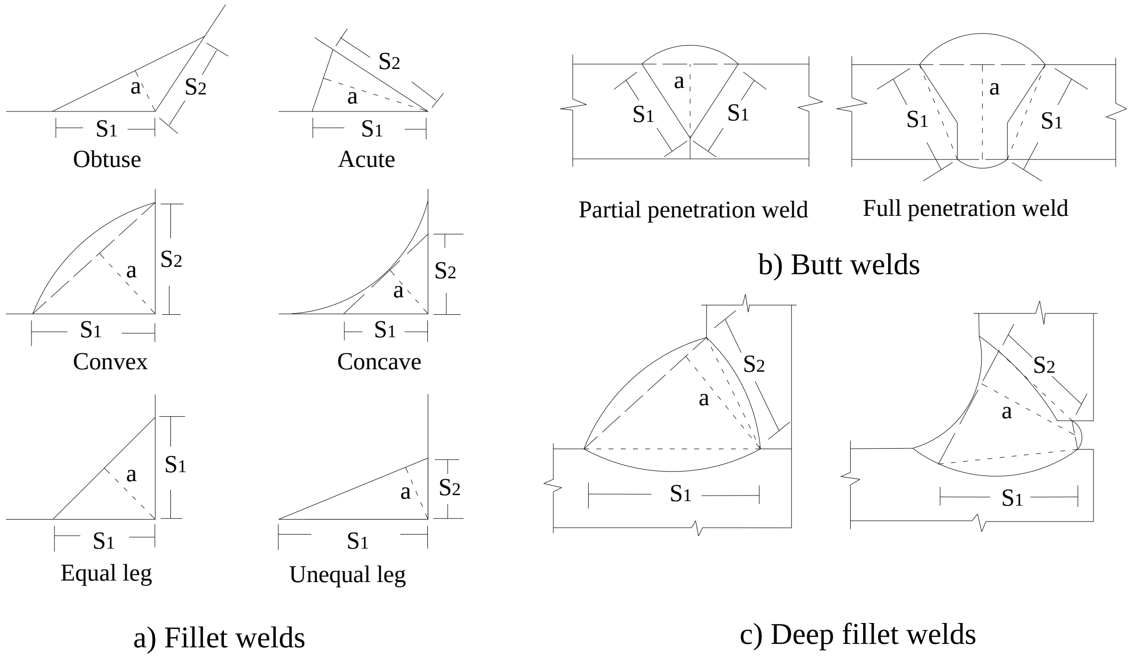


Figure 9.8 Definitions of sizes of fillet weld and butt weld

The strength p_w of fillet weld depends not only on the strength of parent metal, but also the material used in the welding electrode. The strength of different weld grades can be found in *Table 9.2* of the HK Code.

When two different grades of parent materials are joined by fillet welds, the lower grade should be considered in design. The design strength p_w of fillet weld for standard steel grade and common electrode type are tabulated in the Table 9.3 below.

Steel grade	Electrode classification		
	35 (N/mm^2)	42 (N/mm^2)	50 (N/mm^2)
S275	220	220	220
S355	220	250	250
S460	220	250	280

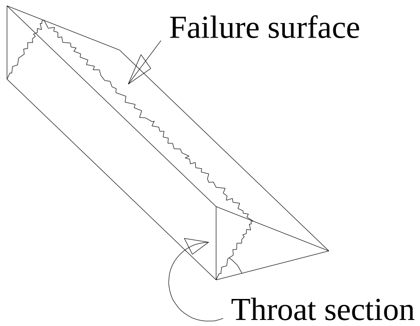
Table 9.3 Design strength of fillet welds p_w to BS EN standards

When the effective length b_e of weld is less than 40mm, the weld length is so small that it cannot be assumed to take any load. Also, the section properties of welded connection should be based on the effective section obtained from the effective length section.

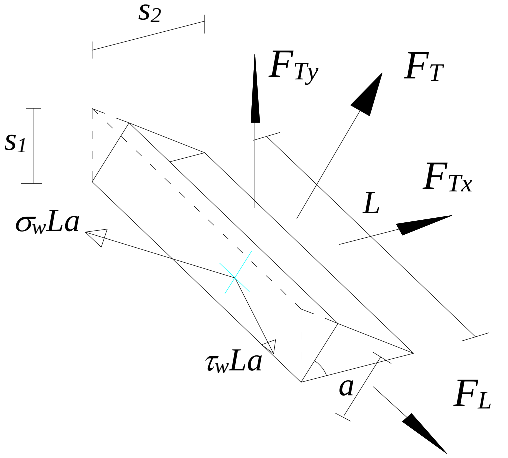
In addition to the design calculation, fillet weld is required to be returned around corners for at least twice of the leg length and the lap length in a lap joint should not be less than 4 times the thickness of the thinner plates.

9.3.5.2 Directional method for capacity of fillet weld

In general, the failure surface of weld is approximately at the throat section under longitudinal and transverse forces as shown in Figure 9.9(a). The strength of fillet weld of length L and throat thickness a is illustrated in Figure 9.9(b).



a) Failure surface at throat section



b) Resultant stresses on throat section

Figure 9.9 Resultant stresses acting on the fillet weld of throat section

The force on a particular weld due to moment and shear in a connection can be resolved into the directions parallel and perpendicular to the weld and then checked against the design capacities of the weld in these two directions as follows. The longitudinal design capacity per unit length of weld P_L is given by the following expression.

$$P_L = p_w a \tag{9.1}$$

in which p_w is the design strength of weld obtained from *Table 9.2* in the HK Code and a is the throat size of the weld.

The capacity per unit length of the weld in the transverse direction P_T is given by,

$$P_T = KP_L \tag{9.2}$$

in which P_L is design capacity per unit length of weld and K is a coefficient given by,

$$K = 1.25\sqrt{\frac{1.5}{1+\cos^2\theta}} \tag{9.3}$$

in which θ is the angle between the resultant and the line bisecting the area of the weld as shown in Figure 9.10(c).

The external force acting on the weld can be resolved into the components in the longitudinal and transverse directions of the weld as shown in Figure 9.10(a) and (b)

and determined as F_L and F_T where $F_T = \sqrt{F_{Tx}^2 + F_{Ty}^2}$ as shown in Figure 9.9(b).
The structural adequacy of the weld can be checked by the conditions as,

$$P_L \geq F_L \tag{9.4}$$

$$P_T \geq F_T \tag{9.5}$$

$$\left(\frac{F_L}{P_L}\right)^2 + \left(\frac{F_T}{P_T}\right)^2 \leq 1 \tag{9.6}$$

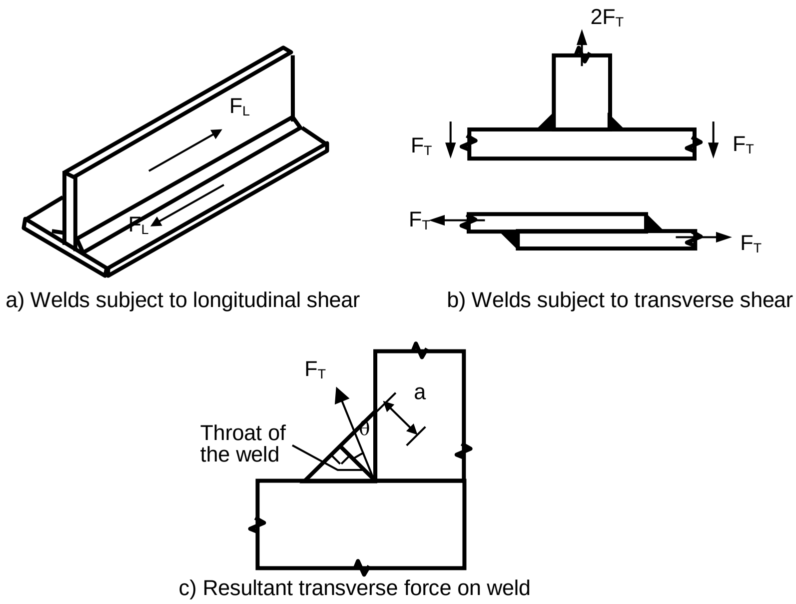


Figure 9.10 Directional approach for capacity of fillet weld

9.3.5.3 The simplified method

This is a simpler but less economical approach of finding the resultant stress acting on weld and checking of this resultant stress against the design strength of weld as,

$$P_L \geq F_R \tag{9.7}$$

in which F_R is the vector resultant stress equal to $\sqrt{F_x^2 + F_y^2 + F_z^2}$ on the weld.

9.3.6 Stress analysis in a welded connection

Before the application of stress check on weld, the stress induced by external loads must first be determined. The stress analysis can be carried out from the first principle and two common connections are adopted for demonstration of the stress analysis of weld group under torsion and shear and under bending and shear.

9.3.6.1 Weld group under torsion and shear

The weld group shown in Figure 9.11(a) is under torsion and vertical shear. Assuming a unit leg length for the weld, the direct shear can be written as,

$$F_S = \frac{P}{\text{length of weld}} = \frac{P}{2x + 2y} \tag{9.8}$$

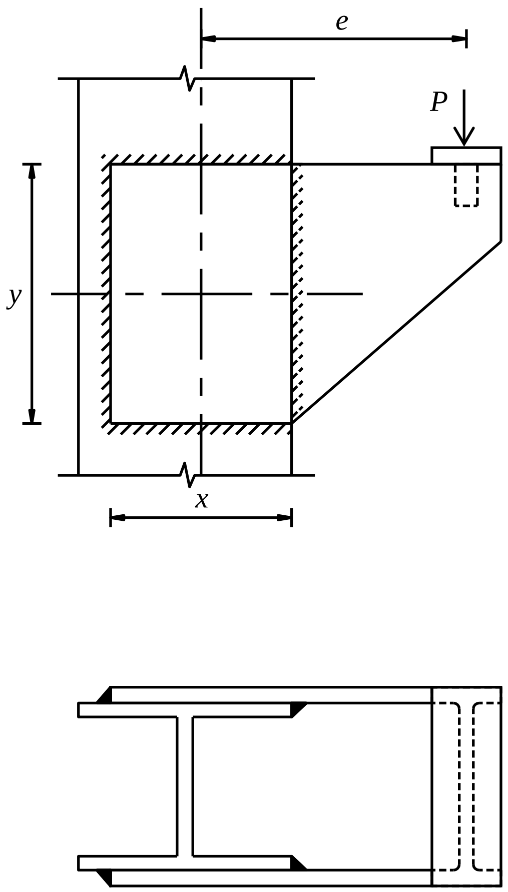


Figure 9.11(a) Torsion and vertical shear

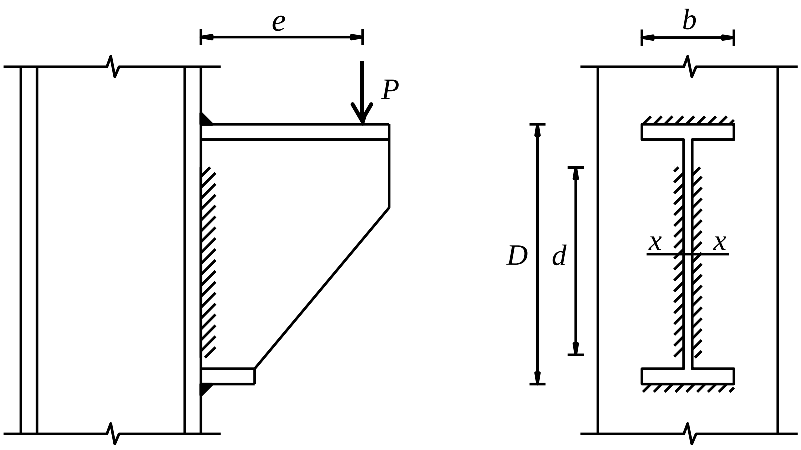


Figure 9.11(b) Bending and vertical shear

Figure 9.11 Connection group subjected to torsion, vertical shear and bending

Shear due to torsion is given by,

$$F_T = \frac{Per}{I_P} \tag{9.9}$$

in which

$$r = \frac{1}{2}\sqrt{x^2 + y^2} \tag{9.10}$$

$$I_P = I_x + I_y = \frac{y^3}{6} + \frac{xy^2}{2} + \frac{x^3}{6} + \frac{x^2 y}{2} \tag{9.11}$$

The resulting shear stress on weld of unit leg length is then given by the resultant of the shear due to vertical force and torsion as $F_R = \sqrt{F_S^2 + F_T^2 + 2F_S F_T \cos\phi}$ in which ϕ is the angle between the two vectors for vertical shear and torsional shear. The required leg length is then equal to $s = \frac{F_R}{0.7\ p_w}$ in which s is the leg length and p_w is the design strength of weld. The factor 0.7 is used for common ratio of leg length to the throat length and it should be varied for special weld geometry.

9.3.6.2 Weld group under bending and shear

For the other common connection shown in Figure 9.11(b) with weld under shear and tension due to bending moment, the shear force on unit leg length of weld can be obtained as follows.

Shear on weld due to vertical load is $F_S = \dfrac{P}{L}$ (9.12)

Tension on weld due to bending is $F_T = \dfrac{M}{I_x}\dfrac{D}{2} = \dfrac{Pe}{I_x}\dfrac{D}{2}$ (9.13)

The resultant stress on weld with unit leg length = $F_R = \sqrt{F_S^2 + F_T^2}$ (9.14)

in which F_S is the induced shear stress, F_T is the induced tension stress, M is the moment at connection, L is the total weld length, D is the distance between the two welds equal to the depth of the I-beam and I_x is the second moment of area about the horizontal x-axis.

The leg length of weld can be obtained similarly as for torsion and shear case as $s = \dfrac{F_R}{0.7\, p_w}$.

9.3.5.5 Welded connections to unstiffened flanges

Owing to the flexibility of connecting plates, the weld length should be reduced in unstiffened plate elements. When the welds connected to the unstiffened plate element of an I-, H- or a box section, a reduced effective length b_e should be used when the effect of weld is also accounted for. For a rolled I- or H-section, the effective length b_e of weld should be as follows.

$b_e = t_c + 2r_c + 5T_c$ (9.15)

but $b_e \le t_c + 2r_c + 5\left(\dfrac{T_c^2}{t_p}\right)\left(\dfrac{p_{yc}}{p_{yp}}\right)$ (9.16)

in which t_c and T_c are the thickness of web and flange of rolled I- or H-section member, respectively, as shown in Figure 9.12(a), r_c is root radius of rolled I- or H-section member, t_p is the thickness of connected plate as shown in Figure 9.12(b), p_{yc} and p_{yp} are respectively design strength of rolled I- or H-column or structural members and connected plate.

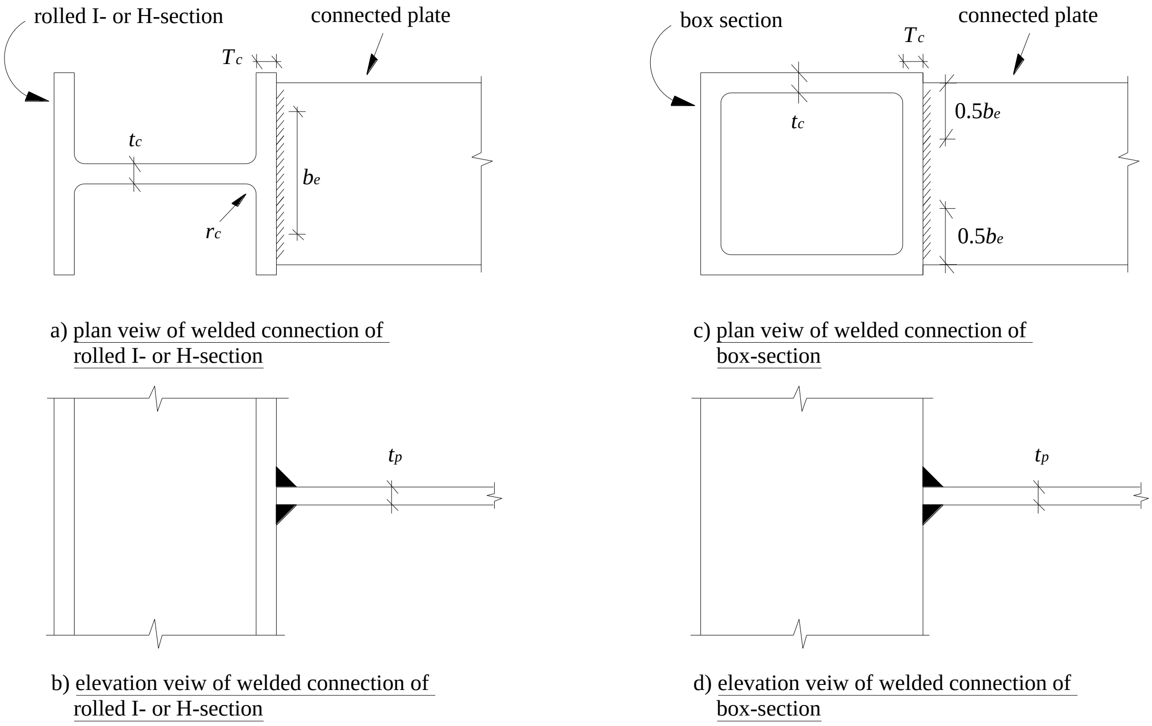


Figure 9.12 Effective length of weld connected to unstiffened plate element

For a box section in Figure 9.12(c) and 9.12(d), the effective length b_e of weld is taken as,

$$b_e = 2t_c + 5T_c \tag{9.17}$$

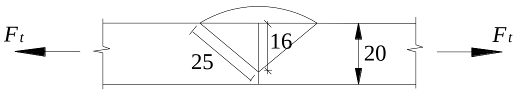
$$\text{But } b_e \le 2t_c + 5\left(\frac{T_c^2}{t_p}\right)\left(\frac{p_{yc}}{p_{yp}}\right) \tag{9.18}$$

where t_c and T_c are the thickness of web and flange of a box section respectively, as shown in Figure 9.12(c) and t_p is the thickness of connected plate as shown in Figure 9.12(d).

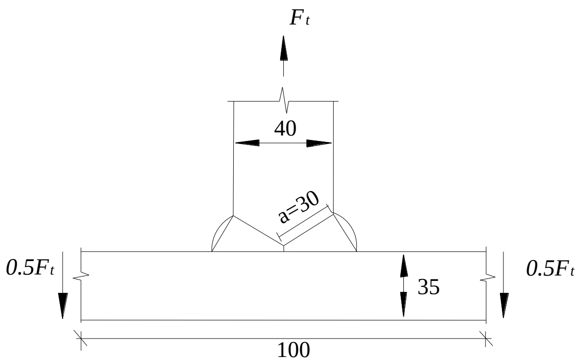
9.4 Worked Examples

9.4.1 Simple welded connection

The connection is formed by joining two plates together by butt weld as shown. The parent metal is in grade S460. They are used to transfer tension only, which are 200kN and 1520kN for case (a) and case (b), respectively. The sizes of butt weld are also given in the figure. The electrode of weld is both E50. Length of connections for both cases in longitudinal direction is 300mm.



a) Partial penetration weld



b) Full penetration weld

Solution

a) Partial penetration weld

TENSION CAPACITY

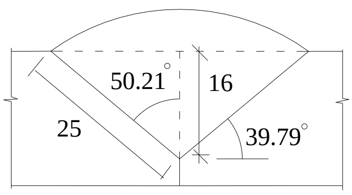
Design strength of parent metal, $p_y = 440\,N/mm^2$ for $16mm < T \le 40mm$ *(Table 3.2)*

Throat size, $a = 16mm$ *(Clause 9.2.5.2.2)*

Eccentricity, $e = 10 - \frac{16}{3} = 4.67mm$

Stress, $\sigma = \frac{F_t}{A} + \frac{F_t \cdot e}{Z} = \frac{200\times10^3}{300\times16} + \frac{200\times10^3\times4.67}{300\times16^2/6} = 114.6\,N/mm^2 < p_y$

(OK)



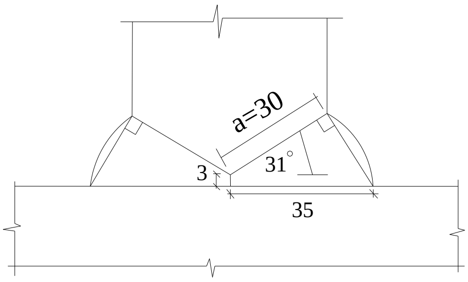
b) Full penetration weld

TENSION CAPACITY

Design strength of parent metal, $p_y = 440 N/mm^2$ for $16mm < T \leq 40mm$ *(Table 3.2)*

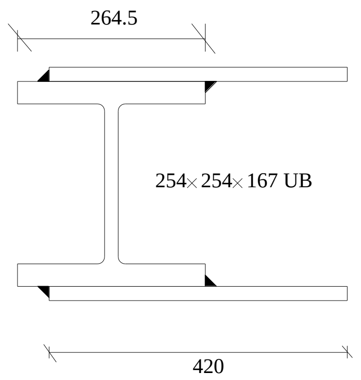
Effective area of vertical plate element, $A_e = 300 \times 40 = 12000 mm^2$

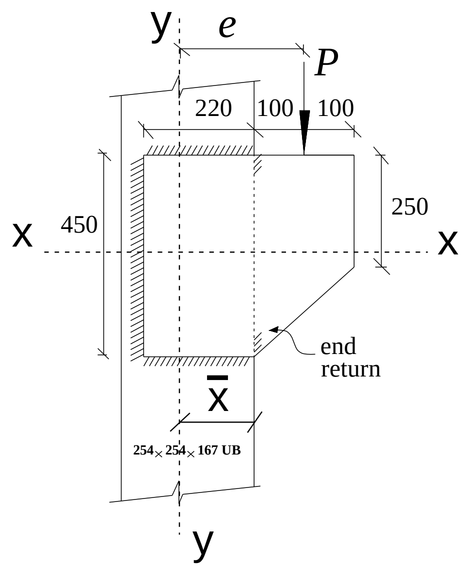
Tension capacity of vertical plate element, $P_t = p_y A_e = 440 \times 12000 = 5280 kN > F_t$ *(8.66)*
(OK)



9.4.2 Bracket connection in typical portal frame

The crane beam in the simple portal frame is supported by bracket connection welded to the steel column. Two gusset plates are welded to the flanges of the steel column to form the bracket connection as shown in the figure. The 20mm thick gusset plate is made of grade S275 steel material. The welded connection is used as this rigid moment connection. The electrode of weld is E35 for the welded connection. Design the size of fillet weld in the bracket connection to enable to take factored shear force of 500kN from crane beam.





<u>Solution</u>

The bracket connection is supported to take point load *P* and the eccentric moment *Pe*. The three side fillet welds are used to withstand the in-plane shear due to both point load and eccentric moment. The structural adequacy of the most outer side weld should be checked.

DESIGN LOAD

Vertical shear, $P = 500kN$

For unit leg length,

Area of weld, $L_w = 450 + 220 \times 2 = 890mm$

Distance to centroid, $\bar{x} = \dfrac{450 \times 220 + 220 \times 110 \times 2}{890} = 165.6mm$

$$r = \sqrt{165.6^2 + 225^2} = 279.4mm$$

Eccentricity of load, $e = 165.6 + 100 = 265.6mm$

Second moment of inertia about x-x axis,

$$I_x = \frac{450^3}{12} + 220 \times 225^2 \times 2 = 2.987 \times 10^7 mm^3$$

Second moment of inertia about y-y axis,

$$I_y = \left[\frac{220^3}{12} + 220 \times (165.6 - 110)^2\right] \times 2 + 450 \times (220 - 165.6)^2 = 4.467 \times 10^6 mm^3$$

Polar moment of inertia about z-z axis,

$$I_z = I_x + I_y = 2.987 \times 10^7 + 4.467 \times 10^6 = 3.434 \times 10^7 mm^3$$

Direct shear, $F_s = \frac{P}{L_w} = \frac{500 \times 10^3}{890} = 561.8\, N/mm$

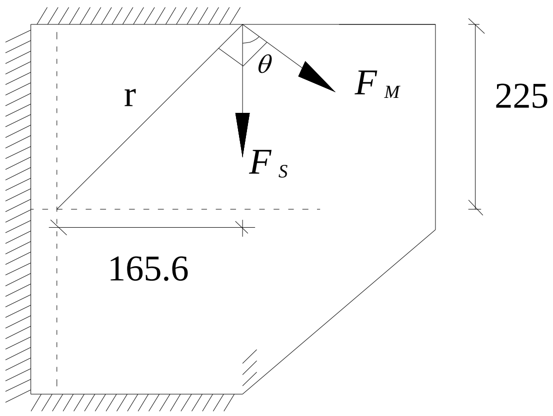
Shear due to torsion, $F_M = \frac{500 \times 10^3 \times 265.6 \times 279.4}{3.434 \times 10^7} = 1080.5\, N/mm$

CAPACITY OF WELD

$\theta = \tan^{-1}\left(\frac{225}{165.6}\right) = 53.6°$

Resultant load, $F_R = \sqrt{F_S^2 + F_M^2 + 2F_S F_M \cos\theta}$

$= \sqrt{561.8^2 + 1080.5^2 + 2 \times 561.8 \times 1080.5 \times \cos 53.6°} = 1484.4\, N/mm$



Design strength of weld, $p_w = 220\, N/mm^2$ *(Table 9.2a)*

Minimum Leg length required, $s = \frac{1484.4}{0.7 \times 220} = 9.6mm$ *(Clause 9.2.5.1.6(a))*

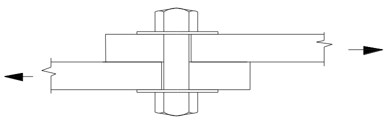
$\therefore$ use 10mm weld

9.5 Bolted connection

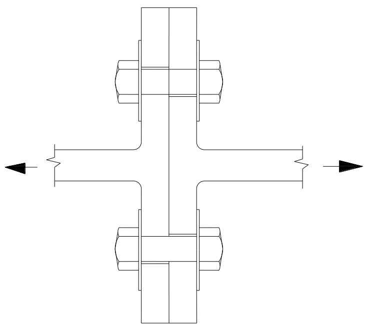
Bolt or fastener is one of the most common methods of connecting two or more members. The advantages of using bolts in place of weld include the easy fabrication on site, avoidance of residual stress for weld, less on-site quality control problem and easy dismantling and re-fabrication of connections. However, bolting on site requires careful planning and positioning and therefore they are less flexible, which is particularly true for construction in Hong Kong where a project is normally executed within a short period of time.

Bolts transfer loads mainly by the actions shown in Figure 9.13 and accordingly the strength of bolts is required to be checked against these actions.

- Tension in thread of bolt
- Shear in bolt shrank or thread
- Bearing of plates containing bolt hole on bolt shrank
- Friction between bolt and clamped plates



Shear connection



Tension connection

Figure 9.13 Bolted connection under shear and tension

It is uncommon to allow bolt to be bent about its own principal axis because of its small second moment of area about its own principal axis.

Steel bolts are required to be adequate not only on their strength, but also on the hardness because insufficiently hard bolts may deform under stress, especially at their thread area leading to slipping of thread and separation of bolts and nuts. Hardness can also be a measure of bolt quality and uniformity. The thread tolerance in bolts is important in making sure no slipping between the thread of bolts and nuts. Mixed use of bolts and nuts from two manufacturing sources should be avoided as their tolerances may not be compatible. The Vickers hardness and Brinell and Rockwell tests are commonly used in bolt standards for measurement of hardness.

There are two major types of bolts as ordinary bolt and high strength friction grip (HSFG) bolt. The ordinary bolt is commonly used because of easy fabrication and simple mechanism in taking loads. It has the advantage of greater ductility. On the other hand, preloaded high strength friction grip (HSFG) bolt normally has a greater strength of the pretension force action and it is normally made of high steel grade. HSFG bolt resists shear by friction between bolts and bearing plates and tension by the pre-loading force.

In the HK Code, bolt grades of 4.6, 8.8 and 10.9 are recommended. The first number for the bolt grade refers to the minimum ultimate strength in hundred N/mm^2 and the second number, after dividing by 10, represents the ratio of yield to ultimate stress. For example, grade 8.8 bolt has an ultimate strength of 800N/mm^2 and the design yield strength as 0.8×800 = 640N/mm^2. However, the design strength needs to be not greater than $0.7 \times U_b$ or 560 N/mm^2 here. The Code does not recommended the use of grade 12.9 bolts, which are occasionally used, because of their nominally lower ultimate strain and lack of ductility.

The size of bolts in thread area and bolt opening cannot be directly calculated from their nominal diameters and Table 9.4 shows the size of common bolt grades of M12 to M36 bolts. M indicates the dimension is in metric unit.

Bolt Size	Nominal diameter (mm)	Shrank area (mm^2)	Tensile stress or thread area (mm^2)
M12	12	113	84.3
M16	16	201	157
M20	20	314	245
M22	22	380	303
M24	24	452	358
M27	27	572	459
M30	30	706	561
M33	33	855	694
M36	36	1017	817

Table 9.4 Sizes of bolts of common bolt grades

The tensile strength of the bolt can then be equal to the product of the tensile stress area and the design strength. For example, M22 grade 4.6 bolt has the capacity equal to 400×0.6×303 = 72.7kN.

The size of bolt opening is slightly greater than bolt size in order to allow bolt installation. As an approximation, for standard hole, bolt diameter greater than 24mm should have a bolt hole with diameter greater than the bolt nominal diameter by 3mm and those bolts with nominal diameter less than or equal to 24mm should have bolt opening 2mm greater than the bolt size.

Tightening of bolts requires sufficient torque but not too high to cause fracture in the bolt or its components. For bolt with faces normal to bolt axes and length not greater than 4 times the diameter, the torque turn should be 1/3 about the bolt axis. For bolt length between 4 and 8 diameters, the angle of turn should be 1/2 turn and for longer bolt length, the angle of turn should be 2/3 turn.

9.5.1 Bolt grades

The most commonly used grades of bolts are grade 4.6, 8.8 and 10.9 bolts. Other grades of bolts are 4.8, 5.6, 5.8, 6.8, and 12.9. Bolt strength outside the range of 400 and 1000 should not be used unless test confirms the applicability. Both ordinary clearance bolts and high strength friction grip (HSFG) bolts are also widely used.

Grade 4.6 bolts made of low carbon high strength steel are used normally for medium and light duty connections such as purlins or sheeting. Holding down bolts also commonly use grade 4.6 bolts because of preferred ductility not only on bolts, but also on base plate of which the design strength is not allowed to be greater than $275N/mm^2$.

Grade 8.8 bolts or higher grade bolts made of high strength alloy should be used for heavy duty connections. HSFG bolts should be used in the load reversal condition and in case when the controlled deflection is very much relying on the connection stiffness, like fixed end in a cantilever beam or moment joints in eave of portals.

9.5.2 Spacing and detailing requirements

Bolts are commonly arranged as a group and positioned in a series of rows. In order to utilize the bolt strength, the spacing and edge distance must be controlled within a certain dimension. Below are some of the common criteria.

9.5.2.1 Maximum spacing

To ensure the load is reasonably shared between the bolts in a group, the spacing cannot be too large. For connection under compression, the spacing should not exceed 12t and 150mm where t is the thickness of the thinner plates in the connection.

9.5.2.2 Minimum spacing

If spacing is insufficient, the material between bolts may be over-stressed. The HK Code has a requirement for control of minimum spacing. The minimum spacing parallel to the load direction is 2.5 times the hole diameter and 3.0 times the hole diameter in the direction perpendicular to the load direction.

9.5.2.3 Minimum edge distance

The end distance e for a bolt measured as the distance between the centre of the opening to the edge of the connecting plate should be referred to *Table 9.3* of the Code.

9.5.3 Behaviour of bolted connections

Appreciation of local behaviour of bolted connection in beam-to-column or beam-to-beam connections is important in detailing. The understanding of load path, which should depend on the arrangement of the components at connection, for transfer of force and moment between members is important in connection design and detailing. When the deformation and stress distribution in the structural component are known, the strength of structural component at the connection should be designed and checked. This section describes the typical behaviour and failure mechanism for beam-to-column and beam-to-beam connections.

9.5.3.1 Beam-to-column connection

An example of the load path for an extended end plate connection showing the behaviour of a bolt group is indicated in Figure 9.14. The connection is used to transmit the vertical shear and moment from beam to column. The point of rotation is assumed at the base of bottom flange of beam member as shown in Figure 9.14. The corresponding deformations at the connection are caused by the loadings of moment and shear transferred from beam member and finally to induce different structural effects, which are tabulated in Table 9.5.

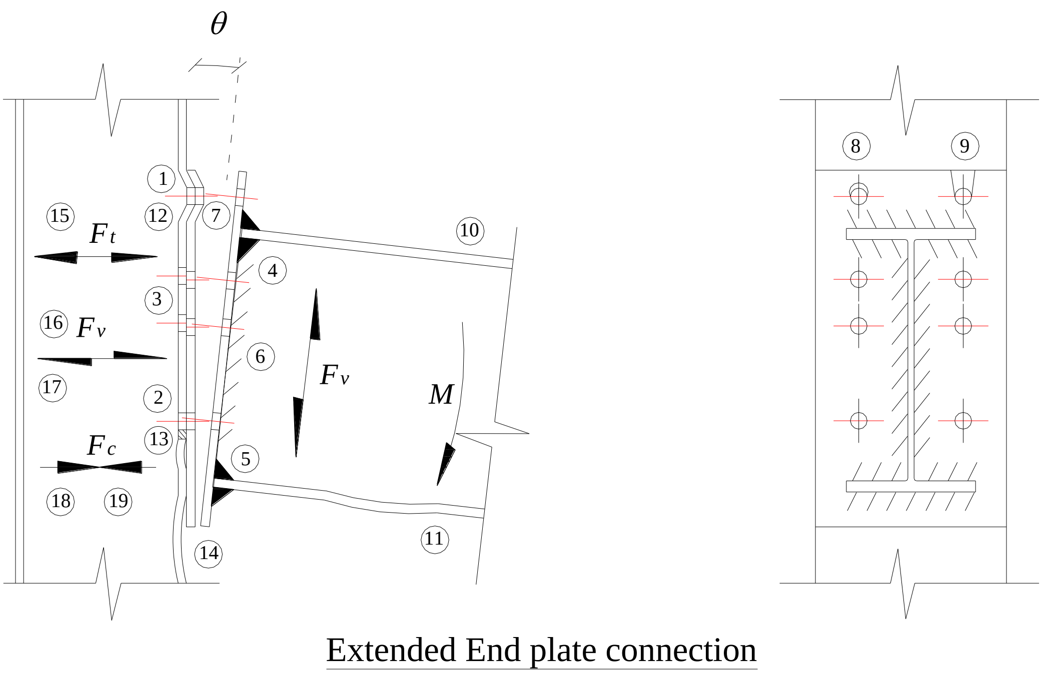


Figure 9.14 Behaviour of different components at beam-to-column connection

Components at connection	Notations	Structural effects
Bolt	①	Yielding due to tension
	②	Yielding due to vertical bearing
	③	Shear failure
Weld	④	Tear off failure
	⑤	Compression failure
	⑥	Shear out failure
End plate	⑦	Prying force due to bending
	⑧	Yielding due to vertical bearing
	⑨	Shear out failure
Flange of beam	⑩	Yielding due to tension on top flange
	⑪	Local buckling on bottom flange
Flange of column	⑫	Prying force due to bending
	⑬	Yielding due to shear and compression
	⑭	Local buckling due to vertical load
Web of column	⑮	Web fracture due to tension
	⑯	Yielding due to shear
	⑰	Shear web buckling
	⑱	Web crushing due to compression
	⑲	Web buckling

Table 9.5 Structural failures at beam column connection

In Table 9.5, there are 19 principal failure modes at the connection to be checked for the components of bolt, weld, end plate, beam member and column. In this chapter, the behaviour of components of bolt, weld or end plate is studied for design. Other structural components, such as column and beam, related to the behaviour at connection should be referred to previous relevant chapters. The interaction effects on bolt and weld are neglected in Table 9.5 but their interactive use to share a load is not recommended because of their different ductility performance. In general, the principal behaviour of these components at connection is similar. In addition, some local effects are also ignored in Table 9.5 and these local effects make connection behaviour more complex and variable. They include the geometric imperfections arising from welding distortion and misalignment of clearance and residual stress and strain due to lack of fits and welding shrinkage. Actually, these local effects are considered in the material strength of weld or bolt. Therefore, the structural design can be carried out according to the behaviour of connection or failure mode at connection as listed in Table 9.5, which is adequate for structural design.

9.5.3.2 Beam-to-beam connections

Secondary beams are commonly connected to primary beams through simple supports indicated in Figure 9.15. In the connection vertical shear load is required to be transferred. In Figure 9.15, the point of rotation is assumed to be at the base of the secondary beam member. The common modes of local failure are tabulated in Table 9.6.

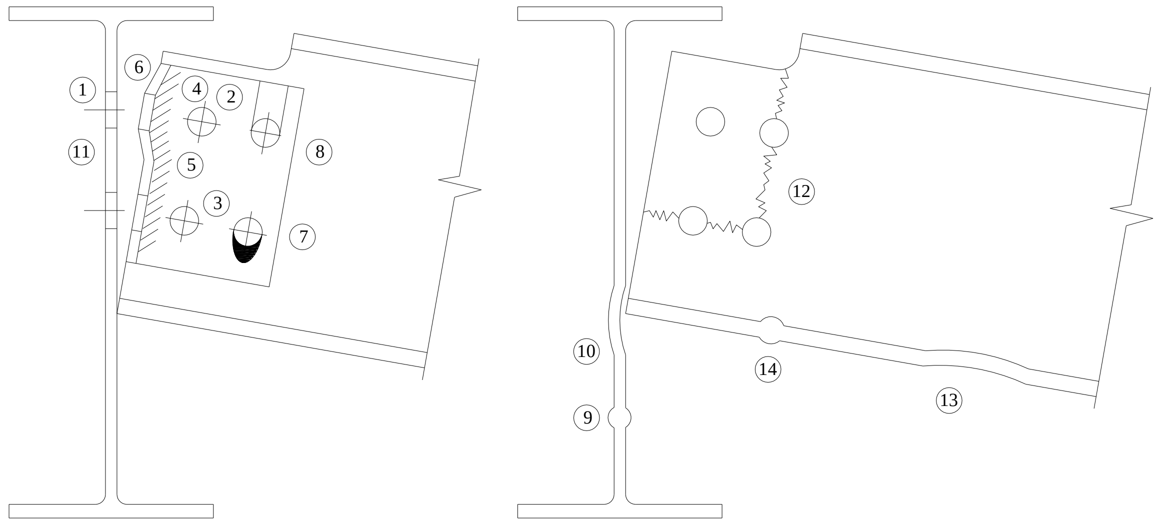


Figure 9.15 Behaviour of different components at beam to beam connection

In Table 9.6, there can be 14 local failure modes at the connection to be checked in design for the components of bolt, weld, end plate, secondary and main beams.

Components at connection	Notations	Structural effects
Bolt	①	Yielding due to tension
	②	Yielding due to vertical bearing
	③	Shear failure
Weld	④	Tear off failure
	⑤	Shear out failure
Connected plate	⑥	Prying force due to bending
	⑦	Yielding due to vertical bearing
	⑧	Shear out failure
Web of main beam	⑨	Crushing due to compression
	⑩	Shear web buckling
	⑪	Web crushing due to vertical shear
Web of secondary beam	⑫	Block shear failure
Flange of secondary beam	⑬	Local buckling due to compression
	⑭	Yielding due to compression

Table 9.6 Structural failures at beam to beam connection

The behaviour of a group of bolts in a connection is relatively more complex than those of a single bolt as discussed above. The in-plane or out-of-plane loads shared by a group of bolts is non-uniform. In a long shear bolted connection, the force at end bolt may be high and up to the material yielding stress. The load on such bolts is to redistribute to other bolts near the centre of connection. Also for bolts in tension connection, the tension loads distribution may not be necessarily uniformly shared by each tension bolt. For simplicity, it is commonly assumed that equal size bolts share equally the load in in-plane and out-of-plane shears. To satisfy this condition, the connection plate is assumed to behave rigidly and elastically and the bolts behave elastically and in a ductile manner in order to shear loads.

9.5.3.3 Prying effect in bolted connection

For bolted connection subjected to external tension F_t, the flexible deformations at unstiffened plate element, such as flange or connected plate, induce additional tensile force in addition to axial force in bolt F_{bt}. This additional tensile force is termed the prying force Q as shown in Figure 9.16 due to deflection of unstiffened plate component. The prying force Q develops because unstiffened plate of member is in contact with the connection. The contact area under compression and at the end of unstiffened plate shown in Figure 9.16 produces an additional force in bolt.

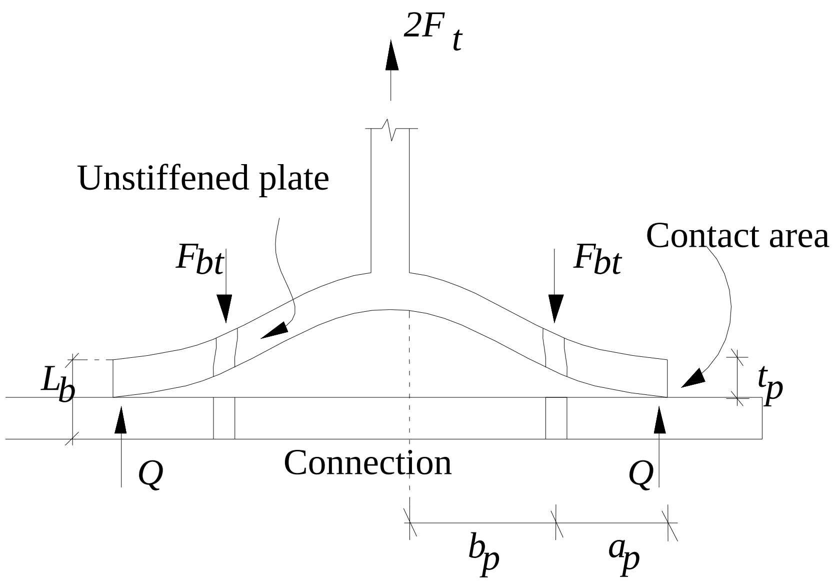


Figure 9.16 Prying force on bolt due to flexural deformation on unstiffened plate

For thicker connected plate where the bending stiffness of the connecting plate is high, the magnitude of prying force is insignificant and vice versa. The prying force for ordinary or pre-loaded bolt for simple bolted arrangement can be derived as follows. Considering symmetrical half of the connection, by the method of virtual work, the displacement at $x = a_p$ is given by,

$$\delta_b = \frac{1}{EI}\left\{\int_0^{a_p} Qx \cdot -x dx + \int_{a_p}^{a_p+b_p}\left[(F_t + Q)a_p - F_t x\right] \cdot -a_p dx\right\} \tag{9.19}$$

in which Q is the prying force shown in Figure 9.16, EI is the flexural constant of the connecting plate, F_t is half of the applied tension and a_p is the distance from the edge of plate for the prying force to the bolt centre and b_p is the distance from the bolt centre to 20% distance into the end plate weld or the root radius as shown in Figure 9.16.

After integrating the above equation, δ_b is given by,

$$\delta_b = \frac{1}{6EI}\left[3F_t a_p b_p^2 - 2Qa_p^2(a_p + 3b_p)\right] \tag{9.20}$$

On the other hand, the bolt axial deformation due to the bolt tension force including prying force is given by,

$$\delta_b = \frac{(F_t + Q - F_{bs})L_b}{EA_s} \tag{9.21}$$

in which A_s is the cross section area of bolt, L_b is grip or total length of the bolt, F_{bs} is the preloaded force on HSFG bolt, if any. For ordinary bolt, the F_{bs} in Equation (9.21) should be set to zero. Substituting δ_b in Equation (9.21) into Equation (9.20), the prying force Q for ordinary bolt can be obtained as follows.

$$Q = \frac{F_t\left(3a_p b_p^2 - \frac{6IL_b}{A_s}\right)}{2a_p^2\left(a_p + 3b_p\right) + \frac{6IL_b}{A_s}} \tag{9.22}$$

9.5.4 Design of ordinary non-preloaded bolts

In simple design where all joints are pinned, connections are required to be designed to take direct forces only and moment are not considered. In other cases moments are unavoidable due to eccentricities of connections, the effect of moment should be considered in finding of bolt forces. The pinned connections should be detailed to allow full rotational ductility.

In design for moment frames where full rotational continuity at connections is assumed, shear, axial forces and moments between members are needed for consideration in finding the forces in bolts. Further, detailing should attempt to provide adequate stiffness at joints. HSFG bolts are recommended when strict control of slipping in joints is required.

In design for semi-rigid connections, partial continuity is assumed between members and connections are required to have adequate strength with sufficient rotational capacity. The moment-rotation characteristics of the connection details are consistently used both in the analysis of the framework and the design of the connections. At present, due to the lack of data in connection stiffness, semi-rigid connection design is uncommon, but it can be included in design and analysis directly and easily when the moment vs. rotation curve such as the ones in Figure 9.4 is available.

9.5.4.1 Shear capacity of ordinary bolts

The ISO hexagon head non-preloaded ordinary bolt shown in Figure 9.17 with washer is most commonly used. The size of bolt hole against the size of bolts is specified in the HK Code.

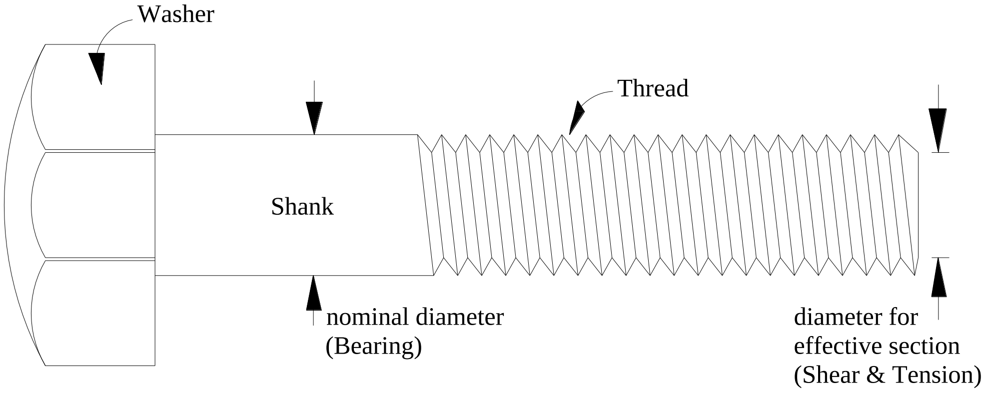


Figure 9.17 Different parts of ordinary bolt and preloaded bolt

When the shear surface is at the thread area, the shear area A_s of ordinary bolt should be taken as the section shown in Figure 9.18. If the shear surface occurs at the shrank, the cross sectional area of the shrank can be used. Table 9.4 shows the area at these locations. With the value of shear area, the shear capacity P_s of an ordinary bolt is then given by Equation (9.23) as,

$$P_s = p_s A_s \tag{9.23}$$

in which p_s is the design shear strength of the bolt given in *Table 9.5* of the HK Code.

For long joints, a reduction factor β_L is required and the determination of this reduction factor is given in *Clause 9.3.6.1.4* of the HK Code.

When the shear action is on two or more surfaces, the shear capacity should be increased by multiplying the single surface shear capacity by the number of shear areas. Figure 9.18 shows the condition of single and double shear capacity in joints.

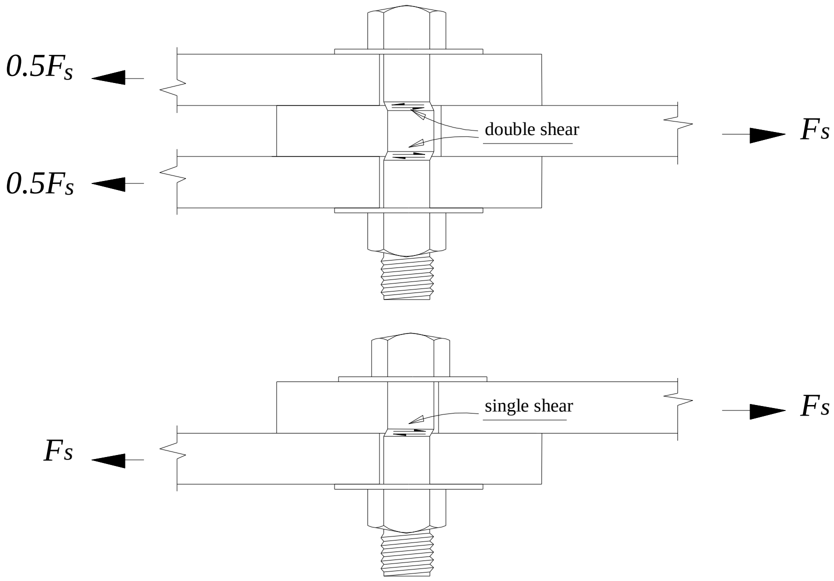


Figure 9.18 Double shear and single shear capacity of ordinary bolt

Apart from the shear failure occurring on a bolt, the block shear failure of a group of bolts is required to be checked. The shear failure surface will be constructed by assuming the minimum length for shearing off of the bolt group shown in Figure 9.19 and the checking eliminates the failure of tearing off in thin plates at connections.

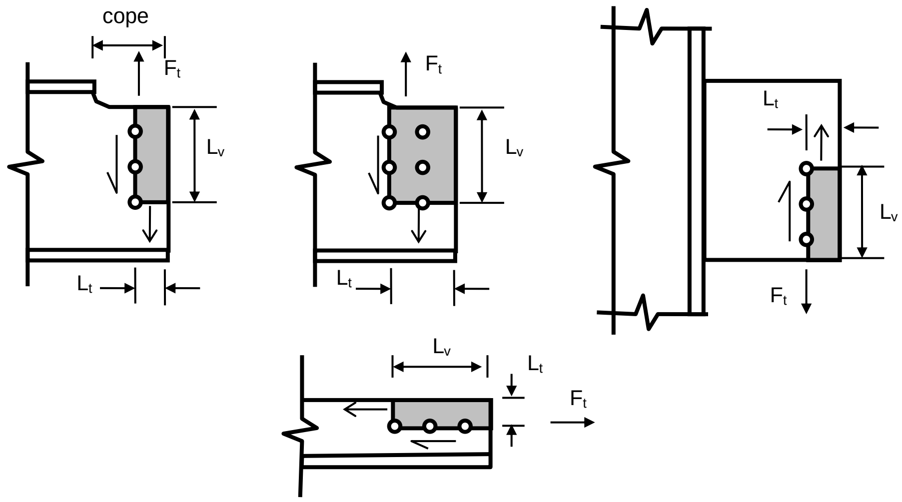


Figure 9.19 Block shear failure through on a group of bolt holes

The combined block shear capacity for both the shear and tension edges or faces in a shear joint shown in Figure 9.19 is given by,

$$P_r = \frac{1}{\sqrt{3}} p_y t \left[L_v + K_e \left(L_t - kD_t \right) \right] \tag{9.24}$$

in which p_y and t are the design strength and thickness of web of beam or bracket, respectively, L_v and L_t are respectively the length of shear face and tension face shown in Figure 9.19, K_e is the effective net area coefficient in *Clause 9.3.4.4* of HK Code and previously discussed, D_t is the diameter of bolt hole along tension face, respectively and k is a factor equal to 0.5 for single row of bolts and to 2.5 for double row of bolts.

If block shear check is not satisfactory, increasing the plate thickness, welding of an additional plate or increasing the length of the failure surface can be considered.

9.5.4.2 Bearing capacity of ordinary bolts

Another possibility of failure of bolts is due to bearing failure of bolts and on bearing plates. The bearing capacity P_{bb} of an ordinary bolt should be taken from Equation (9.25) as,

$$P_{bb} = d t_p p_{bb} \tag{9.25}$$

in which d is the nominal diameter of bolt, t_p is the thickness of thinner connected plate and p_{bb} is the bearing strength of bolt for different grade of bolts indicated in *Table 9.6* of the HK Code.

The bearing capacity P_{bs} of the connected parts should be taken as the least of the followings.

$P_{bs} = k_{bs} d t_p p_{bs}$ (bearing on edge of hole) (9.26)

$P_{bs} = 0.5 k_{ps} e t_p p_{bs}$ (tearing out of connected plate) (9.27) in which p_{bs} is the design bearing strength of the plates, k_{bs} is the hole coefficient for different hole types given in *Clause 9.3.6.1.3* in HK Code and e is the edge distance measured between the centre of the hole and the edge and along the direction of applied force. Figure 9.20 shows the mode of failure for insufficient edge distance.

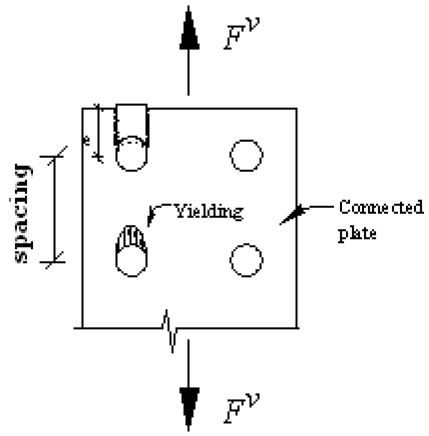


Figure 9.20 Different failure modes of bolted connection subjected to shear

9.5.4.3 Tension capacity of ordinary bolts

Bolts take tension in common connections. For the non-preloaded bolted connection, the tension capacity P_t is written as,

$$P_t = A_t p_t \tag{9.28}$$

in which A_t is tensile stress area and p_t is tension strength given by *Table 9.8* in *Clause 9.3.7.1* of HK Code.

The effect of prying action has been discussed in Section 9.5.3.3 of this chapter. For bolted connections satisfying the requirements in *Clause 9.3.7.2* of HK Code, the effect of prying force Q should be taken into account in calculating the reduced tension capacity P_{nom} as,

$$P_{nom} = 0.8 A_t p_t \tag{9.29}$$

When the condition of using Equation (9.29) is not satisfied, the prying force is required to be calculated explicitly to Section 9.5.3.3 of this chapter. The total applied force F_{tot} in the bolt can be determined as,

$$F_{tot} = F_t + Q \le P_t \tag{9.30}$$

in which F_t is applied tension force required for transmission by bolted connection and Q is the prying force calculated in Section 9.5.3.3 of this chapter and P_t is the design tension capacity of the bolt.

9.5.4.4 Interaction of shear and tension in ordinary bolts

When a bolt is to transfer both shear and tension, the interaction effect of the bolt should be checked, in addition to the separated satisfaction in tension and shear capacity. The additional interaction check can be carried out as follows for the case of no explicit consideration of prying force.

$$\frac{F_s}{P_s} + \frac{F_t}{P_{nom}} \le 1.4 \tag{9.31}$$

in which F_s is shear force on each bolt and F_t is tension force in the bolt. If the prying force Q is calculated explicitly, the condition of interaction effect is given by,

$$\frac{F_s}{P_s} + \frac{F_{tot}}{P_t} \le 1.4 \tag{9.32}$$

9.5.5 Design of high strength friction grip (HSFG) bolts

High strength friction grip (HSFG) bolts or the pre-loaded bolts are commonly used for heavily duty connections and they action of resisting tension and shear are very much different from the ordinary bolts.

9.5.5.1 Shear capacity of HSFG bolts

HK Code considers the design of HSFG to the ultimate load without slipping. In other design codes, the serviceability limit state conditions can be considered alternatively. The design shear capacity of preloaded bolts is given by,

$$P_{SL} = 0.9 K_s \mu P_0 \tag{9.33}$$

in which K_s is a coefficient allowing for different types of hole stated in the HK Code, μ is the slip resistance factor between connected parts for different surface conditions given in *Clause 9.3.6.2* in the HK Code, P_0 is minimum shank tension listed in Table 9.7 below for different size of bolts.

Bolt size (mm)	Minimum shank tension P_0 (kN)
M12	49
M16	91
M20	142
M22	176
M24	205
M30	326
M36	475

Table 9.7 Design capacity for preloaded HSFG bolt

It should be noted that the bolt grade of higher than or equal to grade 8.8 should be used for preloaded HSFG bolt. Moreover, for larger hole size and less skin friction factor, the shear capacity P_{SL} of preloaded HSFG bolt is generally lower, as the slip is allowed for in the bolted connection.

For preloaded HSFG bolted connection, the slip is not allowed in the preloaded bolt connection under factored load such that the preloaded HSFG bolt cannot be in contact with the connected plate. Therefore, the bearing capacity of HSFG bolt is then not critical and it is unnecessary to check the HSFG bolt for bearing failure for the condition under factored loads.

Similar to ordinary bolts, the shear capacity P_{SL} of preloaded HSFG bolt can be double, when there are two interfaces among connected plates in the preloaded bolted connection as shown in Figure 9.21. It implies that the skin friction in this bolted connection among connected plates becomes double. Otherwise, the shear capacity P_{SL} of preloaded HSFG should be same as the value in Equation (9.33).

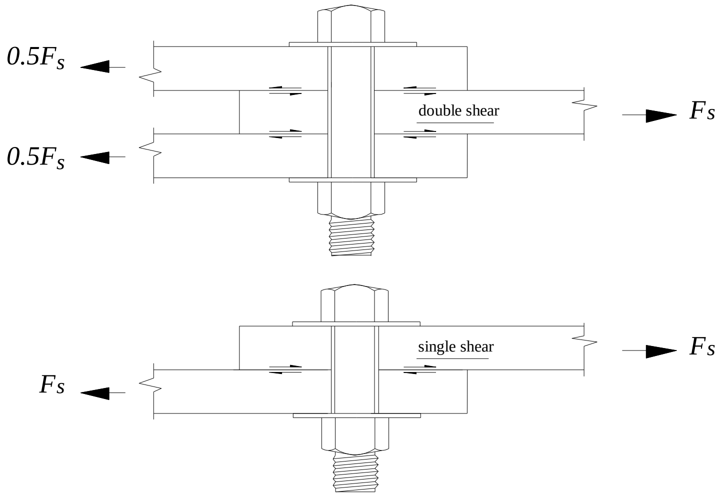


Figure 9.21 Double shear and single shear capacity of preloaded HSFG bolt

9.5.5.2 Tension capacity of HSFG bolts

For the design of tension capacity of preloaded HSFG bolt, the tension capacity P_t of each HSFG bolt should be taken as Equation (9.34) under factored load, when the preloaded force in the HSFG bolt is considered.

$$P_t = 0.9P_0 \tag{9.34}$$

in which P_0 is minimum shank tension as listed in Table 9.7.

9.5.5.3 Interaction of shear and tension in HSFG bolts

The interaction check for preloaded HSFG bolts can be written as,

$$\frac{F_s}{P_{SL}} + \frac{F_{tot}}{0.9P_0} \le 1 \tag{9.35}$$

in which P_{SL} is slip resistance of each preloaded bolt, F_s is shear force distributed on each preloaded HSFG bolt and F_{tot} is the total tension force including prying force Q.

9.5.6 Stress analysis in bolts

Before the application of the above formulae in checking of bolt strength, the load on each bolt should be determined and the principle of structural mechanics can be applied here. Two common types of connections for bolt group under torsion and shear and under bending and shear are selected for illustration of the application of structural mechanics in determination of force in bolts.

9.5.6.1 Bolt under torsion and shear

As shown in Figure 9.22, the bolt group is under torsional moment and shear. The vertical shear for each bolt is given by the simple division as,

$$F_S = \frac{F_V}{\text{No. of bolts}} \tag{9.36}$$

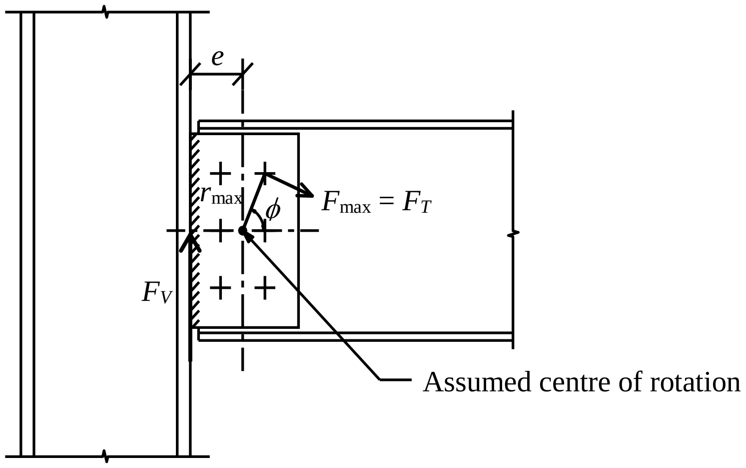


Figure 9.22 Bolt group under torsional moment and shear

The assumption of shear force being proportional to the distance from centre of rotation taken as the centroid of the bolt group is made. For equilibrium, the sum of torsional moment from the shear forces F_i in bolts is equal to the applied torsional moment M_T as,

$$M_T = F_V e = \sum F_i r_i = \sum \frac{F_{\max}}{r_{\max}} r_i^2 = \frac{F_{\max}}{r_{\max}} \sum r_i^2 \quad \text{as} \quad \frac{F_{\max}}{r_{\max}} = \frac{F_i}{r_i}$$

in which F_{max} and r_{max} are respectively the maximum bolt force and the maximum distance of this bolt from the centre of rotation of the bolt group. Noting from geometry that $r_i^2 = x_i^2 + y_i^2$, the maximum bolt force can be obtained as,

$$F_T = \frac{M_T \cdot r_{\max}}{\sum x_i^2 + \sum y_i^2} \qquad (9.37)$$

The resultant force in the bolt under maximum shear can be obtained by the cosine rule for the resultant of shear and torsion forces as,

$$F_R = \sqrt{F_T^2 + F_S^2 + 2F_t F_S \cos\phi} \le P_S \qquad (9.38)$$

in which P_S is the design shear in bolt.

9.5.6.2 Bolts under bending and shear

Bolts can be under direct shear and tension induced by external moment or forces as shown in Figure 9.23. This section shows the typical structural adequacy check for bolts under tension and shear.

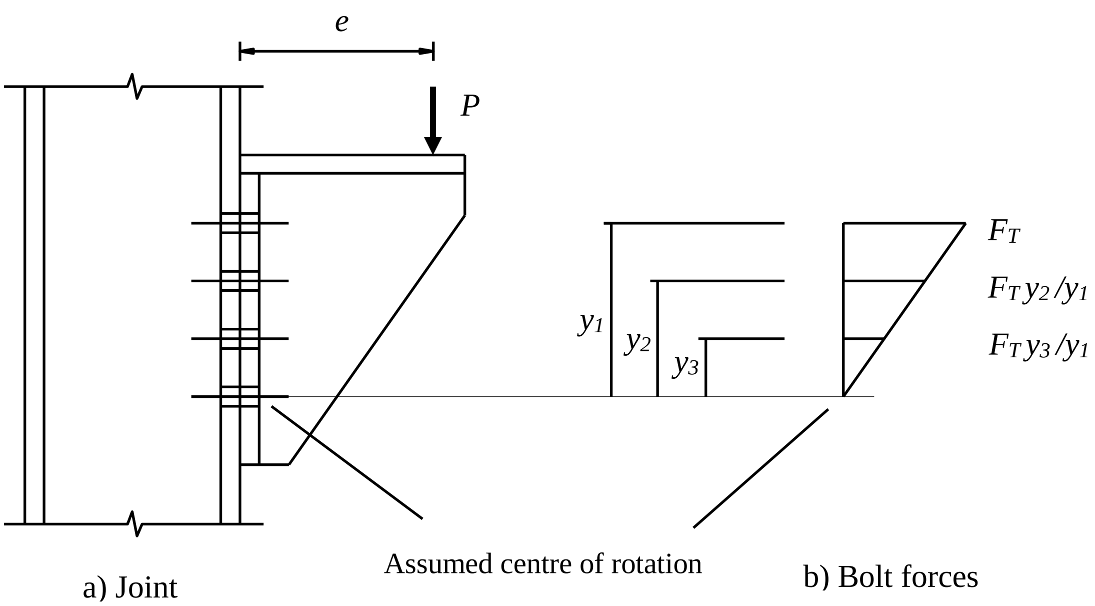


Figure 9.23 Connection group subjected to bending moment and shear

Considering equilibrium against external moment, the tension force induced in the bolt is given by,

$$M = \sum F_i y_i = \sum \frac{F_T}{y_{\max}} y_i^2 = \frac{F_T}{y_{\max}} \sum y_i^2 \text{ as } \frac{F_T}{y_{\max}} = \frac{F_i}{y_i} \qquad (9.39)$$

in which y_{max} and y_i are respectively the maximum and the individual distance of the bolt from centre of rotation and F_T is the maximum bolt force.

The shear due to vertical load can be calculated directly to Equation (9.36).

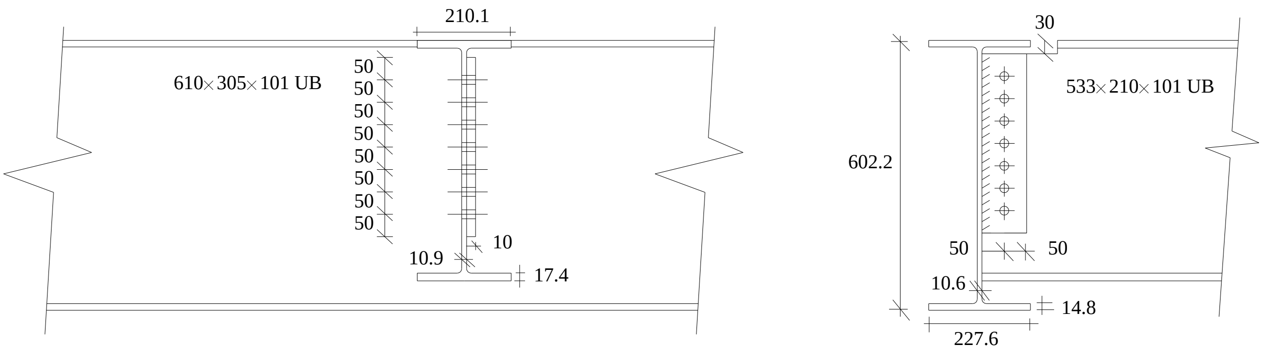
In addition to the separated requirements as $P_S \geq F_S$ and $P_T \geq F_T$, the two force components are required to satisfy the force interaction equation as,

$$\frac{F_S}{P_S} + \frac{F_T}{P_T} \leq 1.4 \qquad (9.40)$$

9.6 Worked Examples

9.6.1 Beam-to-beam connection by single fin plate

A secondary beam is connected to a main beam. The secondary beam is connected by a single fin plate which is welded to the main beam as shown. Only vertical load is transmitted from secondary beam to main beam through a beam-to-beam connection. The factored vertical load is 450kN. The beam-to-beam connection is designed as pinned connection, for which only shear is transferred. And the sections of main and secondary beams are shown in the figure. Steel grade of both main and secondary beams is S275. The bolts in the connection are M20 in grade 8.8 and fillet welds of connection are made of E35 electrode.



Solution

The beam-to-beam connection is assumed as a pinned connection and the bending effect in the bolted connection is neglected.

As a good practice in detailing, the size of fin plate should be at least half of the depth of beam to provide sufficient torsional restraint against twist and the fin plate should be placed near the top flange in order to provide lateral restraint to the top flange.

SHEAR CAPACITY OF BOLT

Shear force on each bolt, $F_s = \frac{F_v}{n} = \frac{450}{7} = 64.3kN$

From Table 9.4, the shear area of an M20 bolt, $A_s = 245mm^2$

Shear strength of bolt, $p_s = 375\,N/mm^2$ *(Table 9.5)*

Shear capacity of bolt, $P_s = p_s A_s = 245 \times 375 = 91.9kN > F_s$ *(OK)* *(9.15)*

BEARING CAPACITY OF BOLT

Thickness of thinner connected plate, $t_p = 10mm$

Nominal bolt diameter, $d = 20mm$

Bearing strength of bolt, $p_{bb} = 1000N / mm^2$ *(Table 9.6)*

Bearing capacity of bolt, $P_{bb} = dt_p p_{bb} = 20 \times 10 \times 1000 = 200kN > F_s$ *(OK)* *(9.16)*

BEARING CAPACITY OF CONNECTED PARTS

Bearing strength of connected parts, $p_{bs} = 460 N/mm^2$ *(Clause 9.3.6.1.3)*

End distance, $e = 50mm$

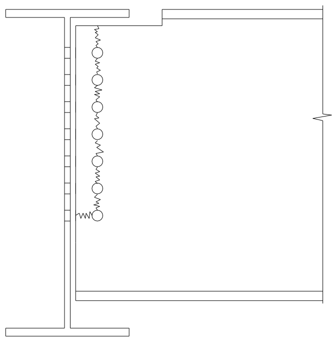
Hole coefficient for standard holes, $k_{bs} = 1.0$ *(Clause 9.3.6.1.3)*

Bearing capacity of connected parts,

$P_{bs} = k_{bs} d t_p p_{bs} = 1 \times 20 \times 10 \times 460 = 92kN > F_s$ *(OK)* *(9.17)*

$P_{bs} = 0.5 k_{bs} e t_p p_{bs} = 0.5 \times 1 \times 50 \times 10 \times 460 = 115kN > F_s$ *(OK)* *(9.18)*

BLOCK SHEAR CAPACITY



Standard bolt hole of M20, $D_t = 22mm$ *(Table 9.4)*

Length of shear force, $L_v = 50 \times 7 = 350mm$

Length of tension force, $L_t = 50mm$

Coefficient of row number of bolts, $k = 0.5$ *(Clause 9.3.5)*

Effective area coefficient, $K_e = 1.2$ *(Clause 9.3.4.4)*

Effective shear area, $A_{v,eff} = t[L_v + K_e(L_t - kD_t)]$ *(9.14)*

$$= 10 \times [350 + 1.2 \times (50 - 0.5 \times 22)] = 3968mm^2$$

Block shear capacity, $P_r = \frac{1}{\sqrt{3}} p_y A_{v,eff} = \frac{1}{\sqrt{3}} \times 275 \times 3968 = 630.0kN > F_v$ *(9.13)*

(OK)

SHEAR CAPACITY OF WELD

A leg length of 6mm is assumed

Design strength of weld, $p_w = 220 N/mm^2$ *(Table 9.2a)*

Effective length of weld, $L_w = (400 - 2 \times 6) \times 2 = 388 \times 2 = 776mm$ *(Clause 9.2.5.1.3)*

Second moment of area of weld, $I_w = \frac{388^3}{12} \times 2 = 9.735 \times 10^6 mm^3$

Shear force per unit width, $F_s = \frac{F_v}{L_w} = \frac{450 \times 10^3}{776} = 579.9 N/mm$

Tension force per unit width, $F_t = \frac{My_{max}}{I_w} = \frac{450 \times 10^3 \times 50 \times 388/2}{9.735 \times 10^6} = 448.4 N/mm$

Resultant force per unit width,

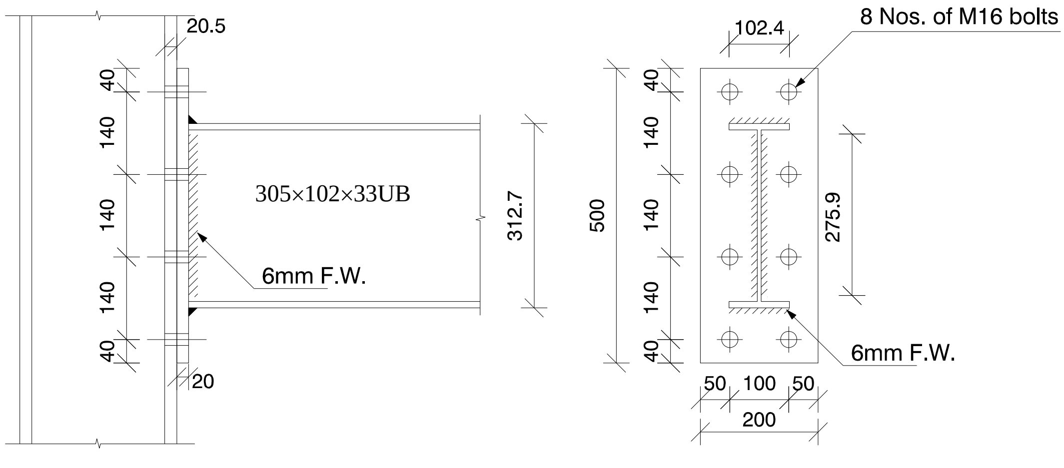
$F_R = \sqrt{F_s^2 + F_t^2} = \sqrt{579.9^2 + 448.4^2} = 733.0 N/mm$

Minimum leg length required, $s = \frac{F_s}{0.7 p_w} = \frac{733}{0.7 \times 220} = 4.8mm < 6mm$ *(Clause 9.2.5.1.6(a))*

$\therefore$ 6mm weld is adequate

9.6.2 Typical extended plate for beam to column connection

An extended end plate connection under both shear force and moment about major axis is shown in Figure below. The factored shear force is 200kN and moment is 35kNm. The configuration of beam column connection and the section of the members are also indicated in the figure. The beam member is welded to the end plate which is then bolted to the column. For welded connection, 6mm fillet weld is used in flange web as shown in the figure below. The end plate and members are in S275 steel material and the bolts are M16 in grade 8.8 and the electrode used for welding is E35.



Solution

The connection is under the action of shear and moment and the bolts and welds are under the action of both shear and tension. The prying action and bearing due to vertical shear on connected plate should be checked here.

CAPACITY OF FILLET WELD

Assume the centre of rotation at the centroid of the connection group and the connected plate is assumed to be stiffened.

Effective length of weld at flange, $b_e = 102.4 - 2\times 6 = 90.4mm$ *(Clause 9.2.5.1.3)*

Effective length of weld at web, $b_e = 275.9 - 2\times 6 = 263.9mm$

For unit length of weld,

Area of weld, $L_w = (90.4 + 275.9)\times 2 = 708.6mm$

Second moment of inertia, $I_w = \left[\dfrac{263.9^3}{12} + 90.4\times\left(\dfrac{312.7}{2}\right)^2\right]\times 2 = 7.483\times 10^6\,mm^3$

Average shear force on horizontal weld, $F_s = \dfrac{F_v}{L_w} = \dfrac{200\times 10^3}{708.6} = 282.2\,N/mm$

Maximum tension on horizontal weld, $F_t = \dfrac{M_x \cdot D/2}{I_w} = \dfrac{35\times 10^6 \times 312.7/2}{7.483\times 10^6} = 731.3\,N/mm$

Simplified method

Design strength of weld, $p_w = 220\,N/mm^2$ *(Table 9.2a)*

Resultant force on horizontal weld,

$F_R = \sqrt{F_s^2 + F_t^2} = \sqrt{282.2^2 + 731.3^2} = 783.9\,N/mm$

Minimum required leg length, $s = \dfrac{F_R}{0.7p_w} = \dfrac{783.9}{0.7 \times 220} = 5.1mm < 6mm$ *(OK)* *(Clause 9.2.5.1.6(a))*

$\therefore$ 6mm weld is ok

Directional Method

$P_L = p_w a = 220 \times 6 \times 0.7 = 924N/mm$ *(9.4)*

$\theta = \tan^{-1}\dfrac{731.3}{282.2} - 45° = 23.9°$

$K = 1.25\sqrt{\dfrac{1.5}{1+\cos^2\theta}} = 1.25\sqrt{\dfrac{1.5}{1+\cos^2 23.9°}} = 1.130$ *(9.6)*

$P_T = KP_L = 1.13 \times 0.92 = 1.04kN/mm > F_R$ *(OK)* *(9.5)*

$\therefore$ 6mm weld is ok

(It should be noted that the simplified method is more conservative than the directional method.)

SHEAR CAPACITY OF BOLT

Shear force on each bolt, $F_s = \dfrac{F_v}{n} = \dfrac{200}{8} = 25kN$

From Table 9.4, $A_s = 157mm^2$

Shear strength of bolt, $p_s = 375N/mm^2$ *(Table 9.5)*

Shear capacity of bolt, $P_s = p_s A_s = 375 \times 157 = 58.9kN > F_s$ *(OK)* *(9.15)*

BEARING CAPACITY OF BOLT

Thickness of thinner connected part, $t_p = 20mm$

Nominal diameter of bolt, $d = 16mm$

Bearing strength of bolt, $p_{bb} = 1000N/mm^2$ *(Table 9.6)*

Bearing capacity of bolt, $P_{bb} = dt_p p_{bb} = 16 \times 20 \times 1000 = 320kN > F_s$ *(OK)* *(9.16)*

BEARING CAPACITY OF CONNECTED PLATE

Hole coefficient, $k_{bs} = 1$ for standard hole *(Clause 9.3.6.1.3)*

Bearing strength, $p_{bb} = 460N/mm^2$ for grade S275

$P_{bs} = k_{bs} dt_p p_{bs} = 1 \times 16 \times 20 \times 460 = 147.2kN$ *(9.17)*

$P_{bs} = 0.5k_{bs} et_p p_{bs} = 0.5 \times 1 \times 40 \times 20 \times 460 = 184kN$ *(9.18)*

$\therefore P_{bs} = 147.2kN > F_s$ *(OK)*

TENSION CAPACITY OF BOLT

$$\sum y_i^2 = (140^2 + 280^2 + 420^2) \times 2 = 5.488 \times 10^5 \, mm^2$$

Maximum tension force on bolt,

$$F_t = \frac{M_x y_{\max}}{\sum y_i^2} = \frac{35 \times 10^6 \times 420}{5.488 \times 10^5} = 26.8kN$$

Tensile strength of bolt, $p_t = 560 N/mm^2$ *(Table 9.8)*

Tensile capacity of bolt, $P_t = p_t A_s = 560 \times 157 = 87.9kN > F_t$ *(OK)* *(9.25)*

PRYING FORCE ON BOLT

Width of connected plate, $B = 200mm$

Distance between centerline of bolts, $G = 100mm < 0.55B = 110mm$

Tension capacity of bolt, $P_{nom} = 0.8 A_t p_t = 0.8 \times 157 \times 560 = 70.3kN > F_t$ *(OK)* *(9.26)*

$\therefore$ design against prying force is not required *(Clause 9.3.7.2)*

INTERACTION BETWEEN TENSION AND SHEAR OF BOLT

$$\frac{F_s}{P_s} + \frac{F_t}{P_{nom}} = \frac{25}{58.9} + \frac{26.8}{70.3} = 0.81 \le 1.4 \text{ (OK)} \quad (9.28)$$

9.7 Base plate

Column base is commonly used to transfer shear, compression and moment from the superstructure to the foundation, as indicated in Figure 9.24. The base plate should be designed with sufficient strength and ductility and the supporting concrete foundation and base should be designed against concrete crushing and yielding in steel plate. Overturning leading to the base taking tension caused by uplift is also required to be considered in the design.

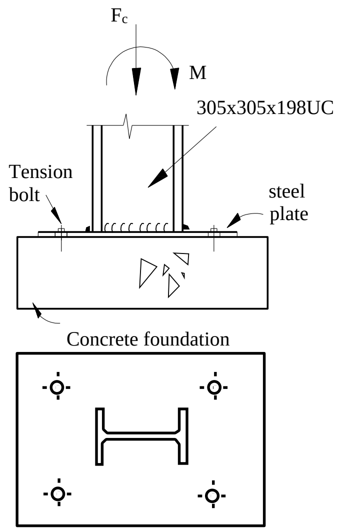


Figure 9.24 Typical column base with steel column, base plate and concrete block

The maximum bearing pressure of the concrete due to both compression and moment should be limited to $0.6f_{cu}$ in which f_{cu} is the 28 day concrete cube strength. The allowable bending stress on the steel base plate should not exceed the design strength p_y of steel plate. In the design, the stress distribution under the base plate is assumed to be linear in practical design.

9.7.1 Column base under concentric force

When the base plate is only under pure compression, the size of base plate should be sufficiently large to carry whole compression force. The effective area method is used for design under concentric axial forces. Figure 9.25 illustrates the effective area as shaded area in order to spread the compressive stress to the design stress. In the construction of effective area the dimension c is the largest perpendicular distance from the face of the column for finding the effective area under concentric force.

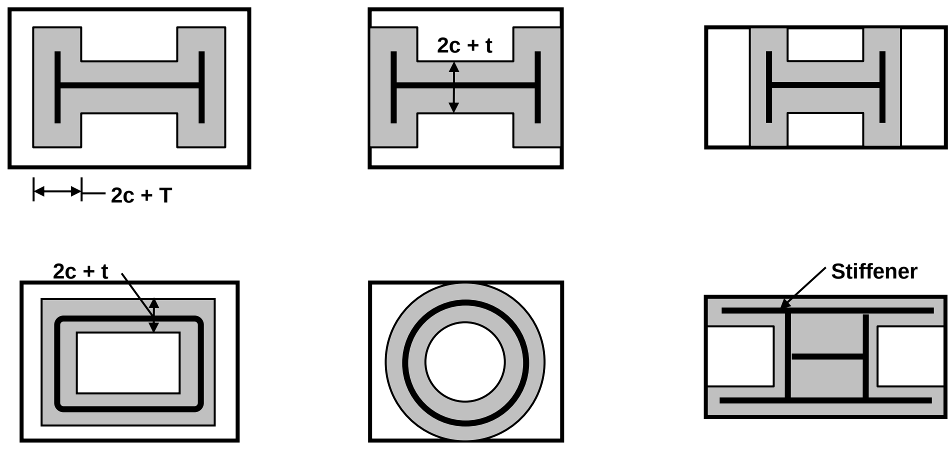


Figure 9.25 Effective areas of different typical base plates

c is determined from the effective area method, which sets equivalent area as shaded area in Figure 9.25 to be equal to F_c/ω for equilibrium under pure compression. Thus, the shaded area times 0.6 of concrete cube strength is set equal to the axial force and the parameter "c" is determined from this equivalent equation. The thickness t_p of the base plate can be obtained directly from being stress analysis as $M = \frac{\omega c^2}{2} = p_{yp} \frac{t_p^2}{6}$ as,

$$t_p = c\sqrt{\frac{3\omega}{p_{yp}}} \qquad (9.41)$$

in which ω is uniform compressive stress, which is assumed as uniform distribution over entire effective portion but limited to $0.6f_{cu}$ and p_{yp} is the design strength of steel base plate.

9.7.2 Column base under eccentric force

When a base plate is subjected to both compression and moment, the effective area method is inapplicable and the linear elastic design is used. The eccentricity e of loading, which is determined from Equation (9.43), should be used to classify whether the tension zone occurring on base plate or not. When e is greater than d/6, part of the area will be in tension and holding down bolts are required to prevent lifting of steel base plate from concrete foundation. The bearing pressure distribution should be determined by an alternative approximate method based on the different assumption and load intensity. There are four different available methods dealing with the design of base plate under eccentric loading.

The checking of the presence or absence of the tension zone under a base plate requires the determination of eccentricity. According to the simple combined stress formula, the condition for no-tension under rectangular base plate of length d and width b is given by,

$$\frac{F}{A}-\frac{M}{Z}=\frac{F}{bd}-\frac{6Fe}{bd^{2}}\geq 0 \tag{9.42}$$

$$e=\frac{M}{F}\leq\frac{d}{6} \tag{9.43}$$

When e is not larger than $d/6$, the base plate is under compression thoroughly and the design can be as follows.

9.7.2.1 Column base under small eccentricity with e ≤ d/6

This condition applies to the loading case where the base plate on concrete is under compression only but the pressure is not uniform.

The length of base plate is larger than 6*e*, where '*e*' is the eccentricity given by Equation (9.43).

According to the linear non-uniform stress distribution under small eccentric force, the maximum stress is given by,

$$\frac{F_c}{bd}+\frac{6F_ce}{bd^2}=0.6f_{cu} \tag{9.44}$$

and the minimum width of base plate b is given by,

$$b=\frac{F_c\left(\frac{1}{d}+\frac{6e}{d^2}\right)}{0.6f_{cu}} \tag{9.45}$$

The maximum and minimum pressure are then derived by the following equations,

$$p_{\max}=\frac{F_c}{bd}+\frac{6F_ce}{bd^2} \tag{9.46}$$

$$p_{\min}=\frac{F_c}{bd}-\frac{6F_ce}{bd^2} \tag{9.47}$$

The bending moment on a 1mm wide strip on the base plate is given by,

$$M=\frac{p_oa^2}{6}+\frac{p_{\max}a^2}{3} \tag{9.48}$$

The maximum moment M should not exceed $1.2p_{yp}Z_x$, where the design strength of base plate $p_{yp}\leq 275N/mm^2$. Compression will be assumed to transmit direct bearing provided that the bearing contact is tight. Welds or fasteners should be provided to transmit shear or tension due to the factored loads.

9.7.2.2 Column base under large eccentricity with e>d/6

When e is larger than $d/6$, the following method can be used.

The method assumes that the base plate has a linear strain distribution between concrete ε_c and steel bolts ε_s as shown in Figure 9.26(a). The modular ratio α_m equal to E_s/E_c is used in the design method of composite structure. The linear strain distribution is expressed as Equation (9.49). Layout of the column base is shown in Figure 9.26(b). In this case, the maximum design strengths in the concrete f_c and bolts f_t are assumed to occur simultaneously.

Linear strain distribution relationship,

$$\alpha_m = \frac{E_s}{E_c} = \frac{y}{d_e - y}\frac{f_t}{f_c} \tag{9.49}$$

$$y = \frac{\alpha_m f_c}{\alpha_m f_c + f_t} d_e \tag{9.50}$$

Moment equilibrium about the centre of the tension bolt, see Figure 9.25(c),

$$M' = M + F_c a \tag{9.51}$$

Compression force in concrete C is given by,

$$C = \frac{M'}{z} \tag{9.52}$$

in which z is the lever arm equal to the distance between the centroid of concrete stress block and the centre of the tension bolt as $z = d_e - y/3$

Stress in concrete is then determined as,

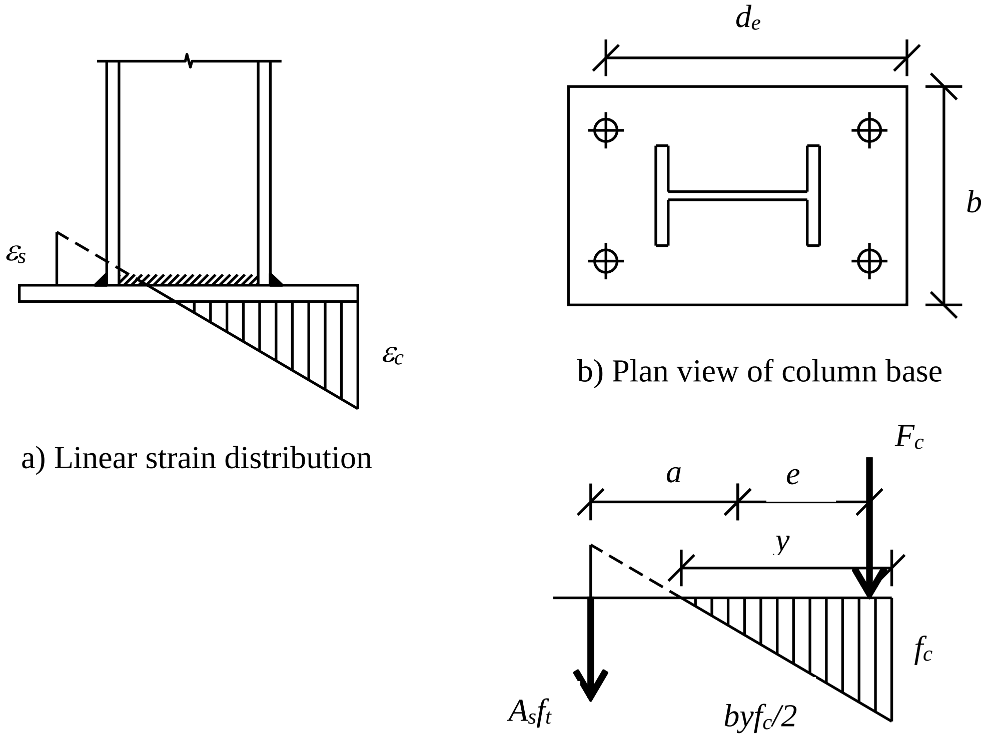
$$f_c = \frac{2F_c}{b \cdot y} \tag{9.53}$$

in which B is the breadth of the base plate and determined to set the concrete stress f_c not greater than $0.6f_{cu}$.

The total force in tension bolts, T, is determined from equilibrium of force as,

$$T = C - F_c \tag{9.54}$$

The capacity of tension bolts is required to be not less than the required tension force and the total area of tension bolts required is then equal to $A_s = T/p_t$.



a) Linear strain distribution

b) Plan view of column base

c) Actual stress distribution

Figure 9.26 Bearing pressure and layout of column base

9.8 Worked Examples

9.8.1 Base plate subjected to eccentric load

A column base plate is under moment of 55kNm and downward compressive force of 780kN. The column size is 254×254×89 UC. The base plate is made of Grade S275 steel. And the allowable compressive concrete stress f_{cu} is 30N/mm^2. Design the dimension of column base plate under eccentric load case.

<u>Solution</u>

Try 450×350×35

Design strength of base plate, $p_{yp} = 265N/mm^2$ for $16mm < T \leq 40mm$ *(Table 3.2)*

Eccentricity of load,

$$e = \frac{M}{P} = \frac{55\times10^6}{780\times10^3} = 70.5mm < \frac{L_p}{6} = 75mm$$

$\therefore$ whole base plate is under compression

Maximum and minimum bearing pressure,

$$p_{\max} = \frac{780\times10^3}{350\times450} + \frac{55\times10^6}{350\times450^2/6}$$

$$= 4.95 + 4.66 = 9.61N/mm^2 < 0.6 f_{cu} = 18N/mm^2 \text{ (OK)}$$ *(Clause 9.4.1)*

$$p_{\min} = 4.95 - 4.66 = 0.29N/mm^2$$

Edge distance from the column face,

$$a = \frac{450-260.3}{2} = 94.9mm$$

Bearing pressure at the column flange is given by,

$$P = 9.61 - \frac{9.61-0.29}{450}\times94.9 = 7.64N/mm^2$$

Maximum moment due to bearing pressure per unit width is given by,

$$M_{\max} = \frac{9.61\times94.9^2}{3} + \frac{7.64\times94.9^2}{6} = 40317Nmm$$

Plastic modulus per unit width is given by,

$$S = 1.2\times\frac{t_p^2}{6} = \frac{35^2}{5} = 245mm^2$$

Therefore, the moment capacity of the base plate per unit width is given by,

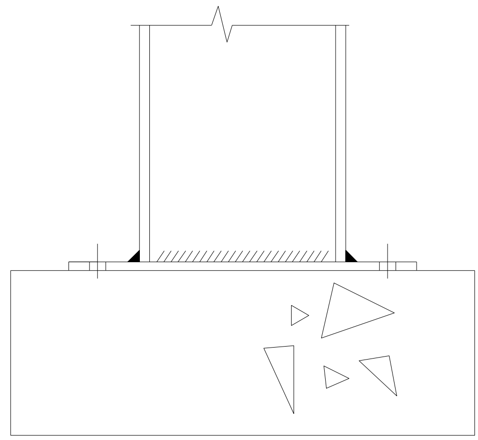
$$M_c = 265\times245 = 64925Nmm > M_{\max} \text{ (OK)}$$

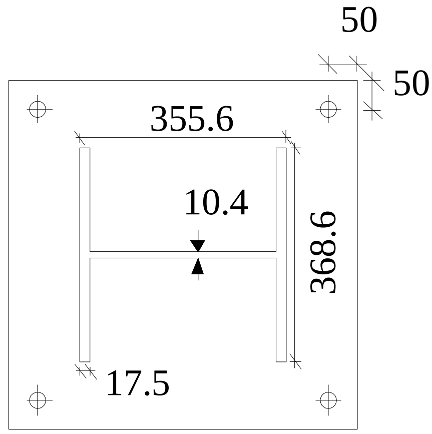
9.8.2 Column base subjected to different loading conditions

The column shown below is to transmit a factored axial compression and bending moment about its major axis to column base, which is then subjected to three critical loading cases. One is concentric load, and the others are respectively eccentric load with and without tensile bearing stress under base plate. The column base is then designed to be grouted to the ground by bolted connection as shown. The section of column is 356×368×129 UC in grade S275 steel and the base plate is also made of S275 steel. The compressive stress of base concrete is 20N/mm^2 for the first two cases and 40N/mm^2 for the last case. The bolt size is selected to be M24 in grade 8.8. The size of the base plate is 600×600. Design the thickness of the base plate for the column base when subjected to concentric loading case by considering compression only and eccentric loading case comprised of both compression and moment.

Case	Axial force (kN)	Bending moment (kN-m)	Concrete cube strength(N/mm^2)
1	1200	0	20
2	1200	100	20
3	1200	400	40

356x368x129UC





Solution

Case 1: Try 600×600×15

Design strength of base plate, $p_{yp} = 275N/mm^2$ for $T \le 16mm$ *(Table 3.2)*

Required area, $A_{req} = \dfrac{F_c}{\omega} = \dfrac{1200\times10^3}{12} = 10^5 mm^2$

Effective area, $A_{eff} = 2(2c+T)(2c+B)+(D-2T-2c)(t+2c)$

$= 2(2c+17.5)(2c+368.6)+(320.6-2c)(10.4+2c)$

$= 2(4c^2+772.2c+6450.5)+(3334.24+620.4-4c^2)$

$= 4c^2+2164.8c+16235.24$

Set effective area equal to required area

$10^5 = 4c^2+2164.8c+16235.24$

$4c^2+2164.8c-83764.76=0$

$\therefore c = 36.3mm$

Length of base plate, $L_p = D + 2c = 355.6 + 2 \times 36.3 = 428.2mm < 600mm$ *(OK)*

Width of base plate, $B_p = B + 2c = 368.6 + 2 \times 36.3 = 441.2 < 600mm$ *(OK)*

Plate thickness, $t_p = c\sqrt{\frac{3w}{p_{yp}}} = 36.3\sqrt{\frac{3 \times 0.6 \times 20}{275}} = 13.1mm < 15mm$ *(OK)* *(9.37)*

The base plate is subjected to concentric compression and the tension bolts here are nominal or to transmit shear only

Case 2: Try 600×600×35

Design strength of base plate, $p_{yp} = 265N/mm^2$ for $16mm < T \le 40mm$ *(Table 3.2)*

Eccentricity of load, $e = \frac{M}{F_c} = \frac{100 \times 10^6}{1200 \times 10^3} = 83.3mm < \frac{L_p}{6} = 100mm$

$\therefore$ whole base plate is under compression and the tension bolts are nominal or to transmit shear only

Maximum bearing pressure,

$$p_{\max} = \frac{1200 \times 10^3}{600 \times 600} + \frac{100 \times 10^6 \times 6}{600 \times 600^2}$$

$$= 3.33 + 2.78 = 6.11 N/mm^2 < 0.6 f_{cu} = 12 N/mm^2 \quad (OK)$$ *(Clause 9.4.1)*

Minimum bearing pressure,

$$p_{\min} = 3.33 - 2.78 = 0.55 N/mm^2$$

Edge distance from the column face,

$$a = \frac{600 - 355.6}{2} = 122.2mm$$

Bearing pressure at the column flange,

$$p = 6.11 - \frac{6.11 - 0.55}{600} \times 122.2 = 4.98 N/mm^2$$

Maximum moment due to bearing pressure per unit width,

$$M_{\max} = \frac{6.11 \times 122.2^2}{3} + \frac{4.98 \times 122.2^2}{6} = 42807Nmm$$

Moment capacity per unit width of base plate,

$$M_c = p_{yp} \times \frac{t_p^2}{5} = 265 \times \frac{35^2}{5} = 64925Nmm > M_{\max} \quad (OK)$$

Case 3: Try 600×600×55

Design strength of base plate, $p_{yp} = 255N/mm^2$ for $40mm < T \le 63mm$ *(Table 3.2)*

Eccentricity of load, $e = \frac{M}{F_c} = \frac{400 \times 10^6}{1200 \times 10^3} = 333.3mm > \frac{L_p}{6} = 100mm$

$\therefore$ part of the base is in tension and holding down bolts are required to resist tension caused by uplift

The eccentric effect due to bending is carried by both compression and tension, for which the compression is taken by bearing pressure of concrete and tension is resisted through holding down tension bolts. The bearing stress distribution assumes to be linear for the method for elastic behavior.

Modular ratio, $\alpha_m = 15$

Distance from the centerline of the bolts in tension to the edge of the base plate in compression,
$d_e = 600 - 50 = 550mm$

Depth of the neutral axis,
$$y = \left(\frac{\alpha_m f_c}{\alpha_m f_c + f_t} \right) d_e = \left(\frac{15 \times 24}{15 \times 24 + 560} \right) 550 = 215.2mm$$

Take moments about the centerline of the bolts in tension,
$M' = 400 + 1200 \times 0.25 = 700kNm$

Lever arm,
$$z = d_e - \frac{y}{3} = 550 - 215.2/3 = 478.3mm$$

Compressive force in the concrete
$$C = \frac{M}{z} = \frac{700 \times 10^3}{478.3} = 1463.5kN$$

Maximum bearing pressure in the concrete,
$$p_{\max} = \frac{2C}{b \cdot y} = \frac{2 \times 1463.5 \times 10^3}{600 \times 215.2} = 22.7N/mm^2 \leq 0.6 f_{cu} = 24N/mm^2 \quad (OK)$$ *(Clause 9.4.1)*

Tensile force for holding down bolts,
$T = C - F_c = 1463.5 - 1200 = 263.5N$

Use 2 nos. of M24 bolts, $A_s = 358mm^2$

Tension force per bolt, $F_t = \frac{263.5}{2} = 131.8kN$

Design tensile strength of bolt, $p_t = 560N/mm^2$ *(Table 9.8)*

Tension capacity of bolt, $P_t = A_s p_t = 358 \times 560 = 200.5kN > F_t$ *(OK)* *(9.25)*

Bearing pressure at the column flange,
$$p = 22.7 \times \frac{215.2 - 122.2}{215.2} = 9.81N/mm^2$$

Maximum moment due to bearing pressure per unit width,
$$M_{\max} = \frac{22.7 \times 122.2^2}{3} + \frac{9.81 \times 122.2^2}{6} = 137407Nmm$$

Maximum moment due to bolt tension per unit width,
$$M_{\max} = \frac{131.8 \times (122.2 - 50) \times 10^3}{(122.2 - 50)\tan 60° + 50} = 54360Nmm$$

Therefore, bending due to bearing pressure is more critical.

Moment capacity of base plate per unit width,
$$M_c = 255 \times \frac{55^2}{5} = 154275Nmm > M_{\max} \quad (OK)$$

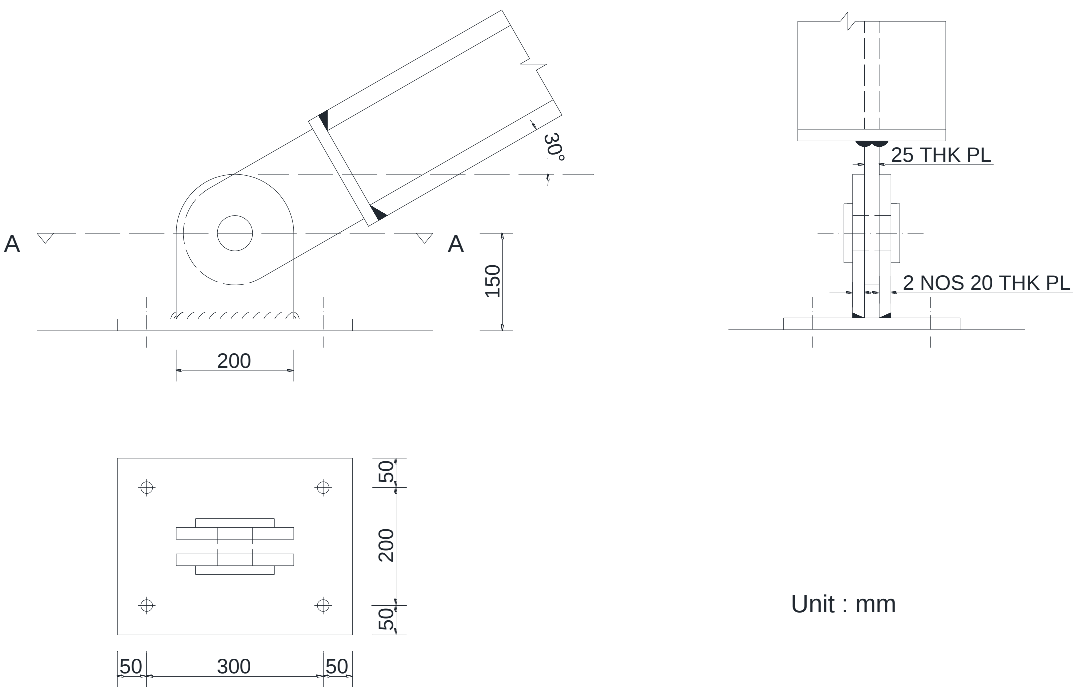
9.8.3 Connection at base of space frame

The pinned connection shown in figure below is under an axial compression force of 200 kN live load and 150 kN dead load in the H-column inclined 30° to the horizontal direction.

(a) Determine the size of the pin required.
(b) Check the adequacy of the base plate of width 300mm and depth 400mm
(c) Determine size of bolts required for the base plate and
(d) Determine the thickness of the base plate required.

Use Grade 8.8 for bolts, S355 for pin and pin plates and S275 for base plate. The concrete grade is C35.

The elastic modulus of pin is $Z = \dfrac{\pi D^3}{32}$.



Solution

DESIGN FORCE OF PIN

Factored compression force, $F_c = 1.4 \times 150 + 1.6 \times 200 = 530kN$ *(Table 4.2)*

(a) Determine the size of the pin

SHEAR CAPACITY OF PIN

Assume $40mm < D \leq 63mm$

$p_{yp} = 335 N/mm^2$ *(Table 3.2)*

The pin is in double shear,

$$F_s = \frac{530}{2} = 265kN$$

$$P_s = 0.6p_{yp}A \qquad \textit{(9.31)}$$

$$F_s = 0.6p_{yp}\frac{\pi D^2}{4}$$

$$D = \sqrt{\frac{4F_s}{0.6\pi p_{yp}}} = \sqrt{\frac{4\times 265\times 10^3}{0.6\times\pi\times 335}} = 41.0mm$$

$\therefore$ try 50 mm pin

BEARING CAPACITY OF PIN

$$p_b = 1.5p_y dt = 1.5\times 335\times 50\times 25 = 628.1kN > F_c \ \ (OK) \qquad \textit{(9.33)}$$

MOMENT CAPACITY OF PIN

$$M = \frac{530\times 0.045}{4} = 5.96kNm$$

$$M_c = 1.5p_{yp}Z = 1.5\times 335\times\frac{\pi\times 50^3}{32} = 6.17kNm > M \ \ (OK) \qquad \textit{(9.35)}$$

$\therefore$ 50 mm pin is adequate

(b) Check the adequacy of the base plate

Design force

$$F_V = 530\sin 30° = 265.0kN$$

$$F_H = 530\cos 30° = 459.0kN$$

$$M = 459\times 0.15 = 68.9kNm$$

$$e = \frac{M}{P} = \frac{68.9\times 10^3}{265} = 260.0mm > \frac{L}{6} = \frac{400}{6} = 66.7mm$$

$\therefore$ the holding-down bolts are in tension

Distance from the centerline of the bolts in tension to the edge of the base plate in compression,

$$d_e = 400 - 50 = 350mm$$

Depth to the neutral axis,

$$y = \frac{f_c\alpha_m}{f_c\alpha_m + f_t}d_e = \frac{21\times 15}{21\times 15 + 560}\times 350 = 126mm$$

Take moments about the centerline of the bolts in tension,

$$M' = 68.9 + 265\times 0.15 = 108.7kNm$$

Lever arm, $z = d_e - y/3 = 350 - 126/3 = 308mm$

Compression force in concrete,

$$C = \frac{M'}{z} = \frac{108.7\times 10^3}{308} = 352.9kN$$

Maximum bearing pressure in the concrete,

$$p_{max} = \frac{2C}{b\cdot y} = \frac{2\times 352.9\times 10^3}{300\times 126} = 18.7N/mm^2 \le 0.6f_{cu} = 21N/mm^2 \ \ (OK) \qquad \textit{(Clause 9.4.1)}$$

$\therefore$ 300 mm × 400 mm base plate is adequate.

(c) Determine the bolt size

Tensile force per each holding down bolt,

$$F_t = \frac{C - F_V}{2} = \frac{352.9 - 265}{2} = 44.0kN$$

Shear force per each holding down bolt,

$$F_s = \frac{459}{4} = 114.8kN$$

Try M24 bolts,

Design tensile strength of bolt, $p_t = 560 N/mm^2$ *(Table 9.8)*

Tension capacity of bolt, $P_t = A_s p_t = 353 \times 560 = 197.7kN > F_t$ *(OK)* *(9.26)*

Design shear strength of bolt, $p_s = 375 N/mm^2$ *(Table 9.5)*

Shear capacity of bolt, $P_s = A_s p_t = 353 \times 375 = 132.4kN > F_s$ *(OK)* *(9.15)*

Combined shear and tension,

$$\left(\frac{V}{V_c}\right)^2 + \left(\frac{F_t}{P_t}\right)^2 = \left(\frac{114.8}{132.4}\right)^2 + \left(\frac{44}{197.7}\right)^2 = 0.8 \le 1 \, (OK) \qquad (9.40)$$

$\therefore$ Use M24 bolt.

(d) Determine the thickness of the base plate

Edge distance from the pin plate,

$$a = \frac{400 - 200}{2} = 100mm$$

Bearing pressure at the edge of pin plate,

$$p = 18.7 \times \frac{126 - 100}{126} = 3.86 N/mm^2$$

Maximum bending due to bearing pressure per unit width,

$$M_{\max} = \frac{18.7 \times 100^2}{3} + \frac{3.86 \times 100^2}{6} = 68767Nmm$$

Maximum bending due to bolt tension per unit width,

$$M_{\max} = \frac{44 \times 10^3 \times 50}{50 \tan 60° + 50} = 16105Nmm$$

Therefore, bending due to bearing pressure is more critical.

Moment capacity of base plate per unit width,

$$M_c = p_{yp} \times \frac{t_p^2}{5}$$

Assume $16mm < t_p \le 40mm$,

Design strength, $p_{yp} = 265N/mm^2$ *(Table 3.2)*

$$t_p = \sqrt{\frac{5M}{p_{yp}}} = \sqrt{\frac{5 \times 68767}{265}} = 36.0mm$$

$\therefore$ use 40mm thk base plate

9.9 Bearing and buckling of webs

When a web in a beam is under a concentrated or point load, the web needs to be checked against crushing and buckling. The stiff bearing lengths of webs are indicated in Figure 9.27 below. The stiff bearing length is defined as the length which does not deform appreciably when under bending. Below is the recommended stiff bearing length.

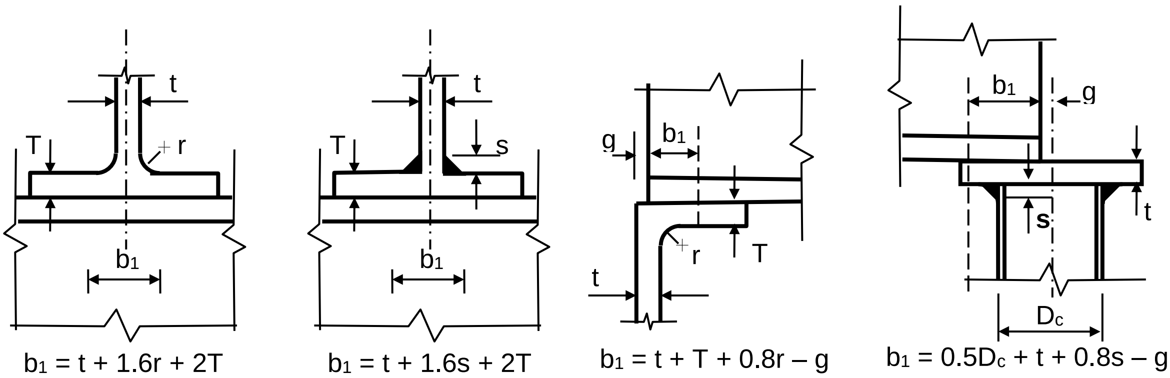


Figure 9.27 Stiff bearing length

9.9.1 Bearing capacity

The bearing capacity of web can be calculated as,

$$P_{bw} = (b_1 + nk)tp_{yw} \tag{9.55}$$

The value of n is given by:

at the ends of a member:

$$n = 2 + 0.6b_e / k \text{ but } n \le 5 \tag{9.56}$$

at other locations

$$n = 5 \tag{9.57}$$

For rolled I- or H-sections: $k = T + r$ (9.58)

For welded I- or H-sections: $k = T$ (9.59)

where

b_1 is the stiff bearing length, see Figure 9.27

b_e is the distance to the nearer end of the member from the end of the stiff bearing;

p_{yw} is the design strength of the web;

r is the root radius;

T is the flange thickness;

t is the web thickness.

9.9.2 Buckling resistance

The buckling resistance of web P_x should be greater than the external point load otherwise stiffeners should be added. P_x is given by the following.

When the flange through which the load or reaction is applied is effectively restrained against both:

a) rotation relative to the web;
b) lateral movement relative to the other flange;

then provided that the distance a_e from the load or reaction to the nearer end of the member is at least 0.7*d*, the buckling resistance of the unstiffened web should be taken as P_x below:

$$P_x = \frac{25\varepsilon t}{\sqrt{(b_1 + nk)d}} P_{bw} \tag{9.60}$$

where

d is the depth of the web;

P_{bw} is the bearing capacity of the unstiffened web at the web-to-flange connection from *Clause 8.4.10.5.1*.

When the distance a_e from the load or reaction to the nearer end of the member is less than $0.7d$, the buckling resistance P_x of the web should be taken as:

$$P_x = \frac{a_e + 0.7d}{1.4d} \frac{25\,\varepsilon\, t}{\sqrt{(b_1 + nk)d}} P_{bw} \tag{9.61}$$

When the condition a) or b) is not met, the buckling resistance of the web should be reduced to P_{xr} given by:

$$P_{xr} = \frac{0.7d}{L_E} P_x \tag{9.62}$$

in which L_E is the effective length of the web, acting as a compression member or a part of a compression member.

Chapter 10 Second-order Direct and Indirect Analysis

10.1 Introduction

The nonlinear analysis, in a more general term to second-order analysis, is a revolutionary approach to the design of not only steel structures, but also any other type of structures including steel-concrete composite, reinforced concrete and other structures including bamboo and pre-tensioned steel truss systems. The basic underlying principle is very different from the first-order linear analysis using the effective length. In the new method, the structure is designed by a simulation process, a truly performance-based approach that the safety is directly checked by the section capacity along the length of every member. The section capacity check approach is used for design of steel and concrete members via the elastic modulus with triangular stress blocks, the plastic modulus with rectangular stress blocks or other functions of modulus used with other stress block assumptions.

Unlike the conventional design method, the P-Δ and the P-δ effects are considered during a second-order direct analysis so there is no need to assume any effective length to account for the second-order effects. Despite its convenience, many structural engineers are still reluctant to switch to this new design method. One major reason is that it requires engineers to learn and get familiar with the new design method. Another major reason is the convenience of using this method is rarely demonstrated. The aim of this chapter is to compare the new design method with the conventional effective length method. Design examples are carried out which include 1) simple columns to demonstrate the analysis with second-order P-Δ and P-δ effects taken into account; 2) two-dimensional frames to illustrate the procedures of conventional design and design using second-order direct analysis; 3) three-dimensional large-scaled structures to demonstrate the advantages of design using second-order direct analysis over conventional analysis and 4) a very slender structure which second-order direct analysis must be used.

The second-order direct analysis method of design is a unified and an integrated design and analysis approach that the effect of fire or elevated temperature effects, seismic, effects of accidental member damage and progress collapse can all be modelled in the design process which integrates with the analysis process. However, this chapter is addressed to the conventional and widely exercised design against static loads. While the concept of the method is essentially the same for all applications under various scenarios, they may require different parameters which will be statutory in future. These parameters include member and frame imperfections under these conditions.

10.2 Background

There exists the P-Δ and P-δ effects in real structures which are due to the global displacement of the structure and the lateral displacement of the member respectively. The consequence of these secondary effects is additional stresses in the member are induced and thus the structure is weakened. A rational design should consider both the P-Δ and P-δ effects. The conventional limit state design method has been used extensively over the past decades. The philosophy of a limit state design can be expressed as follows.

$$\gamma_l F \leq \gamma R \tag{10.1}$$

in which γ_l is the load factor, F is the applied load, γ is the resistance factor and R is the resistance of the structure. Traditionally, F is obtained from the first-order linear analysis in which both geometrical and material nonlinearities are not taken into account while R is calculated based on the specifications so that the second-order P-Δ and P-δ effects and material yielding are considered. Although the analysis procedure is speeded up by the recent rapid development of personal computers, there are still some unavoidable hand calculation processes during the design stage such as calculating the effective length of a compressive column and the amplifications factors for the linear moments. The reliability of the conventional design method depends very much on the accuracy of the assumptions of effective length factors.

In recent years, design method using second-order direct analysis has been developed in which the second-order effects are considered directly during the analysis. There are two major types of second-order analysis, namely second-order elastic analysis and second-order inelastic analysis. The first type does not consider the effect of material yielding therefore section capacity check per member is required to locate the load causing the first plastic moment or first yield moment of the structure. The second type considers the effect of material yielding so the maximum failure load can be directly located by the load deflection plot. The section capacity check is therefore used for assessing the condition of plastic hinge formation. A second-order direct analysis not only facilitates structural design but it also plays a very important role on structural stability problems.

To date, both conventional design method and second-order direct analysis design method are allowed in many national design codes such as LRFD (2016), Eurocode 3 (2005), Code of Practice for Structural Uses of Steel and AS4100(1998). However, despite the convenience of the latter approach, the majority of structural engineers are reluctant to step forward to this state-of-the-art approach. One major reason is most software is programmed for P-Δ-only analysis and extensive manual checking effort is still required. Another major reason is its convenience is rarely illustrated through practical design examples.

Despite this reluctance, the second-order inelastic analysis, or the advanced analysis, will be the major trend in structural design in the future while the second-order elastic analysis can be regarded as a transition from the conventional method to the advanced analysis method. This chapter has two main objectives. The first one is to deliver the idea of how a design can be performed without any effective length. The second objective is to compare the new design method with the conventional method. Design examples are carried out in the hope that through these design examples, engineers will find the merits of design using second-order direct analysis without using effective length and will switch to it.

10.3 Methods of analysis

In the HK Code, both the first-order linear and second-order non-linear analysis methods can be used. However, the effects of change of deformed geometry shall be considered in the design with the elastic critical load factor λ_{cr} not less than 5 otherwise the second-order direct analysis must be used.

Load factor λ in Figure 10.1 represents a scalar multiplied to the set of design load in a particular combined load case. To understand the method, one must first appreciate the behaviour of a structure under an increasing load. Various methods provide an answer of the collapse load under its assumptions, such as plastic collapse load which does not consider any buckling effect and P-Δ-only second-order indirect analysis does not consider member imperfection and member buckling.

The results of these methods are compared with the true collapse or ultimate load of a structure, λ_u in the Figure 10.1 below.

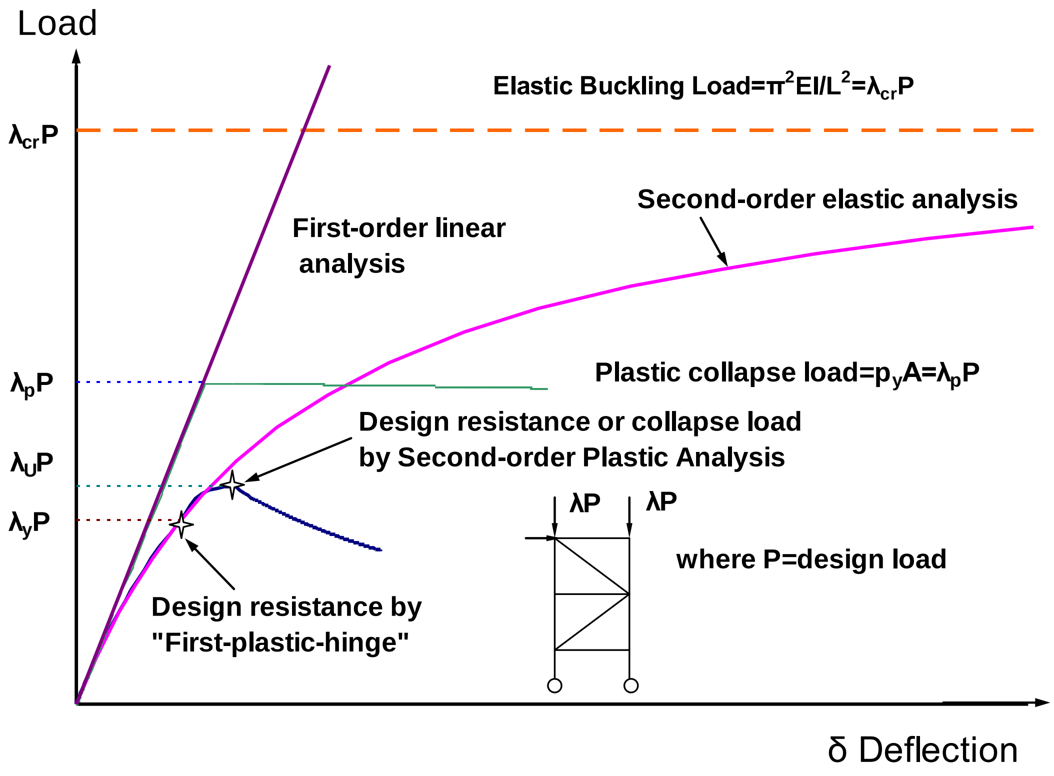


Figure 10.1 Design methods

Some common terms in first-order analysis and second-order analysis are explained as follows.

Elastic critical load factor $\boldsymbol{\lambda_{cr}}$ *is a factor multiplied to the design load to cause the structure to buckle elastically. The deflection before buckling, large deflection and material yielding effects are not considered here and the factor is an upper bound solution that cannot be used directly for design.* λ_{cr} *can be used to measure the instability stage of a frame against sway and buckling. See Equation 10.2 for more explanation.*

Plastic collapse load factor $\boldsymbol{\lambda_p}$ *is a load factor multiplied to the design load to cause the structure to collapse plastically but buckling and second-order effects are not considered. Because of the ignorance of buckling effects,* λ_p *cannot be used for direct design and it is an upper bound solution to the true collapse load of the structure. This load factor was widely used in the past for plastic design because of its simplicity to determine.*

P-delta effects *refer to the second-order effects. There are two types, being P-*Δ *and P-*δ *as shown in Figure 10.2.*

P-*$\boldsymbol{\Delta}$ *effect *is second-order effect due to change of geometry of the structure*

P-*$\boldsymbol{\delta}$ *effect *is second-order effect due to member curvature and change of member stiffness under load. A member under tension is stiffer than under compression.*

Second-order direct analysis for design *is a better design method than the effective length method. The method determines the P-*Δ *effect and the P-*δ *effect with initial imperfections so that effective length need not be assumed.*

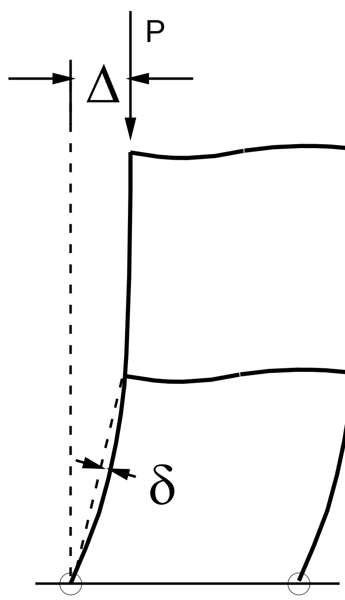


Figure 10.2 The P-Δ and P-δ effects

Linear analysis or first-order linear analysis *is an analysis assuming the deflection and stress are proportional to load. It does not consider buckling nor material yielding and the effects of change of geometry are not considered here.*

Notional horizontal force *is a small force applied horizontally to a structure to simulate lack of verticality and imperfection as illustrated in Figure 10.3. It can also be used to measure the lateral stiffness so that the elastic critical factor can be determined. It is sometimes taken as an alternative to imperfection in considering the imperfect geometry, which may include eqivalent residual stress that geometrical imperfection is enlarged.*

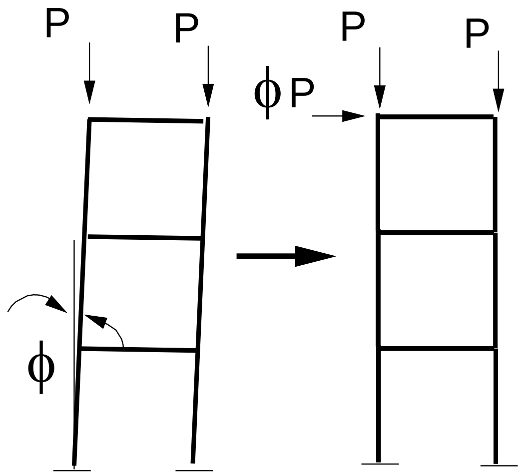


Figure 10.3 Simulation of out-of-plumbness by the notional force

Second-order P-Δ-only <u>indirect</u> analysis for plotting bending moment *is an indirect analysis used to plot the bending moment and force diagrams based on the <u>deformed nodal coordinates</u>. It does not consider member curvature nor the P-δ effect. This method is commonly used in software because of its simplicity in nodel coordinate than member curvature updates. In fact, most software can only do this P-Δ-only analysis which is not qualified for a full second-order analysis accounting for P-Δ and P-δ effects with imperfections at frame and member levels and this is a term first used in HK Steel Code 2005 version and LRFD code 2010 specifying this type of analysis in which computer programs only update the nodel coordinates but not member curvature in their analyses.*

Second-order P-Δ-δ <u>direct</u> analysis with section capacity check *is an analysis which allows for P-Δ effect and the P-δ effect and stops at first plastic hinge. It need not assume an effective length for the buckling strength check, but imperfection must be allowed for.*

The physical meaning of λ_{cr}, named as elastic critical load factor, can be illustrated by the buckling load of a simply supported column as shown in Figure 10.4 of Young's modulus *E*, second-moment of area *I* and length *L*.

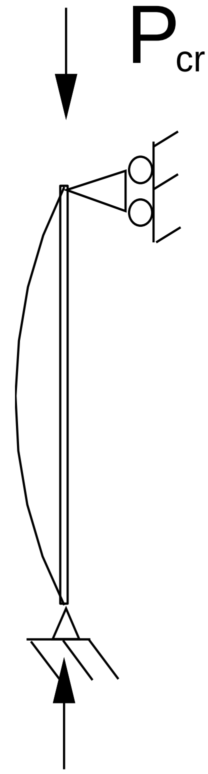


Figure 10.4 Buckling of a pin-pin column

The Euler buckling load is

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{10.2}$$

If the calculated buckling load from Equation (10.2) is 100kN and the factored design load from self-weight, live, wind and dead load is 20 kN, λ_{cr} is then equal to 100/20=5. It should be note that λ_{cr} is not for direct design since it does not consider imperfection and material yielding effects. λ_{cr} is only an indicator of stability stage, for calculating effective length factor (L_E/L) or used for amplification as discussed in Chapter 8.

When using NIDA*, one only needs to use the function of Eigen-Buckling Analysis and select the number of mode as 1 or more but only the first buckling mode is used in NIDA. For higher accuracy, we can just select all members and divide them to 2 elements since NIDA uses cubic element to find the buckling load factor. This division is not needed for second-order analysis in NIDA which use curved element to cater for the P-δ effect and imperfections.

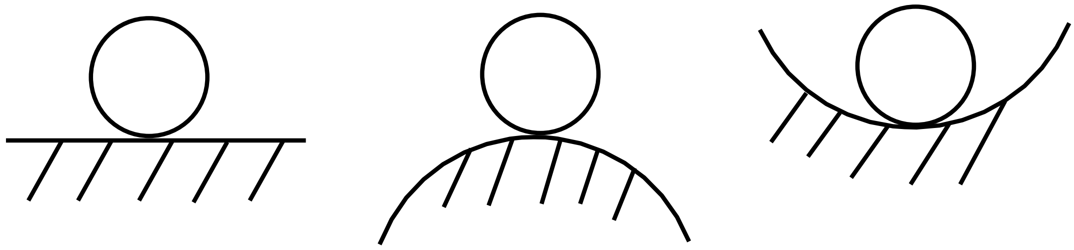
* NIDA is a software approved by the Buildings Department for second-order direct analysis to Code of Practice to Structural Uses of Steel, Hong Kong.

10.3.1 Types of stability

10.3.1.1 Finite element analysis for structural instability

The **principle of minimum potential energy** can be used to solve a buckling problem.

The vanishing of the **first** variation of the total potential energy functional implies the satisfaction of the equilibrium condition. The vanishing of the **second** variation of the energy functional means the structural system is in the state of neutral equilibrium. Figure 10.5 illustrates the concept of three different types of equilibrium.



Neutral equilibrium Unstable equilibrium Stable equilibrium

Figure 10.5 Concept of different types of equilibrium

$$\delta \Pi = 0 \quad \text{for equilibrium} \tag{10.3}$$

$$\delta^2 \Pi \begin{cases} > 0 & \text{for stable equilibirum} \\ = 0 & \text{for neutal equilibirum} \\ < 0 & \text{for unstable equilibirum} \end{cases} \tag{10.4}$$

It should be noted that, after minimisation, the solution cannot be obtained directly. Instead, a set of equations governing the instability condition will be otained

To derive the Euler buckling load of a column, the energy functional of the column can be written as (Tension +ve),

$$\Pi = \frac{1}{2}\int_0^L \left[EI\left(\frac{d^2v}{dx^2}\right)^2 + P\left(\frac{dv}{dx}\right)^2 \right] dx \tag{10.5}$$

In the case of a simply supported column, the assumption of a half sine curve as in Equation (10.6) will satisfy the deflected shape of the column.

$$v = \delta \sin\left(\frac{\pi x}{L}\right) \tag{10.6}$$

Thus, Equation (10.6) is put into Equation (10.5) and after differentiation and integration, the exact value for the Euler buckling load is obtained as in previous case.

That is,

$$\Pi = \frac{EI\delta^2\pi^4}{4L^3} + \frac{P\delta^2\pi^2}{4L} \tag{10.7}$$

$$\delta^2\Pi = \frac{EI\pi^2}{L^2} + P = 0 \tag{10.8}$$

Therefore,

$$P = -\frac{\pi^2 EI}{L^2} \quad \text{(Compression)} \tag{10.9}$$

The steps to develop a finite element for buckling analysis are as follows:

1. Write down the energy functional for the particular type of member. For example, for a general beam-column element as shown in Figure 6, the energy terms corresponding to bending are expressed as,

$$\Pi = \frac{1}{2}\int_0^L \left[EI\left(\frac{d^2v}{dx^2}\right)^2 + P\left(\frac{dv}{dx}\right)^2 \right] dx + M_1\theta_1 + M_2\theta_2 + F_1v_1 + F_2v_2 \tag{10.10}$$

in which Π is the total potential energy, F_1 and F_2 are the conjugate forces, M_1 and M_2 are nodal moments at ends with θ_1 and θ_2 are their conjugate rotations, P is the axial force, v is the lateral deflection with v_1 and v_2 as lateral deflections at ends and v_o as the initial curvature, e is the shortening, δ_0 is the initial imperfection and L and L_0 in figure below are deformed and original undeformed length.

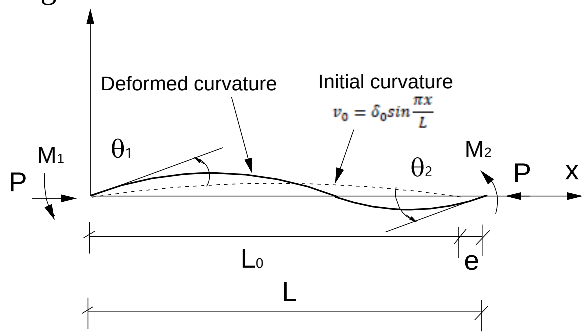


Figure 10.6 Beam-column element

2. Depending on the nodal degree of freedom for an element, write down a polynomial for the deflection of the element. If there are 4 degrees of freedom, a cubic polynomial which has also 4 coefficients is used so that the coefficients can be solved. Thus,

$$v = a_0 + a_1x + a_2x^2 + a_3x^3 \tag{10.11}$$

For $x = 0$, $v = v_1$, $\frac{dv}{dx} = \theta_1$ (10.12)

Therefore,

$$a_0 = v_1 \tag{10.13}$$

$$a_1 = \theta_1 \tag{10.14}$$

For $x = L$, $v = v_2$, $\dfrac{dv}{dx} = \theta_2$ (10.15)

Therefore,

$$a_2 = \frac{3(-v_1 + v_2)}{L^2} - \frac{2\theta_1 + \theta_2}{L} \tag{10.16}$$

$$a_3 = \frac{2(v_1 - v_2)}{L^3} + \frac{\theta_1 + \theta_2}{L^2} \tag{10.17}$$

And, after solving,

$$v = \left[\left(1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}\right) \quad \left(x - \frac{2x^2}{L} + \frac{x^3}{L^2}\right) \quad \left(\frac{3x^2}{L^2} - \frac{2x^3}{L^3}\right) \quad \left(-\frac{x^2}{L} + \frac{x^3}{L^2}\right) \right] \begin{bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix} \tag{10.18}$$

3. Substituting the displacement function, v, in terms of the nodal degree of freedom into the energy functional, we obtain the functional in terms of the nodal degree of freedom. After differentiating the functional with respect to the degree of freedom two by two, we obtain the stiffness matrix as follows,

$$[k_L + k_G] = EI \begin{bmatrix} \frac{12}{L^3} & \frac{6}{L^2} & -\frac{12}{L^3} & \frac{6}{L^2} \\ \frac{6}{L^2} & \frac{4}{L} & -\frac{6}{L^2} & \frac{2}{L} \\ -\frac{12}{L^3} & -\frac{6}{L^2} & \frac{12}{L^3} & -\frac{6}{L^2} \\ \frac{6}{L^2} & \frac{2}{L} & -\frac{6}{L^2} & \frac{4}{L} \end{bmatrix} + P \begin{bmatrix} \frac{6L}{5} & \frac{1}{10} & -\frac{6L}{5} & \frac{1}{10} \\ \frac{1}{10} & \frac{2L}{15} & -\frac{1}{10} & -\frac{L}{30} \\ -\frac{6L}{5} & -\frac{1}{10} & \frac{6L}{5} & -\frac{1}{10} \\ \frac{1}{10} & -\frac{L}{30} & -\frac{1}{10} & \frac{2L}{15} \end{bmatrix} \tag{10.19}$$

Note that the coefficients are given by,

$$k_{ij} = k_{ji} = \frac{\partial^2 \Pi}{\partial x_i \partial x_j} \tag{10.20}$$

in which x_i and x_j are the nodal degrees of freedom.

4. The condition for the structural system to become unstable is the vanishing of the determinant of the matrix. That is,

$$\left| k_L + \lambda_{cr} k_G \right| = 0 \tag{10.21}$$

To this, NIDA has been developed to calculate the value of the load factor, λ_{cr}, as shown in Figure 10.7, for the semi-indefinite condition of the eigenvalue.

Because the cubic Hermite function for lateral deflection represents the exact **linear** solution for the bending for a beam, i.e.

$$EI\frac{\mathrm{d}^2 v}{\mathrm{d}x^2} = M_1\left(1-\frac{x}{L}\right) + M_2\frac{x}{L} \tag{10.22}$$

some researchers do not consider the beam-column element as a finite element which implies that the exact expression for the deflection cannot be obtained but approximated by a series of approximate functions such as cubic polynomial. Moreover, in the present studies of buckling problems, it can be easily seen that the **nonlinear** solution, which is the half sine curve for a simply supported strut, is far from cubic and therefore the use of several elements per member is needed to obtain an accurate solution.

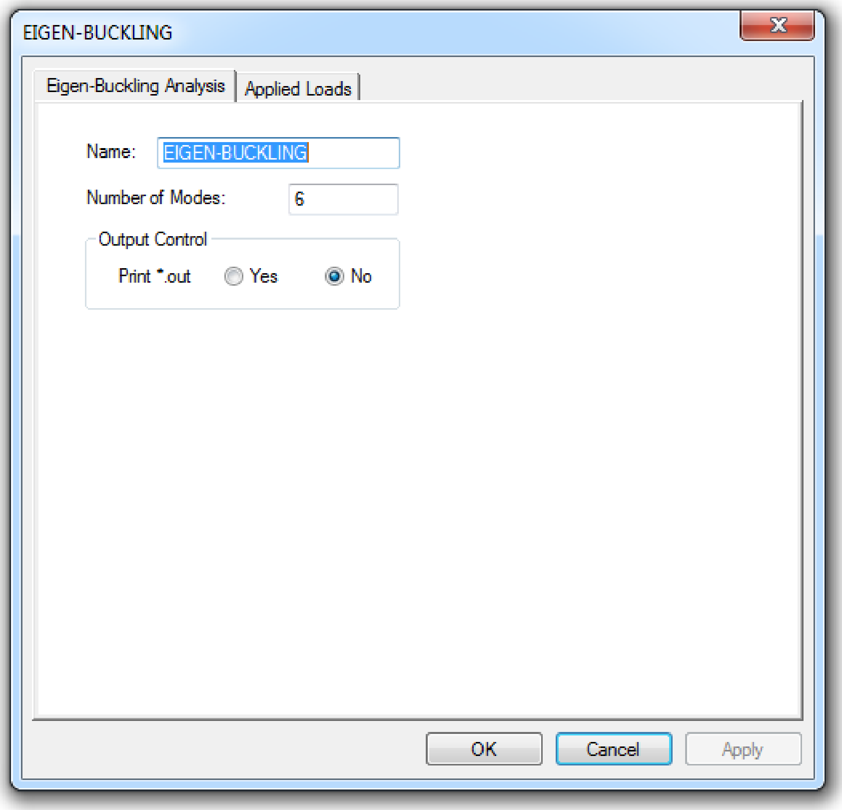


Figure 10.7 The option of buckling and vibration in NIDA

Alternatively, λ_{cr} can be obtained by hand calculation by Equation (10.23), provided that the structure is **regular** portal frame of shallow roof or building frame. For a multi-storey building frame, the formula is applied for each storey and the minimum λ_{cr} is taken as the controlling elastic critical load factor.

$$\lambda_{cr} = \frac{F_N}{F_V}\frac{h}{\delta_N} \tag{10.23}$$

where F_V is the factored dead plus live loads on the floor considered

F_N is the notional horizontal force taken typically as 0.5% of F_V for building frames

h is the storey height and

δ_N is the notional horizontal deflection of the upper storey relative to the lower storey due to the notional horizontal force F_N

In design codes including the HK Code, LRFD (2016) and Eurocode-3(2005), there are in general 3 methods adopted for design, which are listed in the following.

10.3.1.2 First-order linear analysis

The method is a conventional method using effective length in *Chapter 6* of the HK Code. It assumes a linear relationship between force and displacement. It cannot check buckling or material yielding and therefore the output force and moment must be checked to ensure the member is safe. However, the stress due to second-order effects and buckling and stress distribution after yielding are not considered here.

In the first-order linear analysis, the effects of imperfection on member design (i.e. the P-δ effect) shall be incorporated by using appropriate buckling formulae. Curves a_0, a, b, c and d represent different values of member imperfections and *Table 8.7* of the HK Code classifies various types of sections into one of these a_0-d curves.

It can be observed that the linear analysis currently used by most engineers in Hong Kong has already considered imperfection indirectly via uses of curves a_0 to d. **Software claiming to have the ability to do the second order direct analysis without codified way of considering imperfection is therefore unacceptable** and the design could be dangerous**.**

The P-Δ sway effect is considered by multiplying the moment from linear analysis by the amplification factor $\frac{\lambda_{cr}}{\lambda_{cr}-1}$. However, the P-δ effect still needs to be considered by assuming the effective length equal to the member length for checking.

This linear analysis method cannot be used when the structural is irregular or λ_{cr} is less than 5.

10.3.1.3 Second-order indirect analysis (Second-order P-Δ-only elastic analysis)

This analysis method considers the changes in nodal coordinate and sway such that the P-Δ effect is accounted for. The effect of member bowing (P-δ) is not considered here and should be allowed for separately. Member resistance check for P-δ effect to *Clause 8.7* of the HK Code is required and this P-Δ-only method of analysis and design is under the same limitations of use as the linear analysis.

10.3.1.4 Second-order direct analysis, or simply "Direct Analysis" (Second-order P-Δ-δ elastic analysis)

In this method, both the P-Δ and P-δ and imperfections effects are accounted for in the computation of bending moment. Checking the buckling resistance of a structure to *Clause 6.8.3* is sufficient and member check to *Clause 8.9.2* is not needed. The direct analysis here allows an accurate determination of structural response under loads via the inclusion of the effects of geometric imperfections and stiffness changes directly in the structural analysis and *Equations (6.12) to (6.14)* of the HK Code for section capacity check in the structural analysis are sufficient for structural resistance design. As only this method among the three methods in major international codes considers P-Δ-δ effects and qualified as Direct Analysis, this method is commonly referred as such and in LRFD (2016).

This method considers both the P-Δ and P-δ effects such that effective length method for member buckling strength check is not required. This implies significant saving in time as well as improvement in safety.

When the full second-order or P-Δ-δ analysis is used, we use the appropriate imperfections in *Table 6.1* in the HK Code. In this method, one need not consider individual stability check nor effective length at all. Cross section capacity check in Equation (10.24) below is sufficient in checking the stability strength of members as,

$$\frac{P}{p_y A} + \frac{(M_y + P\Delta_y + P\delta_y)}{M_{cy}} + \frac{(M_z + P\Delta_z + P\delta_z)}{M_{cz}} = \phi \le 1 \qquad (10.24)$$

where

Δ	=	nodal displacement due to out-of-plumbness frame imperfections plus sway induced by loads in the frame
δ	=	displacement due to member curvature / bowing due to initial imperfection plus load at ends and along member length of a member. This is calculated using a curved member proposed by Chan and Zhou (1995)
A	=	cross sectional area
p_y	=	design strength
M_{cy}, M_{cz}	=	yield moments about principal *y*- and *z*-axes (i.e. $M_c=p_y Z$); plastic moments can be used by replacing Z by plastic modulus, S
M_y, M_z	=	external moments about principal *y*- and *z*-axes

ϕ	=	section capacity factor. If ϕ>1, member fails in section capacity check. In software NIDA, different values of ϕ are indicated by different colours.

For slender sections, the effective area and moduli should be used in Equation (10.24). For some members influenced by the beam lateral-torsional buckling, the beam buckling moment M_b should be used in place of M_{cz} in Equation (10.24) (see Section 6.5).

Values of global initial imperfection Δ_0 are taken as 0.5% of height or span. Values of member initial imperfection should be taken from *Table 6.1* of the HK Code reproduced below.

Buckling curves referenced in Table 8.7	$\frac{e_0}{L}$ to be used in Second-order P-Δ-δ elastic analysis
a_0	1/550
a	1/500
b	1/400
c	1/300
d	1/200

Table 10.1 Values of member initial bow imperfection used in design

The curve selection should follow Table 8.7 of the HK Code reproduced below.

Type of section	Maximum thickness (see note1)	Axis of buckling	
		x-x	y-y
Hot-finished structural hollow sections with steel grade > S460 or hot-finished seamless structural hollow sections		a_0)	a_0)
Hot-finished structural hollow section < grade S460		a)	a)
Cold-formed structural hollow section of longitudinal seam weld or spiral weld		c)	c)
Rolled I-section	≤ 40 mm > 40 mm	a) b)	b) c)
Rolled H-section	≤ 40 mm > 40 mm	b) c)	c) d)
Welded I- or H-section (see note 2)	≤ 40 mm > 40 mm	b) b)	c) d)
Rolled I-section with welded flange cover plates with 0.25 < U/B < 0.80 as shown in Figure 8.4)	≤ 40 mm > 40 mm	a) b)	b) c)
Rolled H-section with welded flange cover plates with 0.25 < U/B < 0.80 as shown in Figure 8.4)	≤ 40 mm > 40 mm	b) c)	c) d)
Rolled I or H-section with welded flange cover plates with U/B ≥ 0.80 as shown in Figure 8.4)	≤ 40 mm > 40 mm	b) c)	a) b)
Rolled I or H-section with welded flange cover plates with U/B ≤ 0.25 as shown in Figure 8.4)	≤ 40 mm > 40 mm	b) b)	c) d)
Welded box section (see note 3)	≤ 40 mm > 40 mm	b) c)	b) c)
Round, square or flat bar	≤ 40 mm > 40 mm	b) c)	b) c)
Rolled angle, channel or T-section Two rolled sections laced, battened or back-to-back Compound rolled sections		Any axis: c)	

NOTE:

1. For thickness between 40mm and 50mm the value of p_c may be taken as the average of the values for thicknesses up to 40mm and over 40mm for the relevant value of p_y.
2. For welded I or H-sections with their flanges thermally cut by machine without subsequent edge grinding or machining, for buckling about the y-y axis, strut curve b) may be used for flanges up to 40mm thick and strut curve c) for flanges over 40mm thick.
3. The category "welded box section" includes any box section fabricated from plates or rolled sections, provided that all of the longitudinal welds are near the corners of the cross-section. Box sections with longitudinal stiffeners are NOT included in this category.
4. Use of buckling curves based on other recognized design codes allowing for variation between load and material factors and calibrated against Tables 8.8(a_0), (a) to (h) is acceptable. See also footnote under Table 8.8.

Table 10.2 Designation of buckling curves for different section types

For first plastic hinge design, the design capacity is considered to have been reached when ϕ of any member reaches 1. The design capacity is taken as the load causing the formation of the first plastic hinge for members with plastic (Class 1) or compact (Class 2) sections or first yield for member of semi-compact or slender section which further required reduction of cross-sectional area and moduli. If the sections are not class 1 or 2, their ductility can be obtained by a nonlinear finite element plastic analysis and used in a second-order direct analysis. Figure 10.8 shows the option in NIDA of second-order P-Δ-δ elastic analysis.

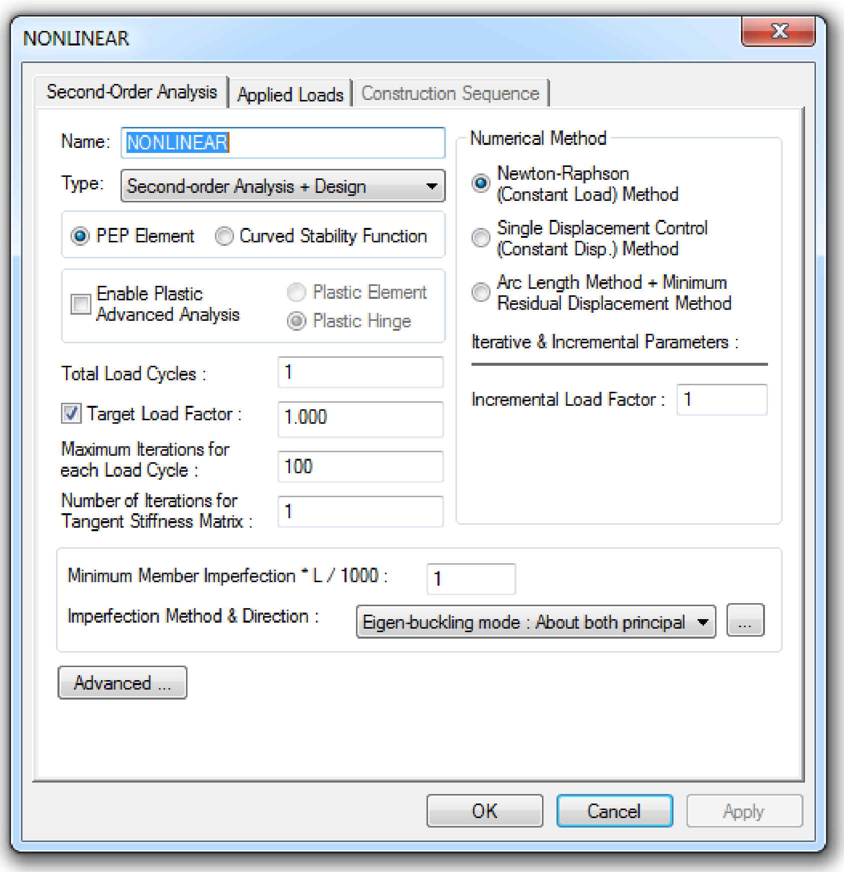


Figure 10.8 The option of second-order P-Δ-δ elastic analysis (note the deactivated “Enable Plastic Advanced Analysis” icon)

10.3.1.5 Second-order direct analysis allowing for beam buckling

For beams, especially open section beams, under moment about major principal axis, it will have a tendency to buckle laterally as shown in Figure 10.9 below. Traditioanlly, a reduction in moment resistance is applied for this type of unrestrained beams or, alternatvely, an increase in $M_x\phi$ moment similar conceptually to P-δ can be applied for beam bucking effect. However, for practicality of simple design, the HK Code does not adopt a $M_x\phi$ analysis for beam buckling check due to simplicity and necesscity as beam buckling check can be done by simple equations and programmed in computer without need of sway or no-sway frame classifications. Nevertheless, this nonlinear beam buckling check could be done easily to date for specific projects using the finite element method.

The reason why normally a P-delta type of analysis for frames with beam buckling is not considered is that, unlike column buckling, the effective length for beam buckling does not rely on the sway sensitivity of the frame and therefore one need not worry too much about the accuracy of beam effective length which can be directly determined from the boundary conditions.

In HK Code, the following equation is used for second-order direct analysis allowing for second-order direct analysis with beam buckling check.

$$\frac{F_c}{A_g p_y} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} = \frac{F_c}{A_g p_y} + \frac{m_{LT}[\overline{M}_x + F_c(\Delta_x + \delta_x)]}{M_b} + \frac{m_y[\overline{M}_y + F_c(\Delta_y + \delta_y)]}{M_{cy}} \leq 1 \tag{10.25}$$

in which M_b is the beam buckling moment determined from *Equations (8.20) to (8.24)* of the HK Code as an additional checking equation (see Trahair and Chan, 2005).

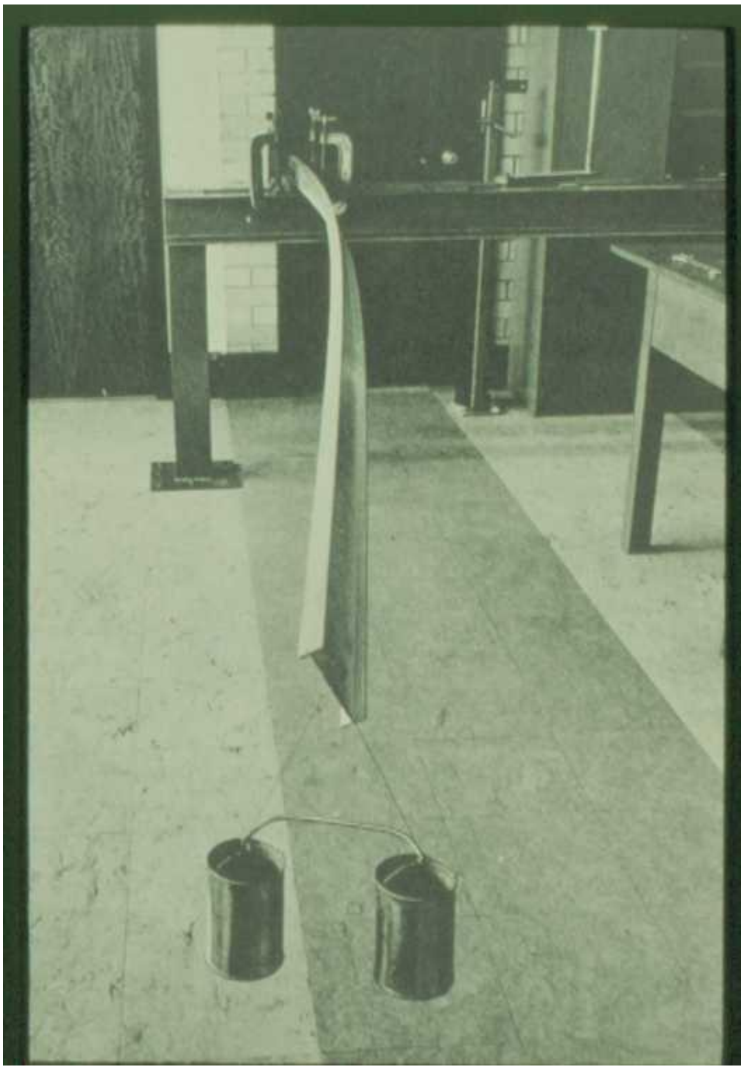


Figure 10.9 Lateral-torsional buckling of beam
(Courtesy to Professor N.S. Trahair of Sydney University)

10.3.1.6 Advanced analysis, Direct Plastic Analysis

For advanced analysis or second-order direct plastic analysis, one or two members yield with $\phi = 1$ do not necessarily indicate structural failure if the structure does not collapse. In Eurocode 3 (2005), plastic analysis can only be used the members are of sufficient rotational capacity to enable redistribution of bending moment. Under *Section 5.6(2)*, this requirement is assumed when plastic (Class 1) section is used and the shear is not larger than 10% of the shear resistance otherwise web stiffeners should be added within a distance $h/2$ from the plastic hinge location where h is the depth of the cross section. As this method was first coded in AS4100, the name advanced analysis from this code is retained here.

Plastic strength reserve of steel material is significant as minimum elongation at breakage of 15% is imposed for qualified steel. Elastic design can be considered as an over-conservative in some cases, especially for highly redundant structures.

According to the limit state design, the ultimate design load of a structure should be smaller than the actual load resistance or computed collapse load of the structure which can allow for plastic yielding in some members. A safe and yet economical design should allow no yielding under working load in order to prevent accumulation of strain energy and no collapse at ultimate load using the ultimate load factors.

For collapse load analysis, a plastic hinge will then be inserted into the member end when Equation (10.24) is satisfied and the analysis continues until a plastic collapse mechanism is formed (see Figure 10.1). The members possessing the plastic hinge must have sufficient rotational capacity which can be insured by plastic (Class 1) and doubly symmetric cross section and all members in the whole frame must be compact (Class 2) or plastic (Class 1). The location behind plastic hinges must be adequately restrained against lateral buckling after formation of plastic hinges. Figure 10.10 shows the option in NIDA of Second-order P-Δ-δ plastic analysis using "plastic hinge" method.

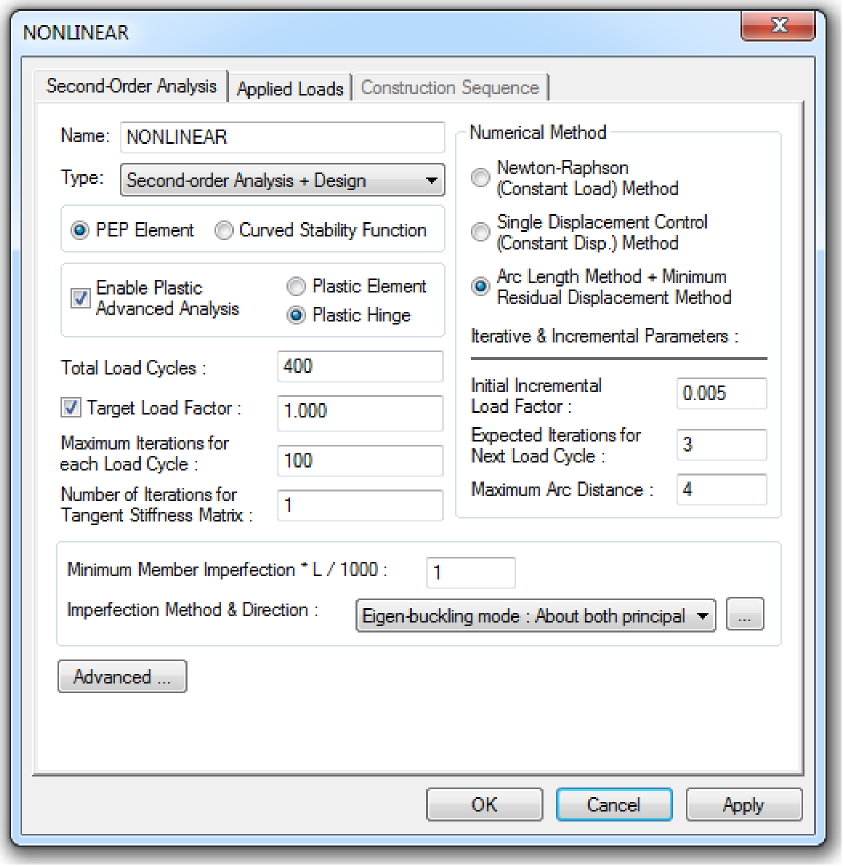


Figure 10.10 Direct plastic analysis using "plastic hinge" method
(note the activated "Enable Plastic Advanced analysis" icon)

10.3.2 Formulation for Nonlinear Numerical Methods

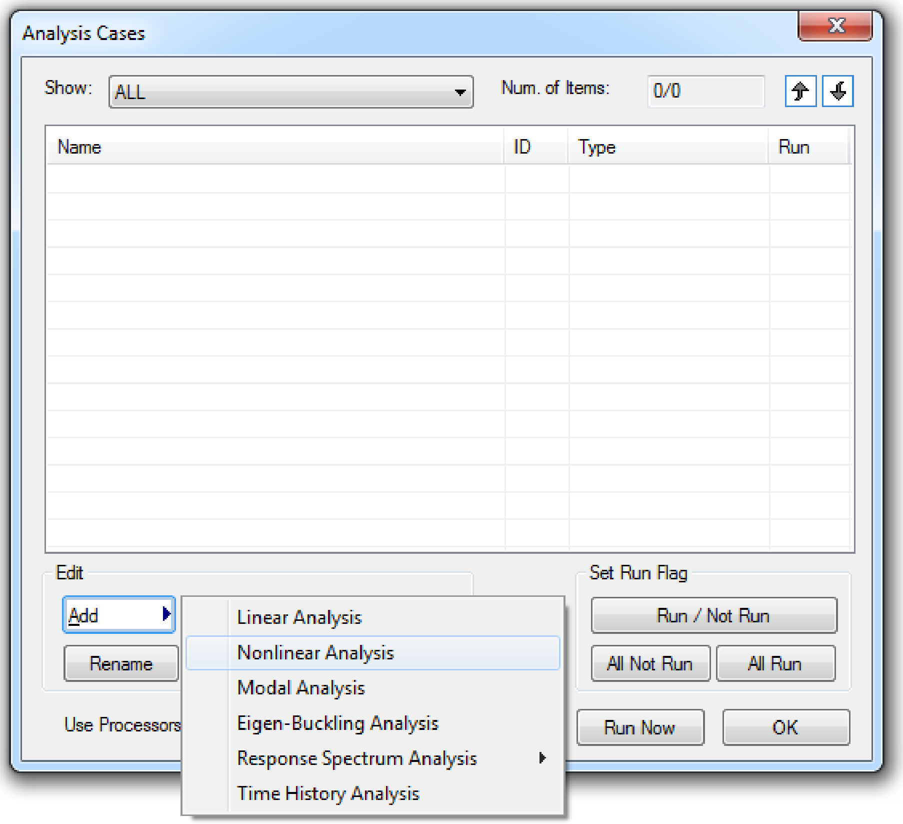
Every nonlinear numerical method has its own merits and limitations. None of them is remarkably superior to others in all cases. Their selection for a particular problem depends heavily on the type and constraint of the problem and the objective of study. For example, to determine the displacement of a structure under specified applied loads as required in most practical design, a load-control scheme should be chosen. If a prescribed displacement is imposed, a displacement-control scheme should be adopted. However, these two methods may not achieve convergence in tracing the snap-through curve or the snap-back curve. To select an appropriate nonlinear numerical method, the user should therefore have a general understanding on the characteristics of these methods. The properties and formulations of some commonly used schemes are briefly described in this section. In Section 10.3.4, a comparison among the schemes is made.

In general, the incremental-iterative equilibrium equation of a system, which is not necessarily controlled by the load, can be written as,

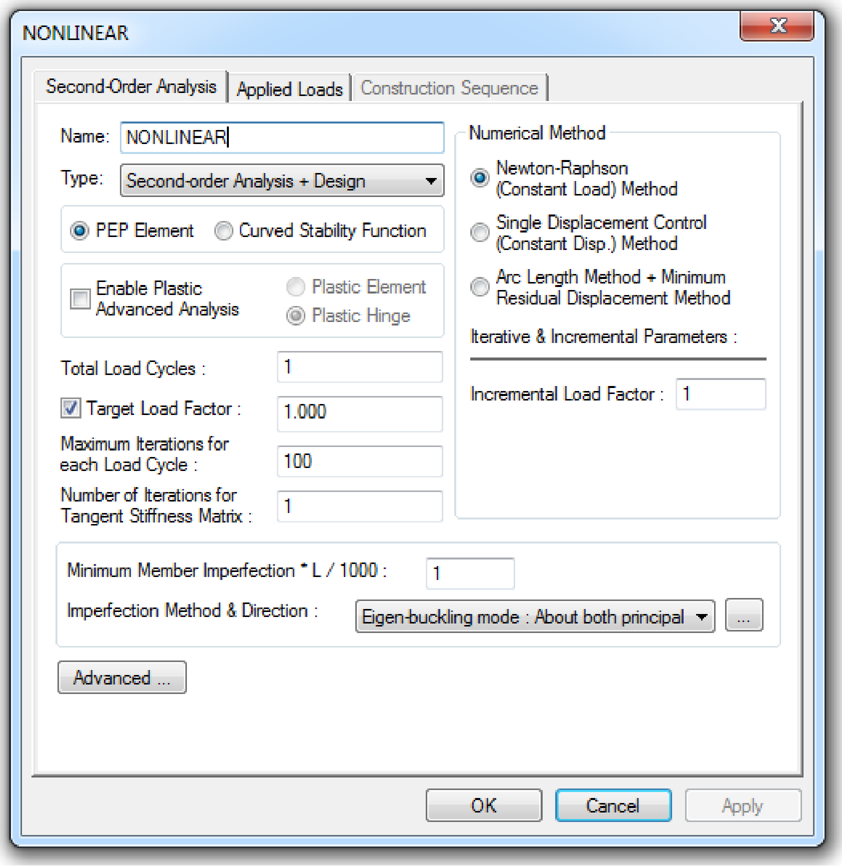
$$\{\Delta F\} + \Delta\lambda_i^k \{\Delta \overline{F}\} = [K]_T \left(\{\Delta u\} + \Delta\lambda_i^k \{\Delta \overline{u}\}\right) \tag{10.26}$$

in which $\{\Delta F\}$ and $\{\Delta u\}$ are respectively the out-of-balance forces and the corresponding displacement increments in the system; $\{\Delta \overline{F}\}$ and $\{\Delta \overline{u}\}$ are respectively the reference load vector and the resulting displacements; and $\Delta\lambda_i^k$ is a control parameter to be determined according to various imposed constrained conditions. The superscript ***k*** refers to the number of load cycle while the subscript ***i*** represents the number of equilibrium iteration within a load cycle. By selecting a suitable numerical scheme for a particular problem considered, the above incremental-iterative equation can used to trace the nonlinear load-deformation curve of the structure. If the selected numerical scheme is successful, the load limit or load-carrying capacity of the structure can be determined from the curve. Furthermore, the structural response for the post-buckling range can also be obtained.

In software NIDA, to use the nonlinear numerical methods, a nonlinear analysis case must be set up first by clicking the <Analysis> <Set Analysis Cases …> in top tool bar and the following template is popped,



To use various numerical methods, click <Add>, followed by <Nonlinear Analysis> and the following template is popped up.



The choices of the numerical methods include Newton-Raphson method, single displacement control method and arc length method + minimum residual displacement method. To select one of the numerical methods, go to the <Numerical Method> selection.

10.3.2.1 The Pure Incremental Method

The pure incremental method for nonlinear analysis is simple and is the earliest nonlinear solution method. Its basic procedure is to divide the total load into a number of small load increments. In each load step, the stiffness of a structure is determined first from the last known structure geometry and the loading state. It is then used to predict the next displacement increment. The sign of the determinant of the updated stiffness matrix will govern the direction of subsequent load step. The linearized displacement increment is calculated by solving the tangent stiffness matrix and the load increment. Once the displacement increment is obtained, the coordinates of structure are updated and then the process is repeated until the desired load level is reached.

In general, this approach is capable of handling both the snap-through and the snap-back problems because it does not require any iteration and thus does not have divergence problem. However, as no equilibrium check or iteration is carried out, unavoidable drift-off error is accumulated in each increment and the error after a number of load steps may make the solution greatly deviated from the true equilibrium path. This drift-off error cannot be estimated and thus the accuracy of the resulting load-deflection curve cannot be assessed. The method to minimize this error is to employ a smaller load step of which the magnitude is, unfortunately, quite difficult to assess. Indeed, there is no guideline suggested for each load step. More importantly, the pure incremental method usually over-estimates the ultimate capacity or the limit load of a structure. This is unsafe and undesirable in practical design. Nevertheless, this simple method is still widely used for nonlinear analysis, especially in commercial packages for nonlinear analyses.

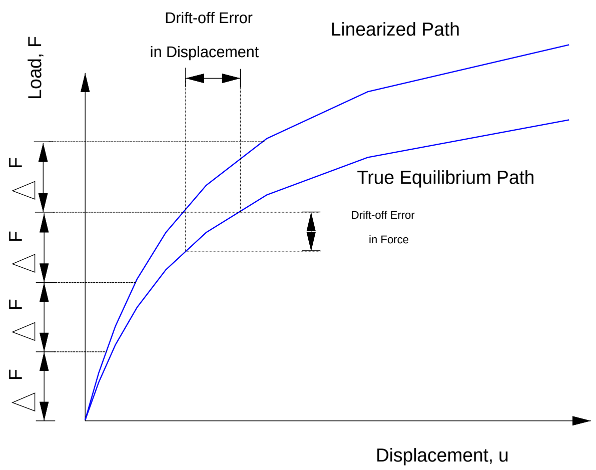
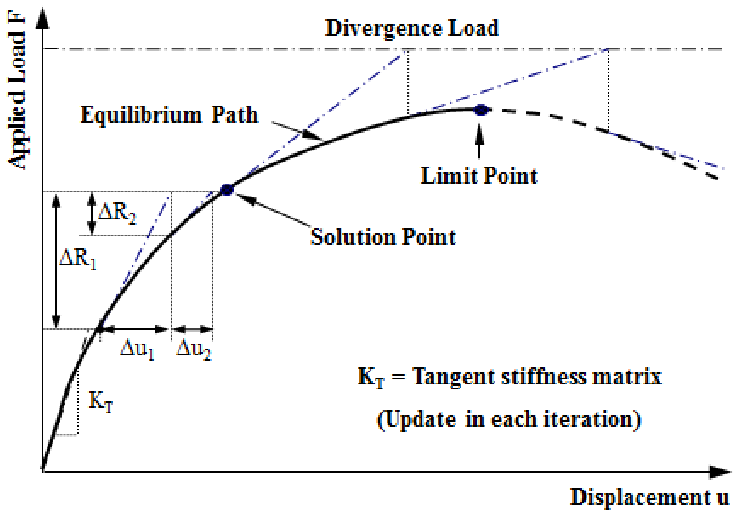


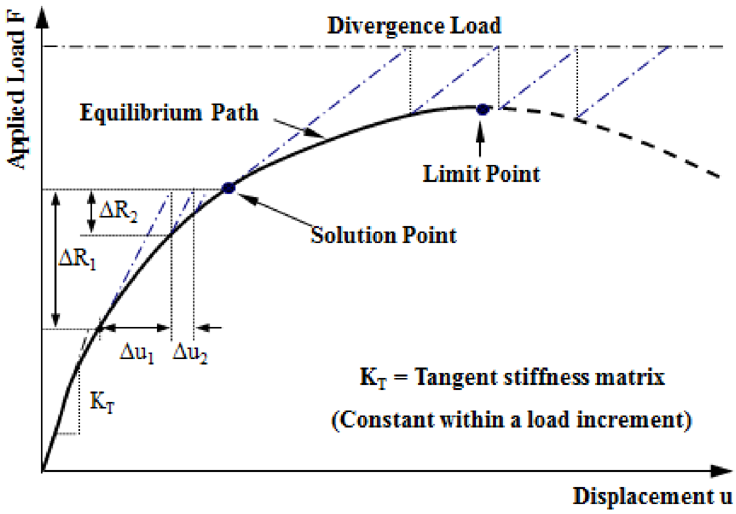
Figure 10.11 Pure incremental method with constant load increment

10.3.2.2 The Newton-Raphson Method

Only the Newton-Raphson method gives the response of a structure at the input load in terms of buckling strength and therefore it should be use when the engineers want to check whether or not a structure is adequate when under a set of factored design loads. In t method, iteration is activated to obtain the equilibrium condition between the applied forces and the internal structural resistance within a load step. Unlike the pure incremental method in which no equilibrium check is performed, the unbalanced force is dissipated via the iterative procedure and can therefore be eliminated by this method. Being free from the drift-off error, the solution is more accurate but the computational time is increased when compared with the pure incremental method.



Conventional Newton-Raphson Method



Modified Newton-Raphson Method

Figure 10.12 The Newton Raphson method

10.3.2.3 The Displacement Control Method

Unlike the load control methods previously described, a constraint equation for displacement is imposed in this approach. This method simultanesouly possesses the capacity of traversing the limit point without destroying the symmetrical property of the tangent stiffness method. A single degree of freedom is chosen to be the steering displacement degree of freedom for control of the advance of the solution for equilibrium path, and the magnitude for each increment must be decided.

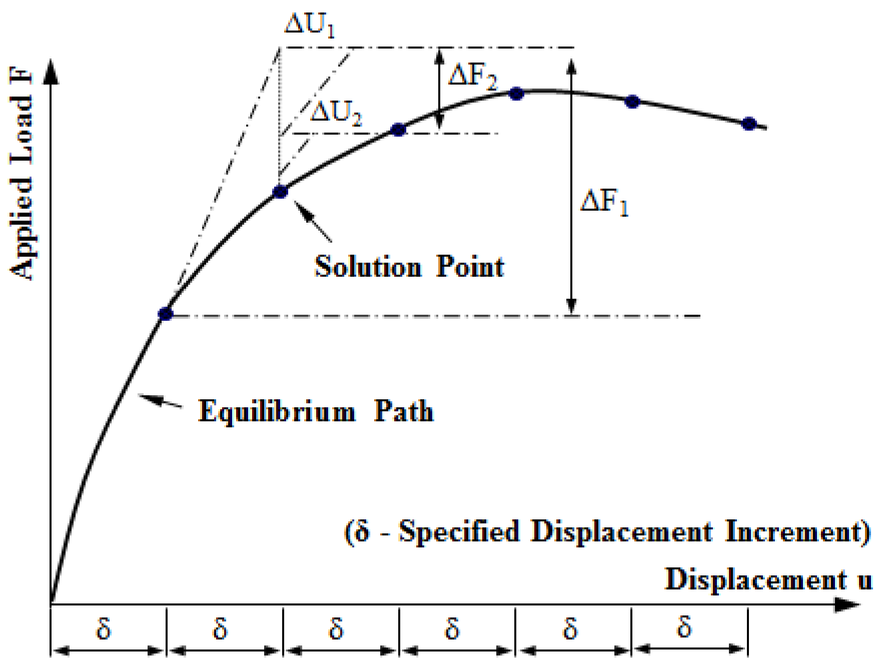


Figure 10.13 The displacement control method

The constant displacement method does not exhibit any difficulty in passing the snap-through limit point but fails to converge in snap-back problems. Thus, it is usually used in conjunction with other solution schemes in order to solve general nonlinear problems.

10.3.2.4 The Arc-Length Method

The basic concept of the spherical arc-length method is to constrain the load increment so that the dot product of displacement along the iteration path remains constant in the 2-dimensional plane of load versus deformation.

The procedure of the spherical arc-length method is illustrated below. Owing to its accuracy, reliability and satisfactory rate of convergence, it is probably the most popular method for nonlinear analysis and it was noted to be robust and stable for pre- and post-buckling analysis.

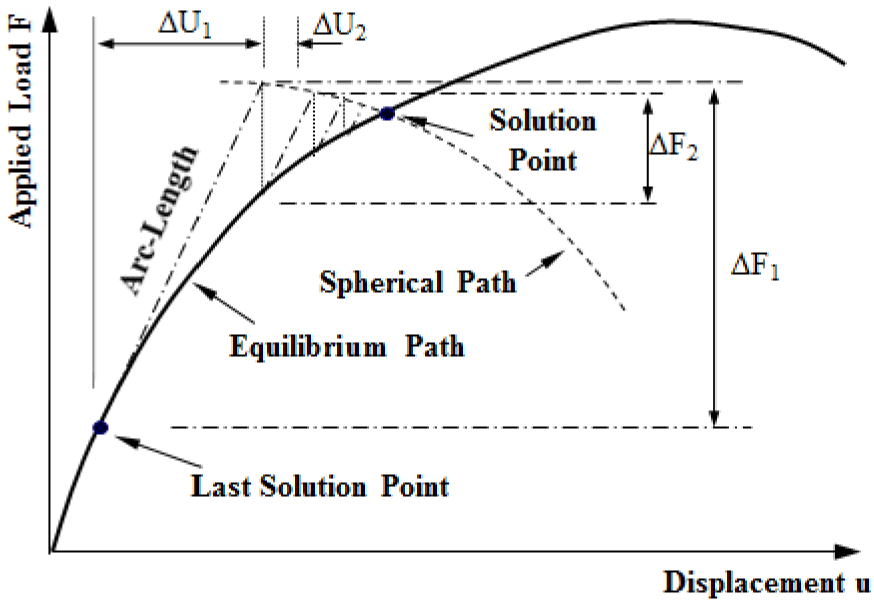


Figure 10.14 The arc-length method

10.3.2.5 The Minimum Residual Displacement Method

The basic idea of this method originally proposed by Chan (1988) is to minimize the norm of residual displacement in each iteration.

The graphical representation of the procedure is demonstrated below. From Figure 10.15, it can be seen that this constraint condition enforces the iteration path to follow a path normal to the load-deformation curve. It adopts the shortest path to arrive at the solution path by error minimization and thus is considered to be an optimum solution. In addition, the procedure is much simpler to use than the arc-length method. Generally speaking, owing to its efficiency and effectiveness in tracing the equilibrium path, the minimum residual displacement method is usually chosen to perform the iterative procedure and combined with the part for load size determination in the first iteration by the arc-length method.

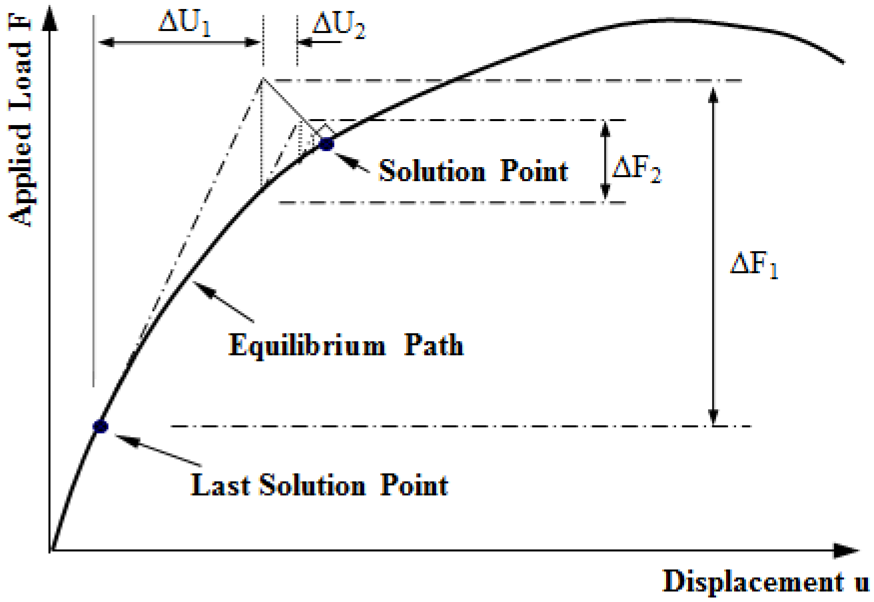


Figure 10.15 The minimum residual displacement method

10.3.3 Convergence criteria

In an effective incremental-iterative method, some criteria should be pre-determined for termination or continuation of iterations. If a tight tolerance is selected, excessive computation effort is spent on unnecessary accuracy. If the tolerance is set too loose, the equilibrium error may be excessive and inaccurate solutions resulted. Further to this, the question of whether the equilibrium tolerance should be set on the unbalanced forces or displacements is debatable.

Through a number of nonlinear analyses by the authors, it was found that a slightly loose tolerance imposed on both the displacement and force error is preferable to a tight tolerance for either the displacement or the force error norm. To this, 0.1% equilibrium error is allowed for each of the maximum unbalanced displacement and force norms. Equilibrium is only assumed when both of the equilibrium checks are satisfied.

Mathematically, the convergence criteria for force and displacement are expressed respectively as,

$$\frac{\{\Delta F\}^{\mathrm{T}}\{\Delta F\}}{\{F\}^{\mathrm{T}}\{F\}} \leq \mathrm{TOL} \tag{10.27a}$$

$$\frac{\{\Delta u\}^{\mathrm{T}}\{\Delta u\}}{\{u\}^{\mathrm{T}}\{u\}} \leq \mathrm{TOL} \tag{10.27b}$$

in which $\{F\}$ and $\{u\}$ are the accumulated force and displacement vectors respectively. TOL is the tolerance for equilibrium condition and is set to 0.1% for the present study.

10.3.4 Comparison among the numerical algorithms

Although the load control Newton-Raphson method is slower in convergence when compared to the arc-length or the minimum residual displacement method or even diverge near the limit point, it is the only solution scheme which allows the analyst to specify an exact load level at which the stresses and deflections are studied. Consequently it is particularly suitable for design of a practical structure allowing for various nonlinearities and under a set of fixed loads, such as the design loads. In addition, since the exciting dynamic load is prescribed as an input data in a nonlinear dynamic analysis, the load control Newton-Raphson method is very suitable for analyzing a structure subjected to cyclic loads.

The displacement control method performs satisfactorily when handling snap-through problems, but it fails at a snap-back point. Moreover, it may be very difficult, in some cases, to select a suitable displacement degree of freedom as the control parameter. Nevertheless, as the user can specify an exact value of displacement in advance, it can be used in the analysis when a specified displacement is required. One example is a structure under differential settlements, in which specified displacements at settled supports are assumed to calculate the settlement effect on the overall structural behaviour.

The arc-length and the minimum residual displacement methods are capable of tracing the nonlinear load-deformation curve with snap-through and snap-back phenomena. It has been generally observed that the minimum residual displacement method gives the most rapid rate of convergence and the highest reliability among these three methods. Better performance may be achieved when the minimum residual displacement iterative scheme is used in conjunction with the arc-length load increment for nonlinear static analysis.

Combining the element stiffness matrix method and the aforementioned numerical schemes, a computer program NIDA capable of handling the large deflection analysis of thin-walled members and frames has been written.

10.4 Imperfections

Unlike the first-order linear analysis, imperfections must be considered in any second-order direct analysis since no real structure is perfect and possesses no residual stress and initial crookedness.

In HK Code and Eurocode 3 (2005), a special feature is about the consideration of frame and member imperfections which are not so explicitly expressed in most other codes.

The effects of imperfections shall be taken into account for two conditions.
Global analysis: P-Δ effect
Member design: P-δ effect

10.4.1 Frame imperfections

The effects of imperfections for typical structures shall be incorporated in frame analysis using an equivalent geometric imperfection in Equation (10.28) based on the lowest global elastic buckling mode(s) or the notional horizontal force of 0.5% for permanent structures. While the approach of using lowest global elastic critical mode as imperfection mode can be applied to all cases, the notional horizontal force should only be used in rectilinear building frames because of the uncertainty in setting these notional forces in irregular frames. The load case with least elastic critical load factor should consider the two modes with lowest elastic critical load factors as two independent load cases to prevent slender members from under-designing.

$$\Delta = \frac{h}{200} \tag{10.28}$$

where

h is the storey height;

Δ is the initial deformation or out-of-plumbness deflection.

The shape of imperfection may be determined using the notional horizontal force for a regular frame or from the elastic critical mode. For regular multi-floor building frames, the shape may be simply taken as an inclined straight line.

10.4.1.1 Elastic critical mode

As an alternative to notional horizontal force in *Clause 2.5.8*, the elastic buckling mode can be used to simulate the global imperfections. The amplitude of such global imperfection can be taken as building height/200 for permanent structures or height/100 for temporary structures. The lowest elastic buckling global mode should be used and at least the two lowest elastic buckling global modes should be used as two load separated load cases for the load case with the lowest elastic critical load factor λ_{cr}. Local member buckling mode should not be used in place of the global buckling mode.

10.4.1.2 Method of notional horizontal force

For regular frames where the buckling mode is in a sway mode and obvious to engineers, a 0.5% of the vertical load should be applied horizontally to a frame which is basically regular in order to simulate imperfection as demonstrated in Figure 10.16. For structures used for other functions and durations, a varied value of notional force is used.

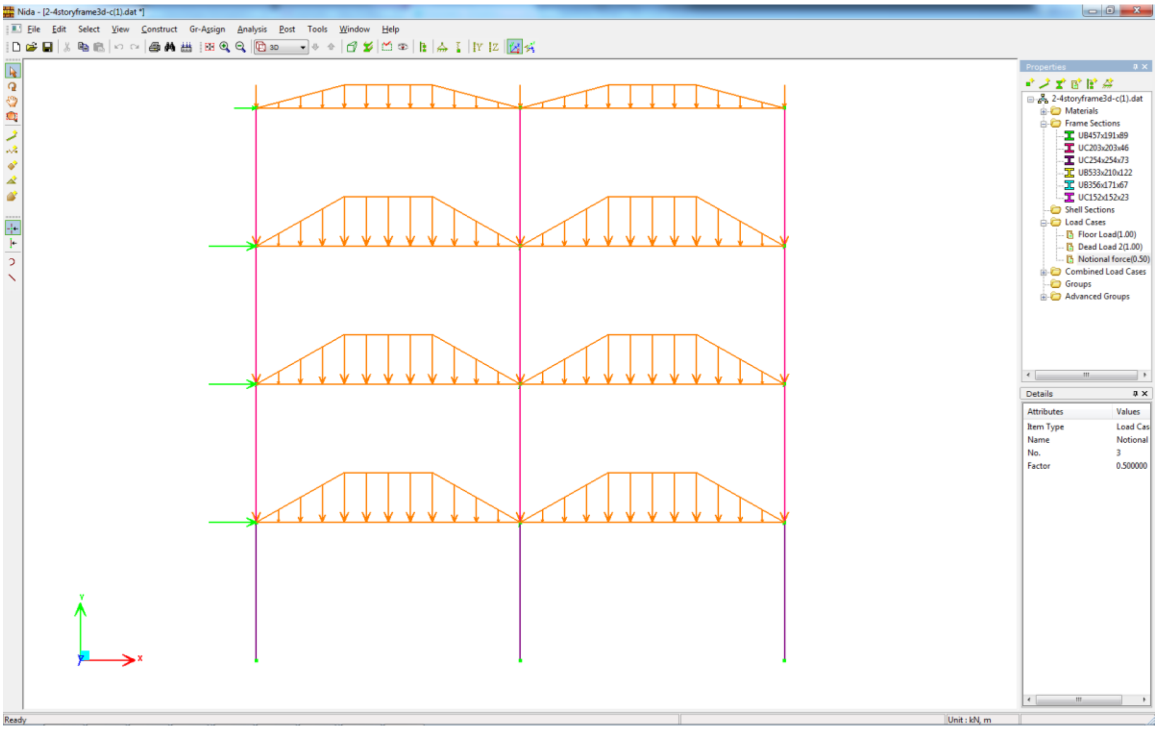


Figure 10.16 Application of horizontal notional forces

10.4.1.3 Imperfection mode as buckling mode

In many structures, the buckling mode shape is not obvious and we need to use computer program to determine the buckling mode. We can use the buckling mode as imperfection mode as the worst scenario as follows. In software, we can specify this eigen-buckling mode option and a magnitude equal to 0.5% multiplied by the height or the longest span or an expected value of imperfection for a particular type of structures. 1% imperfection deflection or notional force is needed for temporary structures and 3% may be needed for structures under demolition.

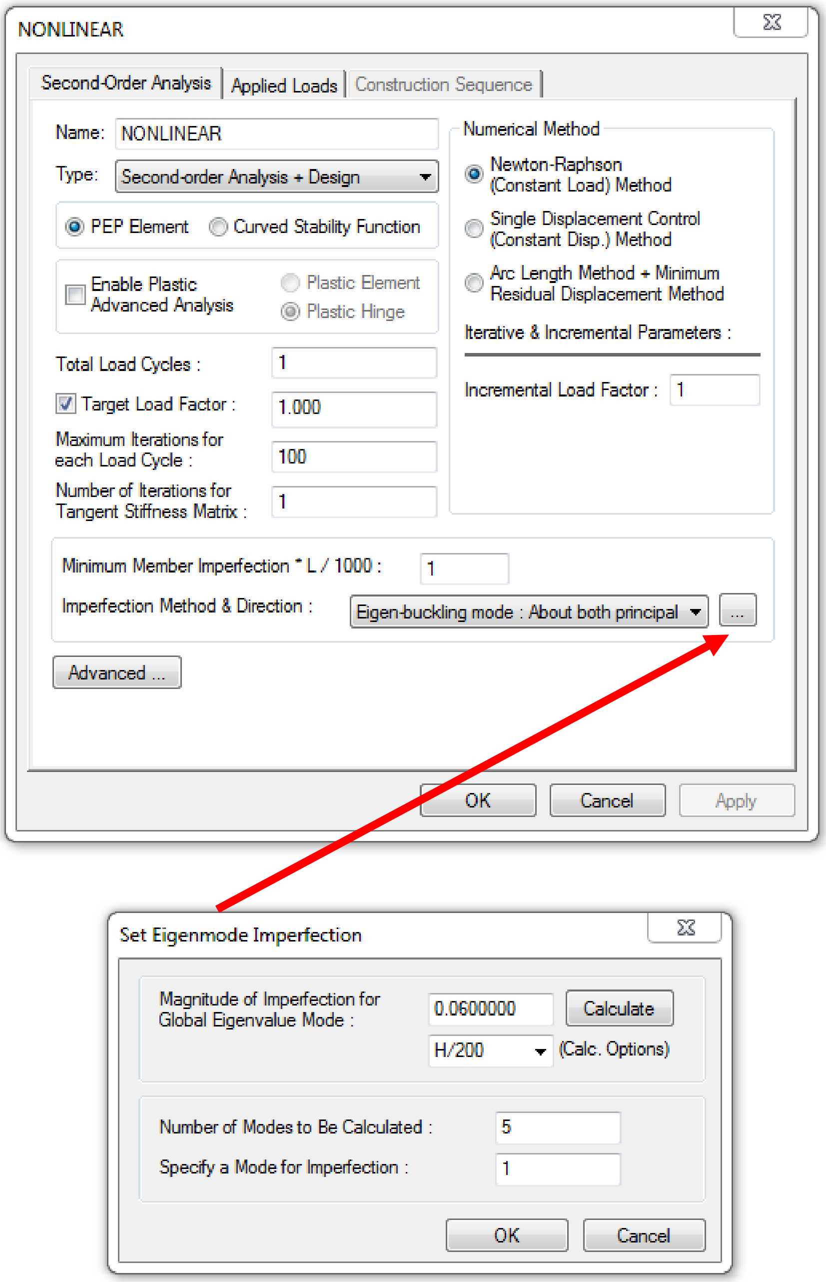


Figure 10.17 Use of buckling mode as imperfection mode

These initial sway imperfections should be applied in all unfavourable horizontal directions, but need only be considered in one direction at a time. For temporary works such as scaffolding, initial deformation should be taken as $\Delta = h/100$. For demolition works, initial deformation equivalent to notional force should be used.

The simulation of out-of-plumbness with notional horizontal force is indicated in Figure 10.18.

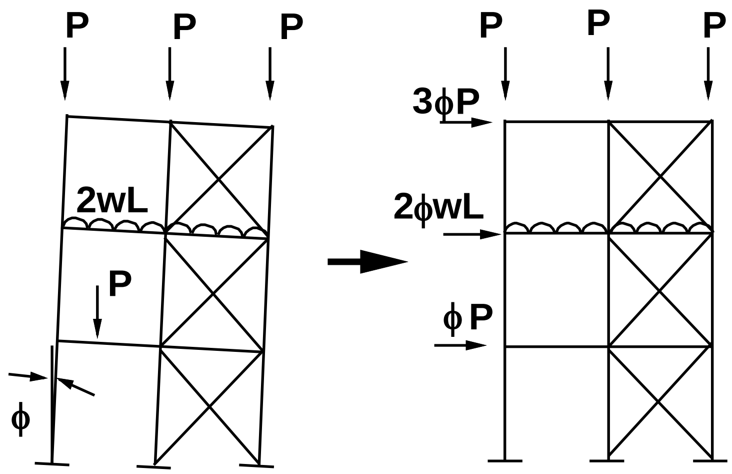


Figure 10.18 Notional horizontal force to simulate out-of-plumbness

10.4.2 Member imperfections

For practical members, initial bow and residual stress are unavoidable and must be considered in the buckling strength determination. Table 6.1 in the HK Code shows the equivalent imperfection for these two sources of imperfections and they are the equivalent imperfection. The value of these imperfections cannot be measured from the initial bow or crookedness of the member but it can be determined by a curve-fitting procedure against the buckling strength vs. slenderness curve. In other words, we can try different values of imperfections to obtain a curve giving a 5% lower bound curve to the experimental curve. Alternatively or more directly, we can calculate the imperfection using the available Perry Robertson constants (Cho and Chan, 2002). For a compression member, the equivalent initial bow imperfection specified in Table 6.1 of the HK Code may be used in a second order analysis of the member.

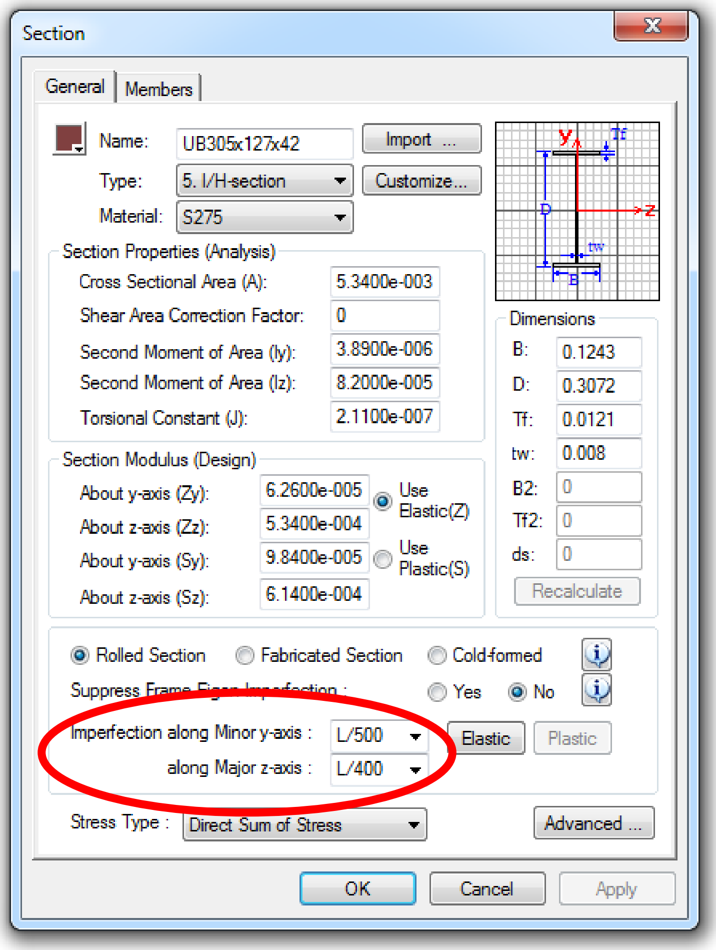


Figure 10.19 Input of member initial imperfections in NIDA

Important Note

The directions of imperfections should be the same as eigen-buckling mode which depends on the loads and thus it varies with different load cases. Thus, it is practically not feasible to use an approach of modelling a member by several elements to simulate imperfect geometry even for a moderate structure for a hundred load cases, because we will then need a hundred data files containing different initial geometries.

The effects of imperfections can be considered in member design when using the effective length method and the moment amplification method. This is the reason we have different buckling curves in the HK Code.

Instead of using different curves, the second-order direct analysis uses different imperfections in *Table 6.1* of the HK Code (2011) or *Table 5.1* of Eurocode 3 (2005). For P-Δ-only analysis, member bucking check based on curves a_0 to d is still necessary. Note that these imperfections cannot be measured directly since they are "equivalent" imperfections considering geometrical initial curvature and residual stress. The geometrical initial curvature is measured to be around L/1000 to L/1500 which is much less than the equivalent initial imperfection allowing for residual stress and geometrical crookedness.

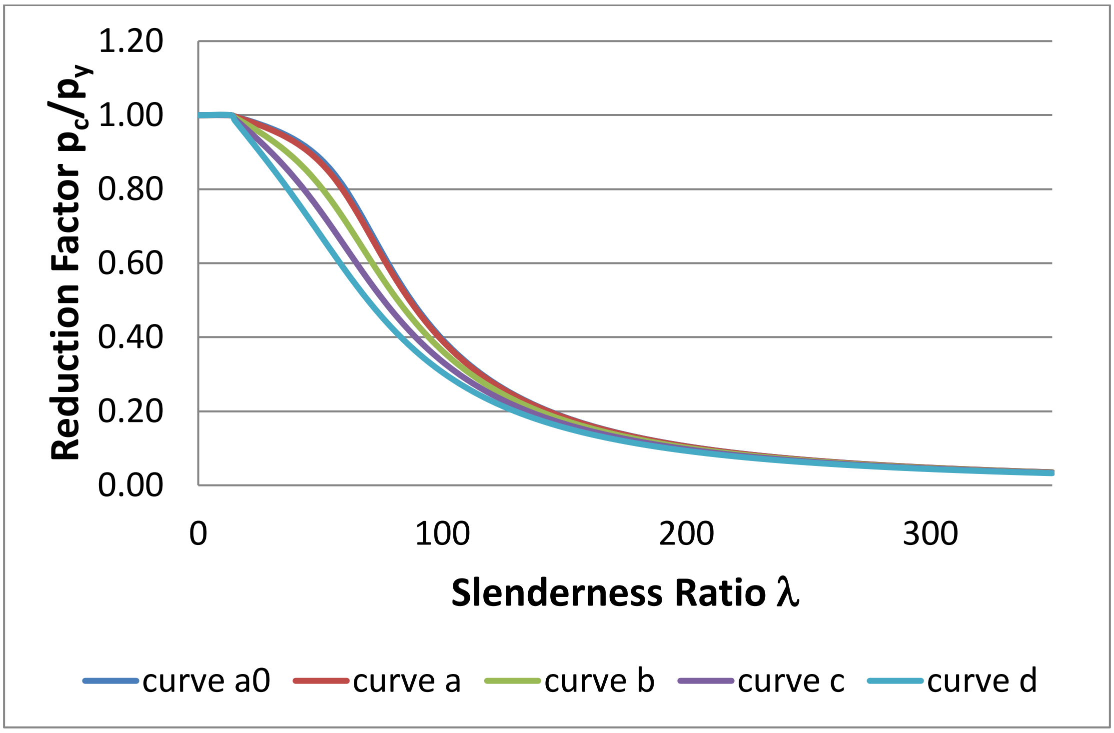


Figure 10.20 Simulation of member imperfection by specifying or using coded values

As can be seen above, a curved member is needed for the simulation. Although the Eurocode 3 (2005) suggests to use equivalent load along a member to simulate the effect, there will be an additional stress induced by this fictitious load which should not exist and it is therefore not allowed in the HK Code which uses the direct method of curved member.

10.5 The effective length method for indirect analysis

In the first-order linear analysis, the analysis finds the load in the columns and the buckling strength is unknown. Unlike the second-order P-Δ-δ analysis which considers the increase in stress due to the second-order and buckling effect, the first-order linear analysis needs to reduce the resistance of the columns when taking the load without considering second-order moment. To calculate the effective length other than making an assumption, we have the following method.

Calculate λ_{cr} by one of the following methods

1. Apply notional horizontal force. λ_{cr} can be determined by Equation (10.23)
2. Use computer programme to find λ_{cr}

For multi-storey frames, the maximum λ_{cr} among all stories should be used to obtain the minimum elastic critical load factor.

λ_{cr} is defined as the factor multiplied to the design load causing the frame to buckle elastically.

Notional force is (1) to simulate lack of verticality of frames and taken as 0.5% of the factored dead and imposed loads applied horizontally to the structure; (2) to calculate the elastic critical load factor λ_{cr} to Equation (10.23). This percentage of notional force may vary for other types of structures like scaffolding where imperfections are expected to be more serious and (3) to classify the frame as non-sway, sway and sway-ultra sensitive frames.

The following section describes the method of using chart to determine the effective length of a member in a regular frame.

10.5.1 Non-sway frame

When $\lambda_{cr} \geq 10$, the frame is considered as non-sway. P-Δ effect can be ignored here and only P-δ effect is needed to be considered. The effective length of members in frames can be designed by chart in *Figure 6.5b* of HK Code or conservatively taken as member length here.

For a sub-frame in a multi-storey frame in Figure 10.21, the distribution factors, k_1 and k_2, are required to be determined as,

$$k = \frac{\text{Total stiffness of the columns at the joint}}{\text{Total stiffness of all the members at the joint}}$$

$$k_1 = \frac{K_c + K_1}{K_c + K_1 + K_{11} + K_{12}} \qquad (10.29)$$

$$k_2 = \frac{K_c + K_2}{K_c + K_2 + K_{21} + K_{22}} \qquad (10.30)$$

and the stiffness coefficients for beams should follow Table 6.2 of Hong Kong Code.

To calculate the load resistance of the column $P_c = A_g p_c$, its effective length is needed to be determined from k_1 and k_2 above with the chart in Figure 10.22.

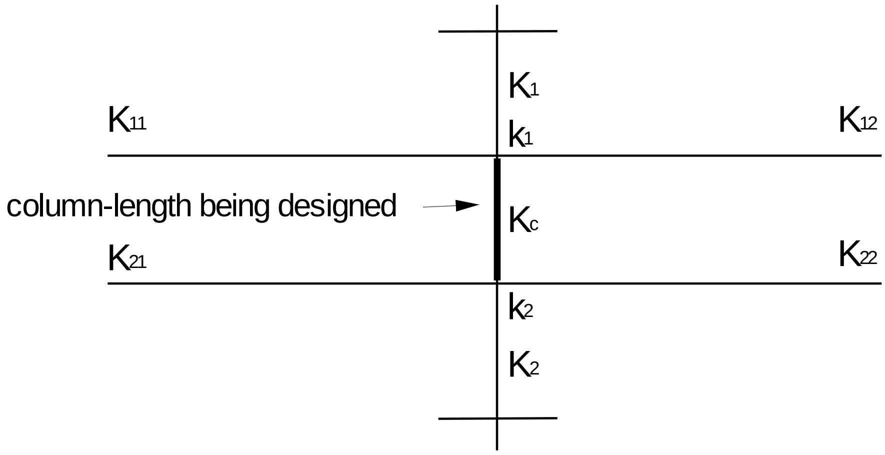


Figure 10.21 Restraint coefficients in a sub-frame

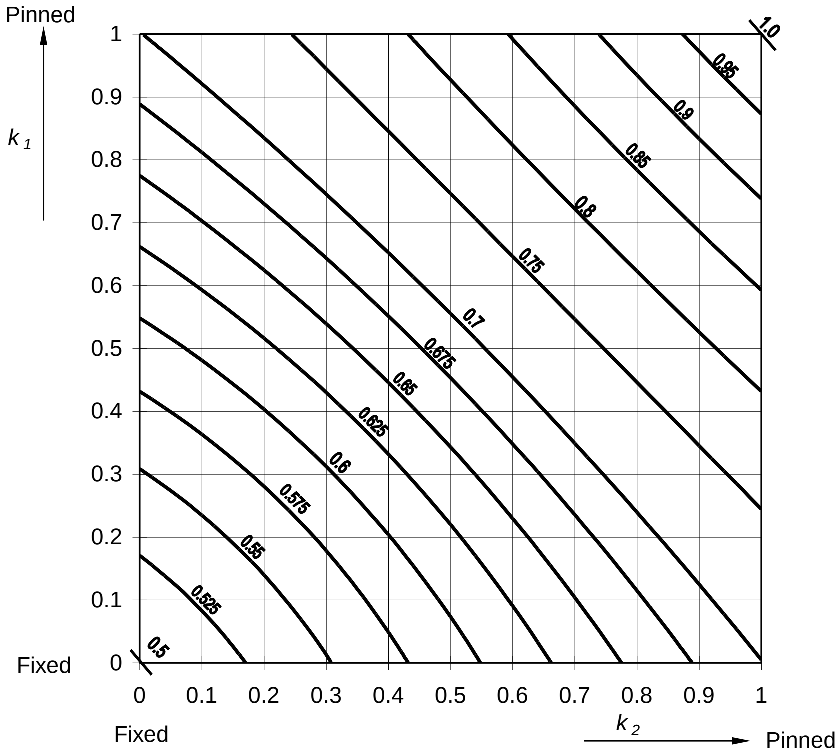


Figure 10.22 Effective length chart for non-sway frame

10.5.2 Sway-sensitive frames

When $10 > \lambda_{cr} \geq 5$, it is a sway frame.

The following chart is used to find the effective length factor. k_1 and k_2 are calculated similarly to the above.

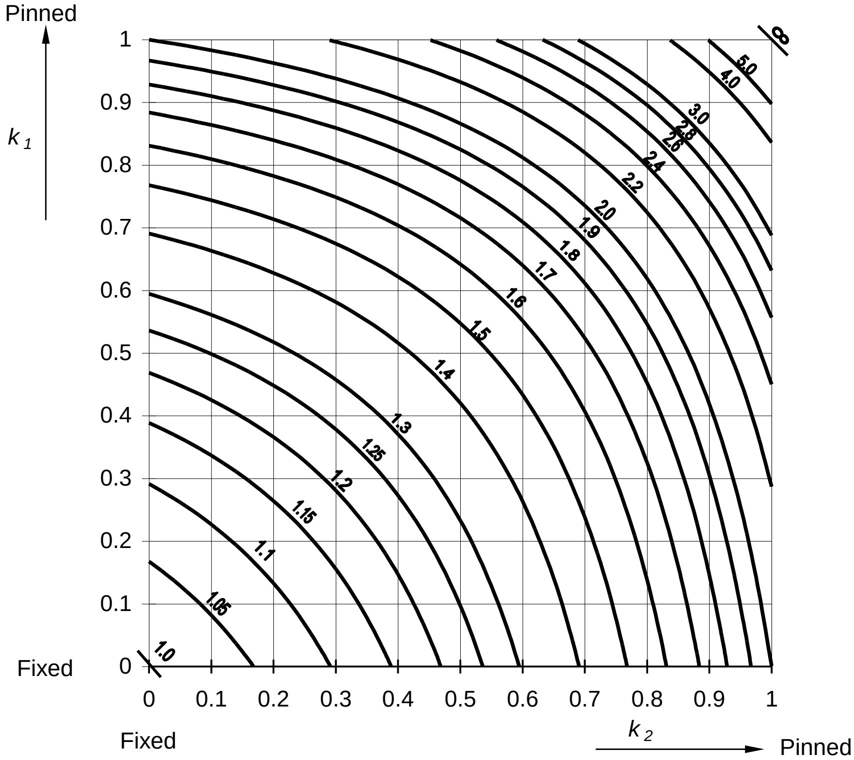


Figure 10.23 Effective length chart for sway frame

Member buckling check to *Equation (8.79)* in HK Code
Using effective length greater than member length to find the P_c is first carried out and the moment is not required to be amplified here because only member buckling check is considered.

$$\frac{F_c}{P_c} + \frac{m_x \overline{M}_x}{M_{cx}} + \frac{m_y \overline{M}_y}{M_{cy}} \leq 1 \tag{10.31}$$

Amplification moment check to *Equation (8.80)* in HK Code
Additionally, moment amplification factor should be used to enlarge the moment due to sway effect as the second checking as

$$\frac{F_c}{\overline{P}_c} + \frac{m_x M_x}{M_{cx}} + \frac{m_y M_y}{M_{cy}} \leq 1 \tag{10.32}$$

Here the buckling resistance $\overline{P}_c$ (note the bar at top while *Equation (8.79)* has no bar at P_c) is determined using effective length equal to member length (i.e. $L_E/L = 1$), but M_x and M_y are needed to be amplified as follows.

$$M = \overline{M}\,\frac{\lambda_{cr}}{\lambda_{cr} - 1} \tag{10.33}$$

Also, the amplified moment should be used for connection design.

As the above considers the P-Δ effect such that the effective length of the column is then taken as its member length or from Figure 10.23.

When the beam may experience beam lateral-torsional buckling, *Equation (8.81)* should also be checked.

10.5.3 Sway ultra-sensitive frames

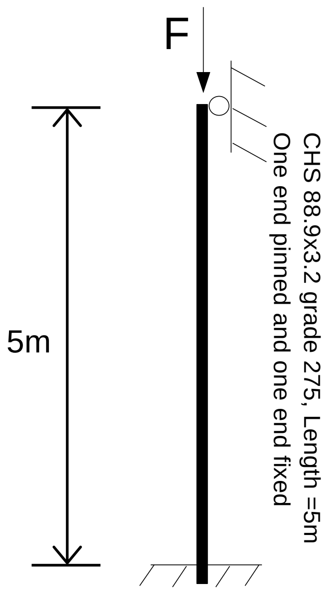
When $\lambda_{cr} < 5$, only second-order direct analysis method can be used.

10.6 Examples

10.6.1 Simple benchmark example for testing of software

A column of CHS 88.9×3.2, grade S275 steel and length 5m, has a boundary condition of one end pinned and one end fixed (i.e. a propped cantilever) as shown in the figure below. The analytical elastic critical load P_{cr} and the compression resistance P_c to the HK Code can be calculated by taking the effective length of this propped cantilever as 0.7 of its true length. When using NIDA, the design load is indicated by the load causing the section capacity factor equal to 1.0 while the elastic critical load is the load when the load vs deflection curve becomes flat. In NIDA, no effective length assumption is required as the P-δ effect is automatically taken into account during the analysis. However, an initial member imperfection of L/500 is imposed as given in *Table 6.1* of the HK Code.

Compare the analytical elastic critical load and the design load to the HK Code with results given by NIDA. Also repeat the example with a boundary condition of one end free and one end fixed (i.e. cantilever) and the effective length factor equal to 2.



SECTION PROPERTIES

$D = 88.9mm$, $t = 3.2mm$, $I = 79.2cm^4$, $r = 3.03cm$, $Z = 17.8cm^3$, $A = 8.62cm^2$

Case 1: Propped cantilever

Effective length, $L_E = 0.7L = 3.5m$

Elastic critical load, $P_{cr} = \dfrac{\pi^2 EI}{L_E{}^2} = \dfrac{\pi^2 \times 205000 \times 79.2 \times 10^4}{3500^2} = 130.8kN$

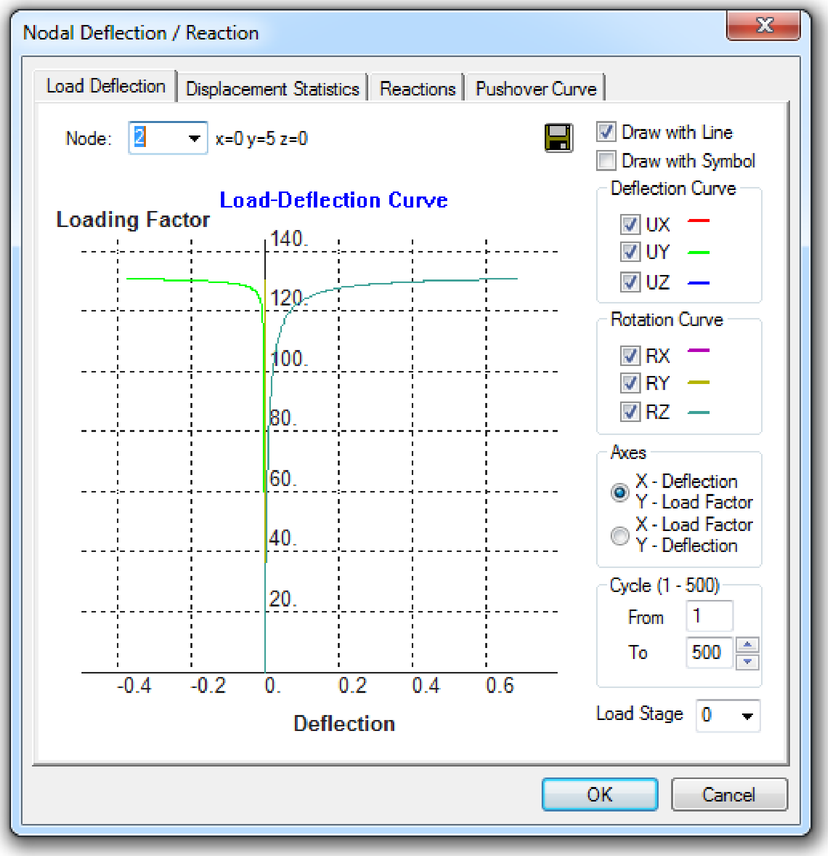
Slenderness ratio, $\lambda = \dfrac{L_E}{r} = \dfrac{3500}{30.3} = 115.5$ *(Clause 8.7.4)*

Compressive strength for hot-rolled hollow section bending about both axes should be obtained from buckling curve a *(Table 8.7)*

Compressive strength, $p_c = 126.0\ N/mm^2$ *(Table 8.8(a))*

Compression resistance, $P_c = p_c A_g = 126 \times 862 = 108.6kN$ *(8.73)*

The elastic critical load and design load found by NIDA are 98.7kN and 131.4kN respectively.



The table below compares the elastic critical loads and compression resistance between different methods.

Methods	Elastic critical load (kN)	Compression Resistance (kN)
NIDA	131.4	98.7
Analytical	130.8	-
HK Code	-	108.6

Case 2: Cantilever

Effective length, $L_E = 2L = 10m$

Elastic critical load, $P_{cr} = \dfrac{\pi^2 EI}{L_E{}^2} = \dfrac{\pi^2 \times 205000 \times 79.2 \times 10^4}{10000^2} = 16.0kN$

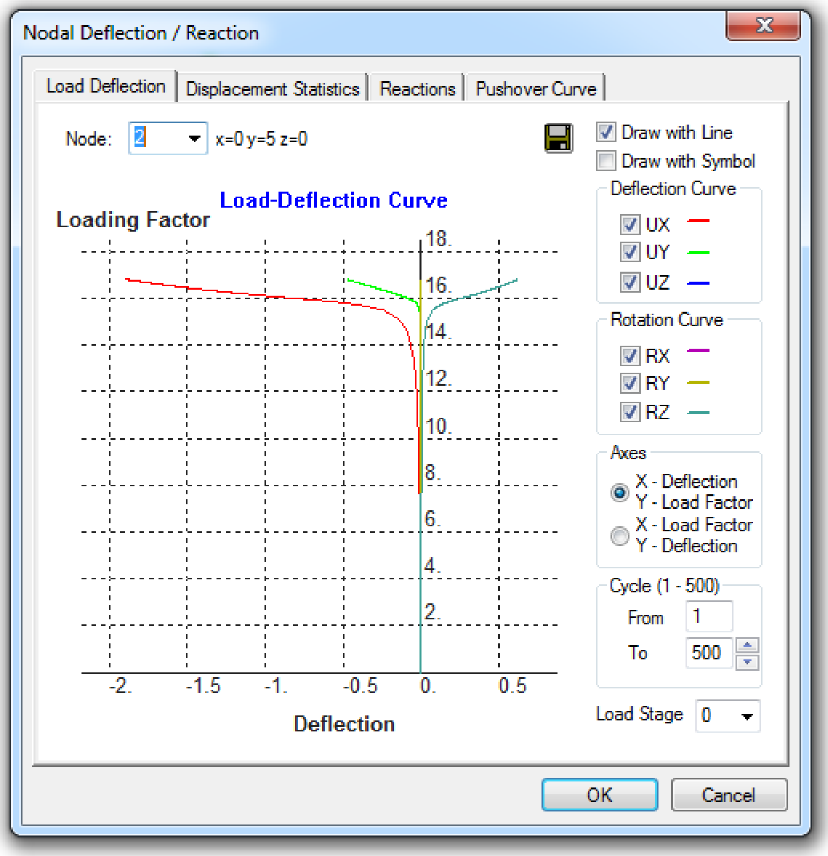
Slenderness ratio, $\lambda = \dfrac{L_E}{r} = \dfrac{10000}{30.3} = 330.0$ *(Clause 8.7.4)*

Compressive strength for hot-rolled hollow section bending about both axes should be obtained from buckling curve a *(Table 8.7)*

Compressive strength, $p_c = 18\,N/mm^2$ *(Table 8.8(a))*

Compression resistance, $P_c = p_c A_g = 18 \times 862 = 15.5kN$ *(8.73)*

The elastic critical load and design load found by NIDA are 15.6kN and 16.8kN respectively.



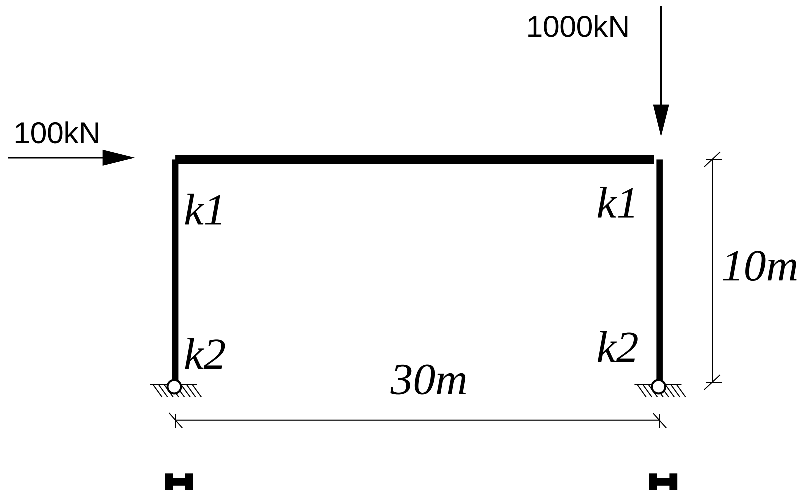
The table below compares the elastic critical loads and compression resistance between different methods.

Methods	Elastic critical load (kN)	Compression Resistance (kN)
NIDA	16.8	15.6
Analytical	16	-
HK Code	-	15.5

These two simple examples show clearly the reliability and capability of NIDA in predicting a single column with resistance dominated by P-δ and P-Δ effects. Imperfection to code value of *L*/500 for hot-rolled tubular sections has been allowed for. While the single column can be verified directly by code assuming an appropriate effective length, the computer method NIDA can be applied to design of complex frames composing of thousands of members by repeating the calculation whereas the HK Code approach can hardly be extended as every member has its own buckling length and some load cases are sway and others are non-sway.

10.6.2 Structural analysis of the portal frame

Check the structural adequacy of the following portal. The section is 686×254×140 UB of grade S355 steel. The frame is rigid-jointed and pin-supported with dimensions shown in the figure below.



DESIGN LOAD

Axial force, $F_c = 1000 + 100 \times 10/30 = 1033.3kN$

Base shear, $V_y = 100/2 = 50kN$

Top moment, $M_x = 50 \times 10 = 500kNm$

SECTION PROPERTIES

$D = 683.5mm$, $B = 253.7mm$, $t = 12.4mm$, $T = 19.0mm$, $d = 615.1mm$, $I_x = 136000cm^4$,

$r_x = 27.6cm$, $Z_x = 3990cm^3$, $S_x = 4560cm^3$, $A = 178cm^2$

SECTION CLASSIFICATION

Design strength, $p_y = 345N/mm^2$ for $16mm < T \le 40mm$ *(Table 3.2)*

$$\varepsilon = \sqrt{\frac{275}{345}} = 0.89$$ *(Table 7.1 Note b)*

Plastic limiting value of b/T for outstand flange of an I-section is 9ε

$$\frac{b}{T} = \frac{253.7}{2 \times 19} = 6.68 \le 9 \times 0.89 = 8.01$$ *(Table 7.1)*

$\therefore$ flange is plastic

Plastic limiting value of d/t for web of an I-section under both axial compression and bending is $80\varepsilon/(1+r_1)$

Stress ratio, $$r_1 = \frac{F_c}{dtp_{yw}} = \frac{1033.3 \times 10^3}{615.1 \times 12.4 \times 345} = 0.393 \le 1$$ *(7.1)*

$$\frac{d}{t} = \frac{615.1}{12.4} = 49.6 \le \frac{80 \times 0.89}{1 + 0.393} = 51.1$$ *(Table 7.1)*

$\therefore$ web is plastic

$\therefore$ the section is Class 1 plastic

FRAME CLASSIFICATION

Deflection due to horizontal force =149.7 mm

(*Note that it is unnecessary to use 0.5% for notional force as the same elastic critical load factor λ_{cr} will be obtained because the deflection δ_N below will be changed proportionally and the final answer will be the same.*)

Elastic critical load factor, $\lambda_{cr} = \frac{F_N}{F_V}\frac{h}{\delta_N} = \frac{100}{1000}\frac{10}{0.1497} = 6.68$ *(6.1)*

$\because 10 > \lambda_{cr} \geq 5$ *(6.4)*

$\therefore$ it is a sway frame *(Clause 6.3.4)*

SHEAR CAPACITY

Shear area, $A_v = tD = 12.4 \times 683.5 = 8475mm^2$ *(Clause 8.2.1)*

Shear capacity, $V_c = \frac{p_y A_v}{\sqrt{3}} = \frac{265 \times 8475}{\sqrt{3}} = 1296.7kN > V$ *(OK)* *(8.1)*

MOMENT CAPACITY

$V \leq 0.6V_c = 778.0kN$ *(Clause 8.2.2.1)*

$\therefore$ It is low shear condition

Moment capacity, $M_{cx} = p_y S_x \leq 1.2 p_y Z_x$ *(8.2)*

$$= 345 \times 4560 \times 10^3 \leq 1.2 \times 345 \times 3990 \times 10^3$$
$$= 1573.2kNm \leq 1651.9kNm$$
$$> M_x \text{ (OK)}$$

COMPRESSION RESISTANCE

Beam stiffness in sway mode should be taken as $1.5\frac{I}{L}$ *(Table 6.2)*

$k_1 = \frac{K_c + K_1}{K_c + K_1 + K_{11} + K_{12}} = \frac{I/10 + 0}{I/10 + 0 + 1.5 \times I/30 + 0} = 0.67$ *(Figure 6.4)*

$k_2 = 1$ for pinned end

Effective length, $L_E = 2.9L = 2.9 \times 10 = 29m$ *(Figure 6.5a)*

Slenderness ratio, $\lambda = \frac{L_E}{r_x} = \frac{29000}{276} = 105.1$ *(Clause 8.7.4)*

Compressive strength for rolled I-section with maximum thickness less than 40mm bending about *x-x* axis should be obtained from buckling curve a *(Table 8.7)*

Compressive strength, $p_c = 156.3 N/mm^2$ *(Table 8.8(a))*

Compression resistance, $P_c = p_c A_g = 156.3 \times 17800 = 2782.1kN > F_c$ *(OK)* *(8.73)*

CROSS-SECTION CAPACITY

$\frac{F_c}{A_g p_y} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} = \frac{1033.3 \times 10^3}{17800 \times 345} + \frac{500}{1573.2} = 0.49 \leq 1$ *(OK)* *(8.78)*

MEMBER BUCKLING RESISTANCE

Buckling check using effective length under sway mode to *Equation (8.79)*

Equivalent moment factor, $m_x = 0.6$ *(Table 8.9)*

$\frac{F_c}{P_c} + \frac{m_x \overline{M}_x}{M_{cx}} + \frac{m_y \overline{M}_y}{M_{cy}} = \frac{1033.3}{2782.1} + \frac{0.6 \times 500}{345 \times 3990 \times 10^{-3}} = 0.59 \leq 1$ *(OK)* *(8.79)*

Buckling check to non-sway mode effective length under amplified moment to *Equation (8.80)*
Effective length, $L_E = L = 10m$

Slenderness ratio, $\lambda = \frac{L_E}{r_x} = \frac{10000}{276} = 36.2$ *(Clause 8.7.4)*

Compressive strength, $p_c = 327.8\,N/mm^2$ *(Table 8.8(a))*

Compression resistance, $\overline{P}_c = p_c A_g = 327.8 \times 17800 = 5834.8kN > F_c$ *(OK)* *(8.73)*

The moment amplification factor is given by the larger of *(8.82)*

$$\frac{\lambda_{cr}}{\lambda_{cr}-1} = \frac{6.68}{6.68-1} = 1.20$$

and $$\frac{1}{1-\frac{F_c L_E^2}{\pi^2 EI_x}} = \frac{1}{1-\frac{1033.3\times 29^2}{\pi^2\times 2.05\times 136000}} = 1.46$$

$\therefore$ The P-δ-Δ amplification factor is taken as 1.46

$$\frac{F_c}{\overline{P}_c} + \frac{m_x M_x}{M_{cx}} + \frac{m_y M_y}{M_{cy}} = \frac{1033.3}{5834.8} + \frac{0.6\times 1.46\times 500}{345\times 3990\times 10^{-3}} = 0.50 \le 1 \text{ (OK)} \qquad (8.80)$$

Note: $\overline{P}_c$ can be calculated by using the effective length found from *Figure 6.5b* for greater accuracy and assuming effective length factor = 1 is only for conservative design. $\overline{P}_c$ can be used for sway frames with amplified moments M_x and M_y because the effect of moment increase has been considered in the moments so economical design can be achieved when compared with use of P_c which is obtained by using effective length factor greater than 1. However, additional check is needed for the case when the frame sways with an effective length factor greater than 1 (i.e. use of P_c in sway frame) but the bending moment is too small to amplify, such as the case when the dominant axial load is concentric to the column being designed.

No beam buckling check to *Equation (8.81)* is needed here as the out-of-plane deflection is restrained.

DESIGN BY SECOND-ORDER DIRECT ANALYSIS

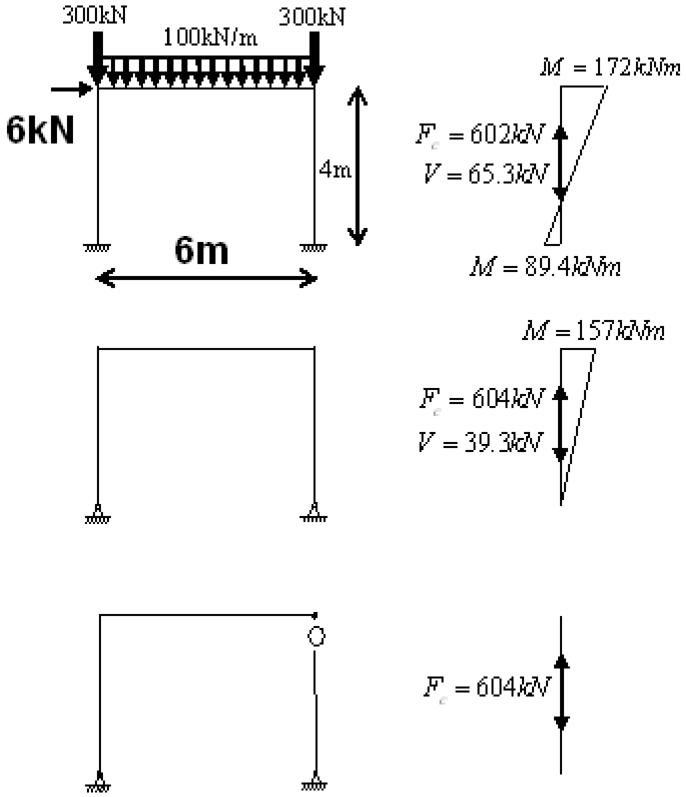
When using second-order direct analysis for design in NIDA, the section capacity factor is 0.60. This is close to the most critical value above.

Comments on second-order direct analysis applied to sway and non-sway frames

The reason for classifying a frame to sway and non-sway is to determine the effect of sway moment. When λ_{cr} is greater than or equal to 10, the sway moment is negligible and when λ_{cr} is less than 10, it is important and the effect need to be and can be considered by the use of second order direct analysis program or sway amplification factor.

10.6.3 Sway and non-sway frame

This example is to demonstrate the design procedures to the HK Code using the conventional approach as well as using second-order P-Δ-δ elastic analysis. Three simple frames subject to same loading condition but three different boundary conditions are shown in the figure below. The columns are 254×254×73 UC and the beams are 406×178×74 UB of S275 steel. Subject to the same notional horizontal forces as required according to *Clause 2.5.8,* the respective internal forces determined from the first-order elastic analysis are also shown in the figure.



SECTION PROPERTIES

For 254×254×73 UC

$D = 254.1mm$, $B = 254.6mm$, $t = 8.6mm$, $T = 14.2mm$, $d = 200.3mm$, $I_x = 11400cm^4$,
$r_x = 11.1cm$, $Z_x = 898cm^3$, $S_x = 992cm^3$, $A = 93.1cm^2$

For 406×178×74 UB

$D = 412.8mm$, $B = 179.5mm$, $t = 9.5mm$, $T = 16.0mm$, $d = 360.4mm$, $I_x = 27300cm^4$,
$r_x = 17.0cm$, $Z_x = 1320cm^3$, $S_x = 1500cm^3$, $A = 94.5cm^2$

SECTION CLASSIFICATION

Design strength, $p_y = 275N/mm^2$ for $T \le 16mm$ *(Table 3.2)*

$$\varepsilon = \sqrt{\frac{275}{275}} = 1$$ *(Table 7.1 Note b)*

Plastic limiting value of b/T for outstand flange of an H-section is 9ε

$$\frac{b}{T} = \frac{254.6}{2\times 14.2} = 8.96 \le 9\times 1 = 9$$ *(Table 7.1)*

$\therefore$ flange is plastic

Plastic limiting value of d/t for web of an H-section under both axial compression and bending is $80\varepsilon/(1+r_1)$

Stress ratio, $r_1 = \dfrac{F_c}{dtp_{yw}} = \dfrac{604\times 10^3}{200.3\times 8.6\times 275} = 1.275 > 1$ *(7.1)*

$\therefore r_1 = 1$

$$\frac{d}{t} = \frac{200.3}{8.6} = 23.3 \le \frac{80\times 1}{1+1} = 40$$ *(Table 7.1)*

$\therefore$ web is plastic

$\therefore$ the section is Class 1 plastic

FRAME CLASSIFICATION

The notional horizontal deflections of the three frames found from linear first-order elastic analysis and the corresponding elastic critical load factor are summarized below.

Boundary condition	Notional horizontal deflection, δ_N (m)	Elastic critical load factor, λ_{cr}
Supports fixed Member joints rigid	8.857×10^{-4}	$\frac{1}{200}\cdot\frac{4}{8.857\times10^{-4}} = 22.6$
Supports pinned Member joints rigid	3.612×10^{-3}	$\frac{1}{200}\cdot\frac{4}{3.612\times10^{-3}} = 5.54$
Supports pinned One member joint pinned	8.919×10^{-3}	$\frac{1}{200}\cdot\frac{4}{8.919\times10^{-3}} = 2.24$

Therefore, the frames can be classified into non-sway, sway and sway ultra-sensitive respectively according to *Clause 6.3*

It should be noted that for sway ultra-sensitive frames, where the elastic critical load factor is less than 5, only second-order direct P-Δ-δ elastic analysis or advanced analysis can be used. In this example, self-weight of the material and lateral-torsional buckling are neglected. For simplicity, this example only shows the design of the column member which involves the following steps.

SHEAR CAPACITY

Shear area, $A_v = tD = 8.6\times 254.1 = 2185mm^2$ *(Clause 8.2.1)*

Shear capacity, $V_c = \dfrac{p_y A_v}{\sqrt{3}} = \dfrac{275\times 2185}{\sqrt{3}} = 346.9kN \ge V$ *(OK)* *(8.1)*

MOMENT CAPACITY

$V \le 0.6V_c = 208.1kN$ *(Clause 8.2.2.1)*

$\therefore$ it is low shear condition

Moment capacity, $M_{cx} = p_y S_x \le 1.2 p_y Z_x$ *(8.2)*

$$= 275 \times 992 \times 10^3 \le 1.2 \times 275 \times 898 \times 10^3$$

$$= 272.8kNm \le 296.3kNm$$

$$> M_x \ (OK)$$

COMPRESSION RESISTANCE

According to *Clause 6.6.3*,

$$k_1 = \frac{K_c}{K_c + K_{11}}$$ *(Figure 6.4)*

where

$$K_c = \frac{I_c}{L_c} = \frac{11400}{400} = 28.5cm^3$$

$$K_{11} = \frac{I_b}{L_b} = \frac{27300}{600} = 45.5cm^3$$

According to Clause 8.7.5, the compression resistance should be obtained from

$$P_c = A_g p_c$$ *(8.73)*

Non-sway frame	**Sway frame**	
Beam stiffness $= 0.75\frac{I}{L}$	Beam stiffness $= 1.0\frac{I}{L}$	*(Table 6.2)*
$k_1 = \frac{28.5}{28.5 + 0.75 \times 45.5} = 0.46$	$k_1 = \frac{28.5}{28.5 + 1.0 \times 45.5} = 0.39$	*(Figure 6.4)*
$k_2 = 0$ for fixed end	$k_2 = 1$ for pinned end	
$L_E = 0.59L = 0.59 \times 4 = 2.36m$	$L_E = 2.3L = 2.3 \times 4 = 9.2m$	*(Figure 6.5)*
$\lambda = \frac{2360}{111} = 21.3$	$\lambda = \frac{9200}{111} = 82.9$	*(Clause 8.7.4)*
$p_c = 270.7 \, N/mm^2$	$p_c = 175.2 \, N/mm^2$	*(Tables 8.7, 8.8b)*
$P_c = 9310 \times 270.7 = 2520.2kN$	$P_c = 9310 \times 175.2 = 1631.1kN$	*(8.73)*

CROSS-SECTION CAPACITY

The cross section capacity check can be carried out as

$$\frac{F_c}{A_g p_y} + \frac{M_x}{M_{cx}} \le 1$$ *(8.78)*

Non-sway frame	**Sway frame**	
$\frac{602 \times 10^3}{9310 \times 275} + \frac{172}{272.8} = 0.87$ *(OK)*	$\frac{604 \times 10^3}{9310 \times 275} + \frac{157}{272.8} = 0.81$ *(OK)*	*(8.78)*

MEMBER BUCKLING RESISTANCE

The resistance of the member can be checked using

$$\frac{F_c}{P_c} + \frac{m_x \overline{M}_x}{M_{cx}} \le 1 \qquad (8.79)$$

$$\frac{F_c}{\overline{P}_c} + \frac{m_x M_x}{M_{cx}} \le 1 \qquad (8.80)$$

in which m_x is the equivalent moment factor as given in *Table 8.9* and M_x is the design moment amplified from the first-order moment $\overline{M}_x$.

For finding $\overline{P}_c$,

Effective length, $L_E = L = 4m$

Slenderness ratio, $\lambda = \frac{L_E}{r_x} = \frac{4000}{111} = 36.0$ *(Clause 8.7.4)*

Compressive strength, $p_c = 254.8\, N/mm^2$ *(Table 8.8(b))*

Compression resistance, $\overline{P}_c = p_c A_g = 254.8 \times 9310 = 2372.2kN$ *(8.73)*

For non-sway frame, the P-Δ can be neglected and the P-δ amplification factor is given by:

$$\frac{1}{1 - \frac{F_c L_E^2}{\pi^2 EI}} = \frac{1}{1 - \frac{602 \times 2.36^2}{\pi^2 \times 2.05 \times 11400}} = 1.01 \qquad (8.83)$$

For sway frame, the P-δ-Δ amplification factor is given by the larger of: *(8.82)*

$$\frac{\lambda_{cr}}{\lambda_{cr} - 1} = \frac{5.54}{5.54 - 1} = 1.22$$

and $\frac{1}{1 - \frac{F_c L_E^2}{\pi^2 EI}} = \frac{1}{1 - \frac{604 \times 9.2^2}{\pi^2 \times 2.05 \times 11400}} = 1.28$

$\therefore$ The P-δ-Δ amplification factor is taken as 1.28

Non-sway frame	**Sway frame**	
$\beta = -\frac{89.4}{172} = -0.52$	$\beta = 0$	
$\therefore m_x = 0.50$	$\therefore m_x = 0.60$	*(Table 8.9)*
	$\frac{604}{1631.1} + \frac{0.60 \times 157}{8.98 \times 2.75 \times 10} = 0.75$ *(OK)*	*(8.79)*
$\frac{602}{2520.2} + \frac{0.50 \times 1.01 \times 172}{275 \times 898 \times 10^{-3}} = 0.59$ *(OK)*	$\frac{604}{2372.2} + \frac{0.60 \times 1.28 \times 157}{8.98 \times 2.75 \times 10} = 0.74$ *(OK)*	*(8.80)*

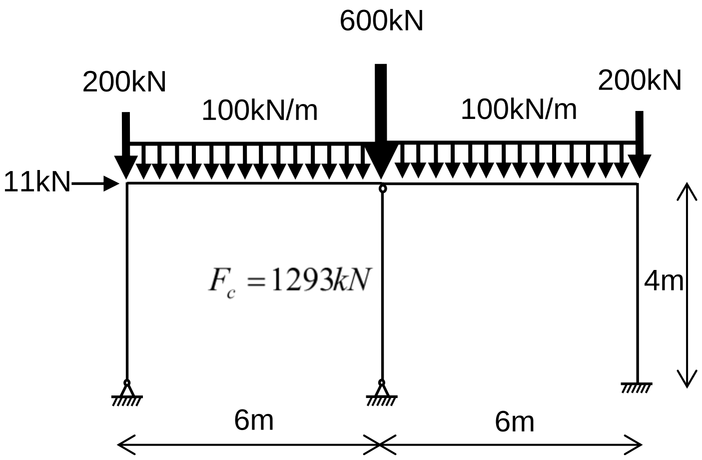
These factors are smaller than the cross section capacity factor and this shows the column is strength controlled partly because of use of low grade steel of S275.

By performing second-order P-Δ-δ elastic analysis, the section capacity factors of the columns of the non-sway, sway and sway ultra-sensitive calculated by Equation (10.24) are respectively 0.85, 0.80 and 0.91*.

*0.91 is calculated using the result of the column opposite to the loaded column. Also, the beam fails with a section capacity factor of 1.34 because the bending moment has been much enlarged by the sway displacement. This shows that the use of linear analysis without amplifying the moment is dangerous.

10.6.4 Leaning column portal

This example is to demonstrate the concept of an effective length paradox for the effective length factor, or the K-factor, of a lean column, which is widely taken as 1.0, but it may be larger or smaller than 1.0 depending on the frame instability. The figure below shows a two-bay frame using the same sections as those in Example 10.6.3. After performing a first-order elastic analysis, the frame is classified as sway frames. Traditionally there are three methods determining the effective length factor.



$\delta_N = 3.143 \times 10^{-3}\, m$

$\lambda_{cr} = 6.36$

$\therefore$ sway

Method 1: Chart method

$k_1 = 1$ for pinned end

$k_2 = 1$ for pinned end

From *Figure 6.5a*, the effective length factor is infinity.

Method 2: Idealized column method

Idealizing the two ends of the column is rotation free and transition fixed, the effective length factor can be taken as 1.00 as recommended in *Table 8.6*.

Method 3: Elastic critical load factor method

The effective length can be calculated by the following equation,

$$L_E = \sqrt{\frac{\pi^2 EI}{F_c \lambda_{cr}}}$$

The elastic critical load factor can be calculated by either the deflection method or the Eigen-buckling analysis.

Deflection Method

$\lambda_{cr} = 6.36$

$$L_E = \sqrt{\frac{\pi^2 \times 2.05 \times 11400}{1293 \times 6.36}} = 5.30m$$

Effective length factor $= \frac{5.30}{4} = 1.33$

Eigen-Buckling Analysis

$\lambda_{cr} = 6.16$

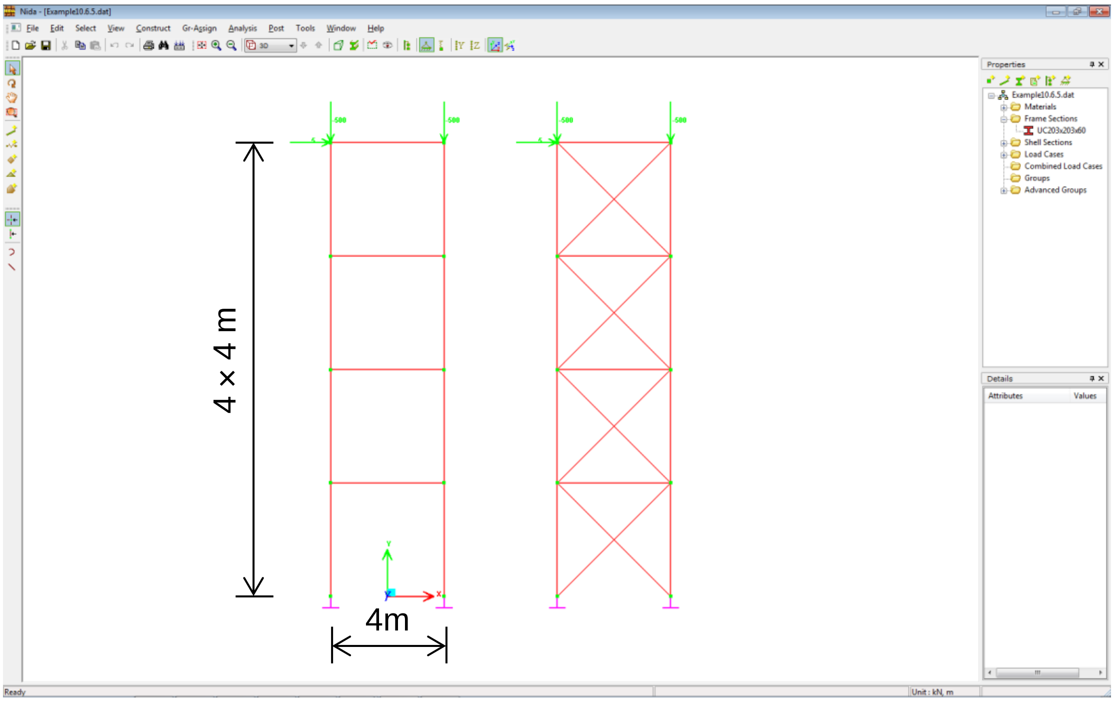
$$L_E = \sqrt{\frac{\pi^2 \times 2.05 \times 11400}{1293 \times 6.16}} = 5.38m$$

Effective length factor $= \frac{5.38}{4} = 1.35$

The effective length factor found by Method 1 is totally unacceptable for design. Most engineers adopt Method 2 for simplicity and take the effective length factor as 1.0. However, since the lean column is part of the sway frame, its actual effective length factor must be greater than 1.0. It appears that only Method 3 provides reasonable estimates of the effective length factor. However, Method 3 is not recommended in the HK Steel Code because the effective length found from this method is only true for the most critical column but not for other columns. For other non-critical columns, this method is inappropriate. The effective length factor can also be traced back from a second-order P-Δ-δ elastic analysis. It is found that the failure load of the lean column is 2308kN. Therefore the compressive strength of the column is 248N/mm^2. From *Table 8.8(b)*, the equivalent slenderness ratio is roughly 42.0 meaning an effective length factor of 1.17. It should be noted that the effective length factor found here is meaningless to a second-order direct analysis but it is used to support the result found from Method 3 and for comparison only. To avoid this effective length paradox, the best way is to adopt second-order direct analysis which avoids the use of effective length factor in column design.

10.6.5 Braced and unbraced frames

The 4-storey frame shown below is designed. All members are 203×203×60 UC. The structure is under a pair of factored vertical point loads of 500kN at top, with a notional force of 0.5% applied horizontally at the same level. The design strength is 275 N/mm^2. In the original study, all members are loaded about their principal minor axes.



SECTION PROPERTIES

$D = 209.6mm$, $B = 205.8mm$, $t = 9.4mm$, $T = 14.2mm$, $d = 160.8mm$, $I_x = 6120cm^4$, $I_y = 2060cm^4$, $r_x = 8.96cm$, $r_y = 5.20cm$, $Z_x = 584cm^3$, $Z_y = 201cm^3$, $S_x = 656cm^3$, $S_y = 305cm^3$, $A = 76.4cm^2$

FRAME CLASSIFICATION

The structure is under a pair of factored vertical point loads of 500kN at top, with a notional force of 0.5% applied horizontally at the same level. In the first study, the members are loaded about their principal minor axes, the second and third studies change the orientation and bracing conditions as shown in Table below.

Using the method of sway index, the elastic buckling load factor, λ_{cr}, is calculated in Case 1 as follows.

Storey	Deflections (mm) / sway indices ϕ_s		
	Case 1 (Bent about minor axis, unbraced)	Case 2 (Bent about major axis, unbraced)	Case 3 (Bent about minor axis, fully braced
1	5.923 / 1.481×10^{-3}	2.027 / 5.180×10^{-4}	0.063 / 1.575×10^{-5}
2	14.99 / 2.267×10^{-3}	5.164 / 7.843×10^{-4}	0.202 / 3.475×10^{-5}
3	24.33 / 2.335×10^{-3}	8.425 / 8.153×10^{-4}	0.392 / 4.750×10^{-5}
4	32.39 / 2.015×10^{-3}	11.28 / 7.138×10^{-4}	0.611 / 5.475×10^{-5}

Note: The sway index is given by:

$$\phi_s = \frac{\delta_i - \delta_{i-1}}{h}$$

Case 1 Bent about minor axis, unbraced

The maximum ϕ_s is 2.335×10^{-3} and the λ_{cr} is given by

$$\lambda_{cr} = \frac{1}{200}\cdot\frac{1}{\phi_s} = \frac{1}{200}\cdot\frac{1}{2.335\times10^{-3}} = 2.141$$

$\because \lambda_{cr} < 5$ *(6.6)*

$\therefore$ it is a sway ultra-sensitive frame *(Clause 6.3.5)*

Since λ_{cr} is less than 5 here, the effective length method cannot be used in the HK Code. There are two methods to solve this problem. The first is to use the major principal axis of members to resist loads, which is considered as Case 2. The other option is to add bracings members which is designated as Case 3.

Case 2 Bent about major axis, unbraced

Referring to the table above, the critical ϕ_s is 8.153×10^{-4} and the coressponding λ_{cr} is given by

$$\lambda_{cr} = \frac{1}{200}\cdot\frac{1}{\phi_s} = \frac{1}{200}\cdot\frac{1}{8.153\times10^{-4}} = 6.133$$

$\because 10 > \lambda_{cr} \geq 5$ *(6.4)*

$\therefore$ it is a sway frame *(Clause 6.3.4)*

Beam stiffness in sway mode should be taken as $1.5\frac{I}{L}$ *(Table 6.2)*

$$k_1 = \frac{K_c + K_1}{K_c + K_1 + K_{11}} = \frac{\frac{I}{L}+\frac{I}{L}}{\frac{I}{L}+\frac{I}{L}+1.5\frac{I}{L}} = 2/3.5 = 0.57$$ *(Figure 6.4)*

$$k_2 = \frac{K_c + K_2}{K_c + K_2 + K_{21}} = \frac{\frac{I}{L}+\frac{I}{L}}{\frac{I}{L}+\frac{I}{L}+1.5\frac{I}{L}} = 2/3.5 = 0.57$$

Effective length, $L_E = 1.6L = 1.6\times 4 = 6.4m$ *(Figure 6.5a)*

Slenderness ratio, $\lambda = \frac{L_E}{r_x} = \frac{6400}{89.6} = 71.4$ *(Clause 8.7.4)*

Compressive strength for rolled H-section with maximum thickness less than 40mm bending about *x-x* axis should be obtained from buckling curve b *(Table 8.7)*

Compressive strength, $p_c = 199.2\,N/mm^2$ *(Table 8.8(b))*

Compression resistance, $P_c = p_c A_g = 199.2\times 7640 = 1521.9kN > F_c$ *(OK)* *(8.73)*

Case 3 Bent about minor axis, fully braced

Referring to the table above, the critical ϕ_s is 5.475×10^{-5} and the coressponding λ_{cr} is given by

$$\lambda_{cr} = \frac{1}{200}\cdot\frac{1}{\phi_s} = \frac{1}{200}\cdot\frac{1}{5.475\times 10^{-5}} = 91.3$$

$\because \lambda_{cr} \geq 10$ *(6.2)*

$\therefore$ it is a non-sway frame *(Clause 6.3.3)*

Beam stiffness in sway mode should be taken as $0.5\frac{I}{L}$ *(Table 6.2)*

$$k_1 = \frac{K_c + K_1}{K_c + K_1 + K_{11}} = \frac{\frac{I}{L}+\frac{I}{L}}{\frac{I}{L}+\frac{I}{L}+0.5\frac{I}{L}} = \frac{2}{2.5} = 0.8$$ *(Figure 6.4)*

$$k_2 = \frac{K_c + K_2}{K_c + K_2 + K_{21}} = \frac{\frac{I}{L}+\frac{I}{L}}{\frac{I}{L}+\frac{I}{L}+0.5\frac{I}{L}} = \frac{2}{2.5} = 0.8$$

Effective length, $L_E = 0.86L = 0.86\times 4 = 3.44m$ *(Figure 6.5b)*

Slenderness ratio, $\lambda = \frac{L_E}{r_y} = \frac{3440}{52} = 66.2$ *(Clause 8.7.4)*

Compressive strength for rolled H-section with maximum thickness less than 40mm bending about *y-y* axis should be obtained from buckling curve c *(Table 8.7)*

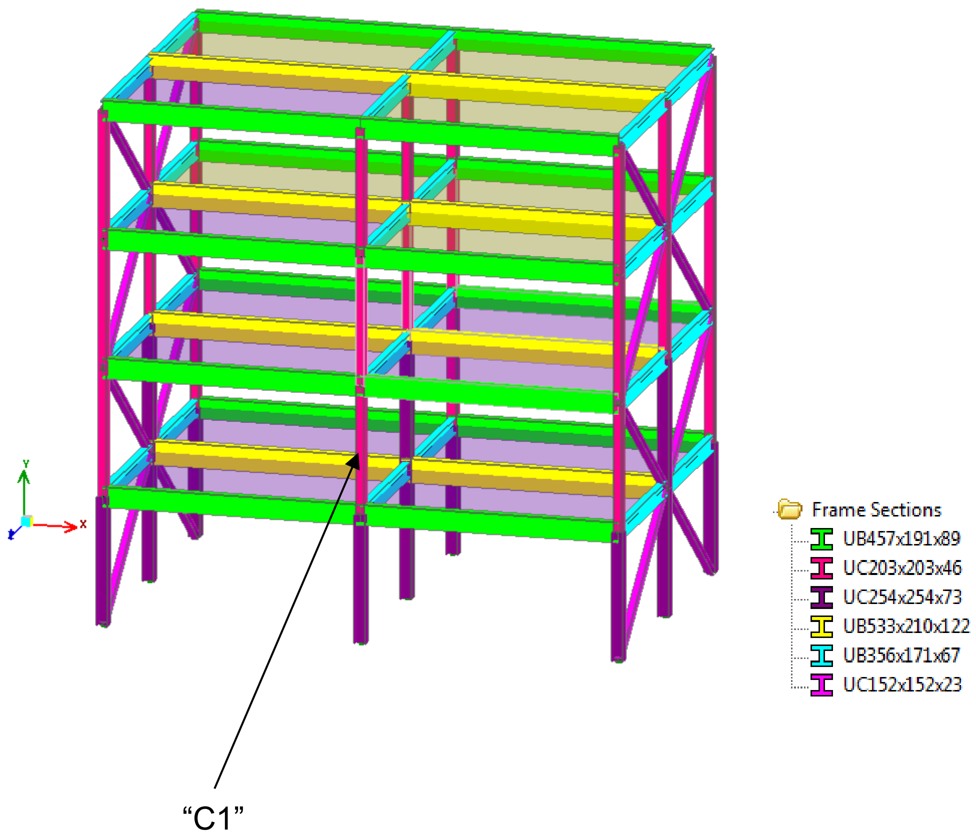
Compressive strength, $p_c = 188.6\,N/mm^2$ *(Table 8.8(c))*

Compression resistance, $P_c = p_c A_g = 188.6\times 7640 = 1440.9kN > F_c$ *(OK)* *(8.73)*

10.6.6 3-Dimensional steel building

This example is for the extension to a three-dimensional structure and to demonstrate the application of a second-order direct analysis dealing on a three-dimensional problem. The figure below shows a three-dimensional four-storey frame with steel grade S355. The loadings on the floors and the roof are 24kN/m^2 and 8kN/m^2 respectively plus the self-weight of the material. The sections used are shown in the figure. In this example, Column "C1" of 203×203×46 UC is to be designed. The frame is 4-storey high of dimensions 12m(H)×12m(L)×8m(W) with story height of 3m. All base connections are assumed pinned and beam to column connections are assumed rigid. Concrete floor is assumed as rigid diaphragm and the stiffness in the Z-axis is strengthened by cross bracings at the two end bays while the sway stiffness in the X-axis is provided by moment frame action. The major axis of the columns is located about the Z-axis in order to provide a larger resistance against sway in the X-axis for which bracings are not provided.

Column "C1" is selected for demonstration. Other members follow the same procedural check.



DESIGN LOAD

From linear analysis, the internal forces of "C1" are:

$F_c = 824.1kN$, $M_{x1} = -3.9kNm$, $M_{x2} = 4.1kNm$, $M_{y1} = 12.1kNm$, $M_{y2} = -12.2kNm$

SECTION PROPERTIES

$D = 203.2mm$, $B = 203.6mm$, $t = 7.2mm$, $T = 11.0mm$, $d = 160.8mm$, $I_x = 4570cm^4$, $I_y = 1550cm^4$, $r_x = 8.82cm$, $r_y = 5.13cm$, $Z_x = 450cm^3$, $Z_y = 152cm^3$, $S_x = 497cm^3$, $S_y = 231cm^3$, $u = 0.847$, $x = 17.7$, $A = 58.7cm^2$

SECTION CLASSIFICATION

Design strength, $p_y = 355N/mm^2$ for $T \le 16mm$ *(Table 3.2)*

$$\varepsilon = \sqrt{\frac{275}{355}} = 0.88 \quad \textit{(Table 7.1 Note b)}$$

Semi-compact limiting value of b/T for outstand flange of an H-section is 15ε

$$\frac{b}{T} = \frac{203.6}{2 \times 11} = 9.25 \le 15 \times 0.88 = 13.2 \quad \textit{(Table 7.1)}$$

$\therefore$ flange is semi-compact

Plastic limiting value of d/t for web of an H-section under both axial compression and bending is $80\varepsilon/(1 + r_1)$

$$\text{Stress ratio, } r_1 = \frac{F_c}{dtp_{yw}} = \frac{824.1 \times 10^3}{160.8 \times 7.2 \times 355} = 2.01 > 1 \quad \textit{(7.1)}$$

$\therefore r_1 = 1$

$$\frac{d}{t} = \frac{160.8}{7.2} = 22.33 \le \frac{80 \times 0.88}{1+1} = 35.2 \quad \textit{(Table 7.1)}$$

$\therefore$ web is plastic

$\therefore$ the section is Class 3 semi-compact

FRAME CLASSIFICATION

The elastic buckling load factor for the unbraced plane is 7.20 and for the braced plane is greater than 10 so that bucking about member major *x*-axis is classified as sway and about member minor *y*-axis as non-sway.

MOMENT CAPACITY

Moment capacity,

$$M_{cx} = p_y Z_x \quad \textit{(8.3)}$$
$$= 355 \times 450 \times 10^3$$
$$= 159.8kNm$$
$$M_{cy} = p_y Z_y \quad \textit{(8.3)}$$
$$= 355 \times 152 \times 10^3$$
$$= 54.0kNm$$

COMPRESSION RESISTANCE

For bending about major x-axis (Designed as sway-frame)

Beam stiffness $= 1.0\dfrac{I}{L}$ *(Table 6.2)*

$$K_C = K_1 = \frac{I}{L} = \frac{4570}{300} = 15.23$$

$$K_{11} = K_{12} = \frac{I}{L} = \frac{41000}{600} = 68.33$$

$$k_1 = \frac{15.23+15.23}{15.23+15.23+68.33+68.33} = 0.18$$ *(Figure 6.4)*

$$K_2 = \frac{I}{L} = \frac{11400}{300} = 38.0$$

$$K_{21} = K_{22} = \frac{I}{L} = \frac{41000}{600} = 68.33$$

$$k_2 = \frac{15.23+38}{15.23+38+68.33+68.33} = 0.28$$ *(Figure 6.4)*

$$L_E = 1.15L = 1.15\times 3 = 3.45m$$ *(Figure 6.5)*

$$\lambda_x = \frac{3450}{88.2} = 39.1$$ *(Clause 8.7.4)*

$$p_{cx} = 319.6\, N/mm^2$$ *(Tables 8.7, 8.8)*

$$P_{cx} = 5870\times 319.6 = 1876.1kN$$ *(8.73)*

For bending about minor y-axis (Designed as non-sway frame)

Beam stiffness $= 1.0\dfrac{I}{L}$ *(Table 6.2)*

$$K_C = K_1 = \frac{I}{L} = \frac{1550}{300} = 5.17$$

$$K_{11} = \frac{I}{L} = \frac{19500}{400} = 48.75$$

$$k_1 = \frac{5.17+5.17}{5.17+5.17+48.75} = 0.17$$ *(Figure 6.4)*

$$K_2 = \frac{I}{L} = \frac{3910}{300} = 13.03$$

$$K_{21} = \frac{I}{L} = \frac{19500}{400} = 48.75$$

$$k_2 = \frac{5.17+13.03}{5.17+13.03+48.75} = 0.27$$ *(Figure 6.4)*

$$L_E = 0.57L = 0.57\times 3 = 1.71m$$ *(Figure 6.5)*

$$\lambda_y = \frac{1710}{51.3} = 33.3$$ *(Clause 8.7.4)*

$$p_{cy} = 316.7\, N/mm^2$$ *(Tables 8.7, 8.8)*

$$P_{cy} = 5870\times 316.7 = 1859.0kN$$ *(8.73)*

CROSS-SECTION CAPACITY

The cross section capacity check can be carried out as

$$\frac{F_c}{A_g p_y} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} \le 1 \qquad (8.78)$$

$$\frac{824.1\times 10^3}{355\times 5870} + \frac{4.1}{159.8} + \frac{12.2}{54} = 0.65 \le 1 \ \ (OK)$$

MEMBER BUCKLING RESISTANCE

For bending about major z-axis

$$\beta = -\frac{3.9}{4.1} = -0.95$$

$$\therefore m_x = 0.41$$ *(Table 8.9)*

For bending about minor y-axis

$$\beta = -\frac{12.1}{12.2} = -0.99$$

$$\therefore m_y = 0.40$$ *(Table 8.9)*

Buckling check using effective length under sway mode to *Equation (8.79)*

$$\frac{F_c}{P_c} + \frac{m_x \overline{M}_x}{M_{cx}} + \frac{m_y \overline{M}_y}{M_{cy}} = \frac{824.1}{1859} + \frac{0.41\times 4.1}{159.8} + \frac{0.40\times 12.2}{54} = 0.54 \le 1 \ \ (OK) \qquad (8.79)$$

Buckling check to non-sway mode effective length under amplified moment to *Equation (8.80)*
Effective length, $L_E = L = 3m$

Slenderness ratio, $\lambda_x = \frac{L_E}{r_x} = \frac{3000}{88.2} = 34.0$ *(Clause 8.7.4)*

Compressive strength, $p_c = 328.6\, N/mm^2 > p_{cy}$ *(Table 8.8(b))*

Compression resistance, $\overline{P}_c = P_{cy} = 1859kN$

For sway frame, the P-δ-Δ amplification factor is given by the larger of:

$$\frac{\lambda_{cr}}{\lambda_{cr}-1} = \frac{7.2}{7.2-1} = 1.16$$

and
$$\frac{1}{1-\frac{F_c L_E^2}{\pi^2 EI}} = \frac{1}{1-\frac{824.1\times 3.45^2}{\pi^2\times 2.05\times 4570}} = 1.12$$

∴ The P-δ-Δ amplification factor is taken as 1.16

For non-sway frame, the P-Δ can be neglected and the P-δ amplification factor is given by:

$$\frac{1}{1-\frac{F_c L_E^2}{\pi^2 EI}} = \frac{1}{1-\frac{824.1\times 1.71^2}{\pi^2\times 2.05\times 1550}} = 1.08 \qquad (8.83)$$

$$\frac{F_c}{\overline{P}_c} + \frac{m_x M_x}{M_{cx}} + \frac{m_y M_y}{M_{cy}} = \frac{824.1}{1859} + \frac{0.41\times 1.16\times 4.1}{159.8} + \frac{0.40\times 1.08\times 12.2}{54} = 0.55 \le 1 \ \textit{(OK)} \qquad (8.80)$$

Buckling check to lateral-torsional buckling mode to *Equation (8.81)*

$$\beta = -\frac{3.9}{4.1} = -0.95$$

$\therefore m_{LT} = 0.44$ *(Table 8.4a)*

For finding M_b,

Assumed effective length, $L_E = 0.5L = 0.5\times 3 = 1.5m$

Slenderness ratio, $\lambda = \frac{L_E}{r_y} = \frac{1500}{51.3} = 29.2$ *(8.26)*

$$v = \frac{1}{\left[1+0.05(\lambda/x)^2\right]^{0.25}} = \frac{1}{\left[1+0.05(29.2/17.7)^2\right]^{0.25}} = 0.969 \qquad (8.27)$$

$$\beta_w = \frac{Z_x}{S_x} = \frac{450}{497} = 0.905 \qquad (8.28)$$

$$\lambda_{LT} = uv\lambda\sqrt{\beta_w} = 0.847\times 0.969\times 29.2\times\sqrt{0.905} = 22.8 \qquad (8.25)$$

$p_b = 355\, N/mm^2$ *(Table 8.3a)*

$M_b = p_b Z_x = 159.8kNm$ *(8.21)*

$$\frac{F_c}{P_{cy}} + \frac{m_{LT} M_{LT}}{M_b} + \frac{m_y \overline{M}_y}{M_{cy}} = \frac{824.1}{1859} + \frac{0.44\times 1.16\times 4.1}{159.8} + \frac{0.40\times 12.2}{54} = 0.55 \le 1 \qquad (8.81)$$

(OK)

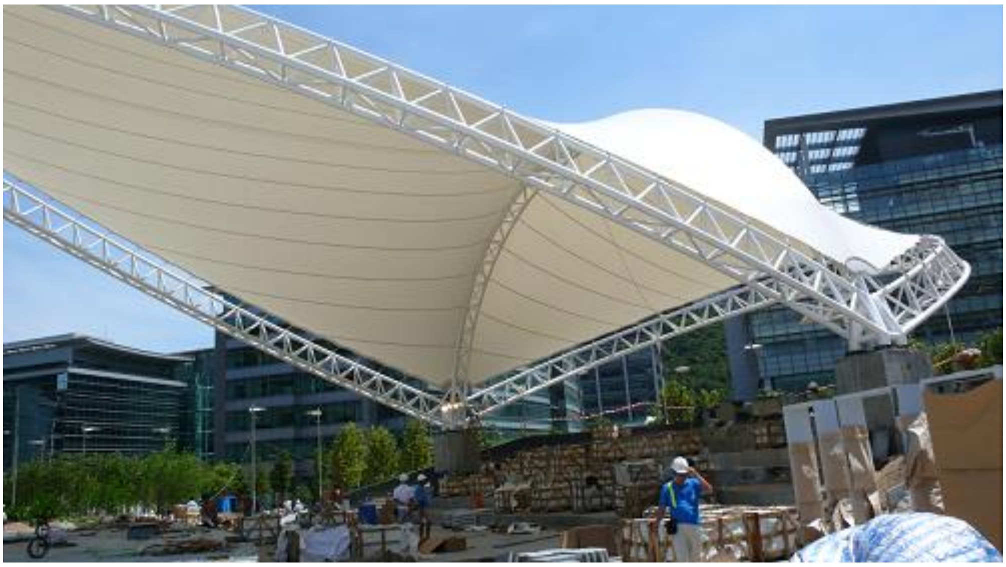
We follow a logic of using either the sway effective length (which is greater than member length) OR the amplified moment in a single equation check, but not both at a time.

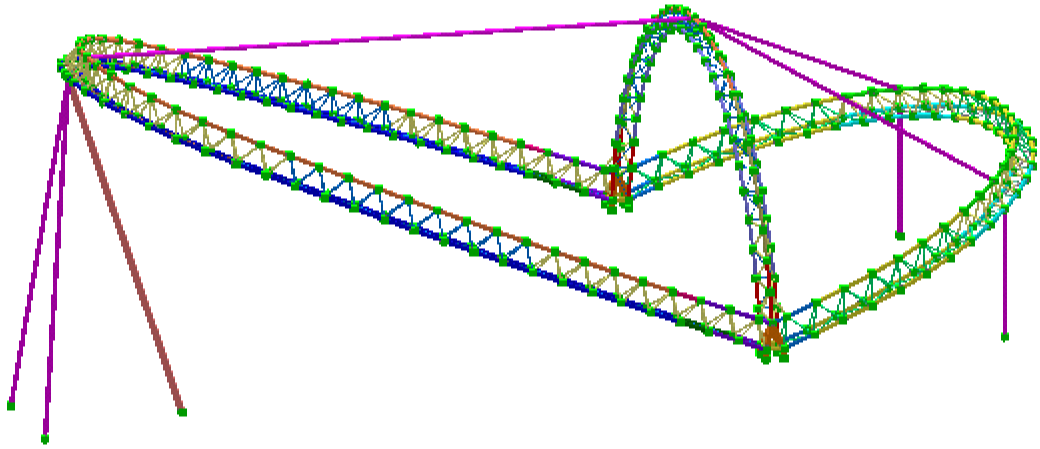
Using Second-order direct analysis, the section capacity factor ϕ is 0.76 with
$F_c = 820.0kN$, $M_{x,\max} = -4.4kNm$, $M_{y,\max} = 18.2kNm$

It can be seen that the first-order analysis method has underestimated the amplified moment about the minor axis by 27.6%. Therefore design using first-order analysis can lead to an unconservative result.

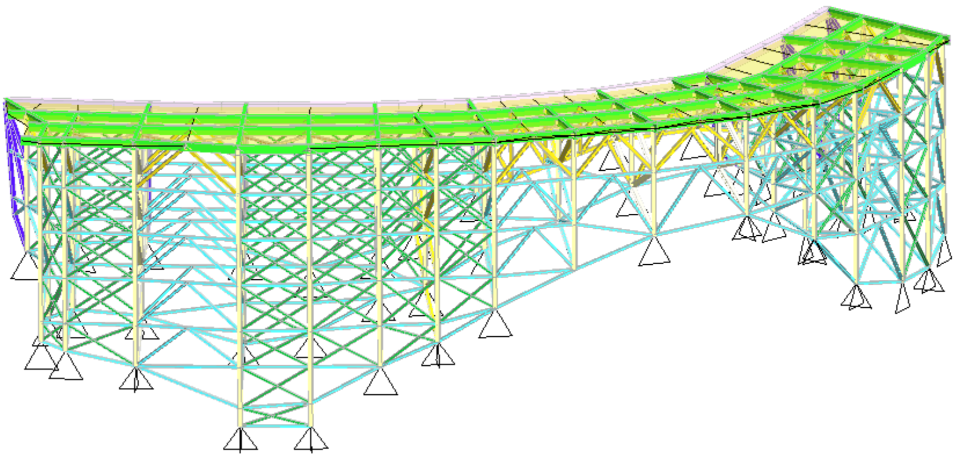
10.6.7 Some selected structures designed by Direct Analysis in practice

The example below demonstrates the design of a space frame without assuming any effective length. The space frame shown in below has been designed without assuming any effective length. All expected loadings have been allowed for in the analysis and design. The figure shows the structure near completion and the computer model.

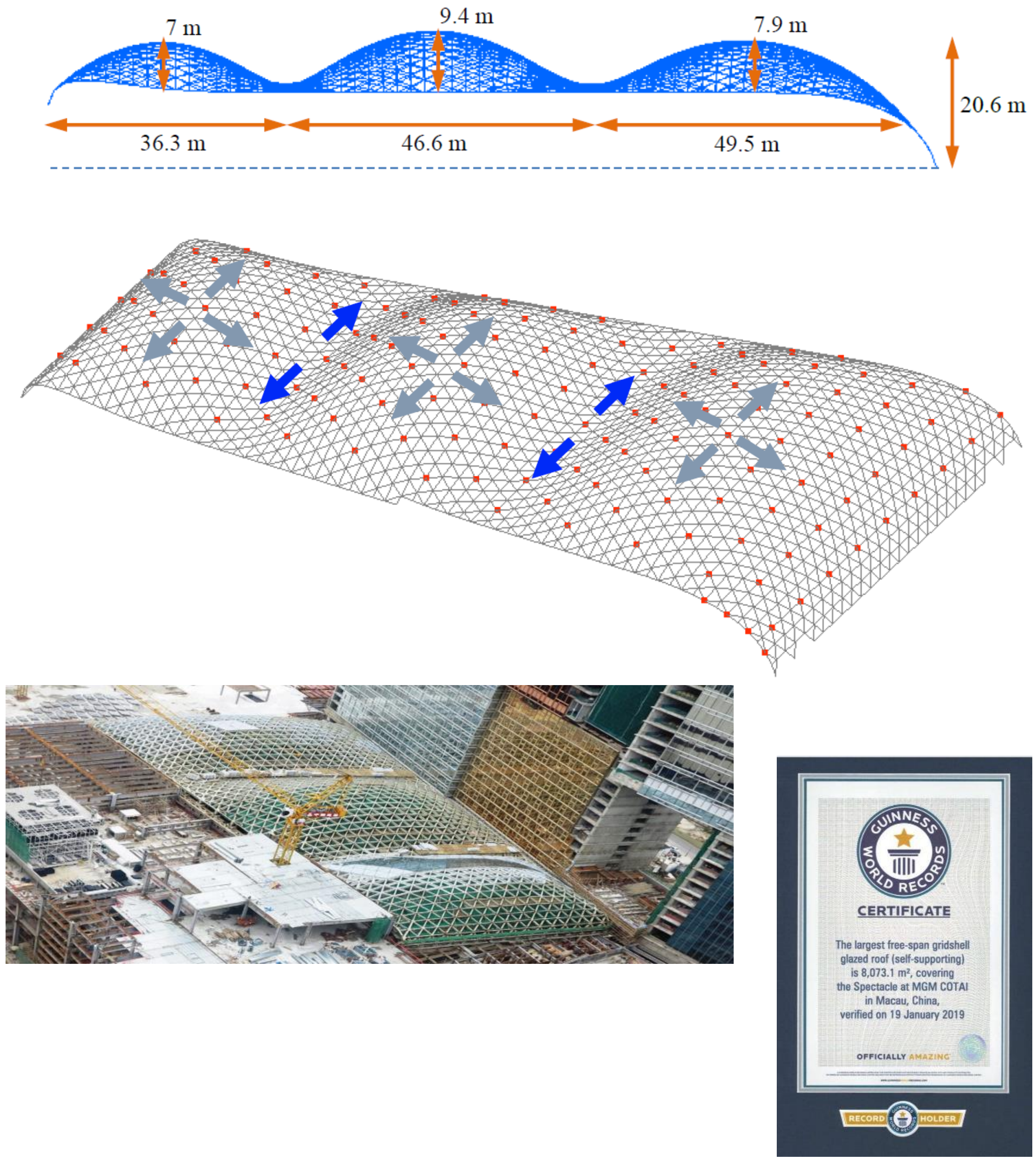




A slender skeleton supporting membrane designed by second-order direct analysis without assumption of effective length (courtesy to HK Science and Technology Parks Corporation)



A heavy platform with mobile crane loads
designed by second-order direct analysis w/o effective length
(courtesy to Sun Hung Kai Development Limited)



World's longest single layer roof at MGM, Macau, designed by direct analysis (courtesy to MGM China Holding Ltd., Siu Yin Wai and Associates Ltd. and China Steel Construction Ltd.)

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